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Robust Control of Uncertain Stochastic Recurrent Neural Networks with Time-varying Delay

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Abstract In this paper, robust control of uncertain stochastic recurrent neural networks with time-varying delay is considered. A novel control method is given by using the Lyapunov functional method and linear matrix inequality (LMI) approach. Several delay-independent and delay-dependent sufficient conditions are then further derived to ensure the global asymptotical stability in mean square for the uncertain stochastic recurrent neural networks, and the estimation gains can also be obtained. Numerical examples are constructed to verify the theoretical analysis in this paper.

Keywords Time-varying delays · Lyapunov functional · Robust control · LMI approach · Stochastic neural networks · Global asymptotical stability

1 Introduction

Recently, a lot of attention has been devoted to the study of artificial neural networks due to the fact that neural networks can be applied in signal processing, image processing, pattern recognition and optimization problems. Some of these applications require the knowledge of dynamical behaviors of neural networks, such as the uniqueness and asymptotical stability of equilibrium point of a designed neural work. Therefore, the problem of stability analysis of neural networks has been an important topic for researchers.

The desired stability properties of neural networks are customarily based on imposing constraint conditions on the network parameters of the neural system. However, the toler-

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ances of electronic components in hardware implementation employed in the design. In such cases, it is desired that the stability properties of neural network should be affected by the small deviations in the values of the parameters. In other words, the neural network must be globally robust stable. Also, there are some research works [1–20] about robust stability of the dynamical system.

In recent years, stability of stochastic delayed system [21–26] has been a focal subject for research due to the uncertainties exist in the real system. Stochastic modelling has come to play an important role in many branches of science and industry. A real system is usually affected by external perturbations which in many cases are of great uncertainty and hence may be treated as random, as fluctuations from the release of neurotransmitters, and other probabilistic causes. Therefore, it is significant and of prime importance to consider stochastic effects to the stability property of the delayed networks.

An area of particular interest has been the automatic control [1–3,27–29] of delayed stochastic systems. To the best of our knowledge, however, the control of uncertain delayed stochastic neural network has been hardly considered yet. Also, few works have discussed delay-dependent stability of stochastic systems with time-varying delays. Actually, control theory for stability of uncertain delayed stochastic neural network has attracted interesting attention.

In the literature, stability analysis for delayed systems can be classified into two catalogs according to their dependence on the information about the size of delays, namely delay-independent stability criterion and delay-dependent stability criterion. The delay-independent stability is independent of the size of the delays and delay-dependent stability analysis is concerned with the size of delays. In general, for small delays, delay-independent criterion is likely to be conservative.

Up to now, most works on delayed neural networks have focused on the stability analysis problem for neural networks with constant or time-varying delays. Sufficient conditions, either delay-dependent or delay-independent, have been proposed to guarantee the asymptotical or exponential stability for neural networks. However, analysis on the uncertain delayed stochastic delayed neural networks are less investigated. Thus in this paper we consider delay-independent and delay-dependent criteria for the uncertain delayed stochastic delayed neural networks. Moreover, the derivative of the time varying delay can take any value. It is a complex task while significant in the design and implementation of the neural networks.

The left parts of this paper is organized as follows: In Sect. 2, a uncertain stochastic delayed neural network model is proposed and preliminaries for our main results is briefly outlined. Some delay-independent (Theorem 1) and delay-dependent (Theorem 2) conditions are presented to ensure the global asymptotical stability of uncertain stochastic neural network with time-varying delay in Sect. 3. In Sect. 4, numerical examples are constructed to show the effectiveness of the proposed stability criteria. The conclusions are finally drawn in Sect. 5.

2 Model Formulation and Preliminaries

In this section, we will give preliminary knowledge for our main results. Recently, some works have been focused on the stability of delayed stochastic neural networks. However, uncertainties may exist in the real neural network, thus we consider the robust stability of delayed stochastic neural network. In this paper, a controller is added to the uncertain delayed stochastic neural network to ensure the global asymptotical stability of the neural network. Some delay-independent and delay-dependent conditions are derived.

In this paper, we consider the following uncertain stochastic neural network:

$$dx(t) = [-(C + \Delta C)x(t) + (A + \Delta A)f(x(t)) + (B + \Delta B)f(x(t - \tau(t))) \\ +u(t)]dt + \sigma(t, x(t), x(t - \tau(t)))d\omega(t), \quad (1)$$

namely,

$$dx_i(t) = [-(c_i + \Delta c_i)x_i(t) + \sum_{j=1}^{n}(a_{ij} + \Delta a_{ij})f_j(x_j(t)) \\ + \sum_{j=1}^{n}(b_{ij} + \Delta b_{ij})f_j(x_j(t - \tau(t))) \\ +u_i(t)]dt + \sum_{j=1}^{n}\sigma_{ij}(t, x(t), x(t - \tau(t)))d\omega_j(t), \quad i = 1, 2, \ldots, n, \quad (2)$$

where n denotes the number of neurons in the network, $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in \mathbb{R}^n$ is the state vector associated with the neurons, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T \in \mathbb{R}^n$ corresponds to the activation functions of neurons, $\tau(t)$ is the time-varying delay, and the initial conditions are given by $x_i(t) = \phi_i(t) \in \mathcal{C}([-r, 0], \mathbb{R})$ with $r = \max_{t\geq 0}\{\tau(t)\}$ and $\mathcal{C}([-r, 0], \mathbb{R})$ denoting the set of all continuous functions from $[-r, 0]$ to $\mathbb{R}$. Moreover, $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are the known connection weight matrix and the delayed connection weight matrix, respectively. The matrices ΔA, ΔB and ΔC represent the uncertainties in the system parameters, respectively, which are possibly time-varying or random. Moreover, u is a feedback controller.

$\omega(t) = (\omega_1(t), \omega_{2,}(t), \ldots, \omega_n(t))^T$ is a n dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ generated by $\{\omega(s) : 0 \leq s \leq t\}$, where we associate Ω with the canonical space generated by $\omega(t)$, and denote $\mathcal{F}$ the associated σ-algebra generated by $\{\omega(t)\}$ with the probability measure $\mathbb{P}$. Here the white noise $d\omega_i(t)$ is independent of $d\omega_j(t)$ for mutually different i and j, and $\sigma : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^{n\times n}$ is called the noise intensity function matrix. This type of stochastic perturbation can be regarded as a result from the occurrence of random uncertainties from the neural network. It is assumed that the right-hand side of system (1) is continuous so as to ensure the existence and uniqueness of the solution for every well-posed initial condition. Our objective is to design a controller u to ensure system (1) to be globally asymptotically stable about its equilibrium point.

To establish our main results, it is necessary to make the following assumptions:

A_1: Each function $f_i : \mathbb{R} \to \mathbb{R}$ is nondecreasing and globally Lipschitz with a constant $F_i > 0$, i.e.

$$|f_i(u) - f_i(v)| \leq F_i|u - v|\forall u, \quad v \in \mathbb{R}, \quad i = 1, 2, \ldots, n, \quad (3)$$

also, $f_i(0) = 0, i = 1, 2, \ldots, n$.

A_2: $\sigma : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^{n\times n}$ is locally Lipschitz continuous and satisfies the linear growth condition [30]. Moreover, σ satisfies

$$trace[\sigma^T(t, x(t), x(t - \tau(t))\sigma(t, x(t), x(t - \tau(t))] \leq \|Mx(t)\|^2 + \|M_1x(t - \tau)\|^2, \quad (4)$$

where M and M_1 are matrices with appropriate dimensions.

A_3:$\tau(t)$ is a bounded differential function of time t, and the following condition is satisfied:

$$r = \max_{t\in\mathbb{R}}\{\tau(t)\}, \quad 0 \le \dot{\tau}(t) \le h < 1, \quad t \in \mathbb{R}, \tag{5}$$

where r and h are positive constants.

A_4: The unknown matrices ΔC, ΔA and ΔB are norm bounded:

$$\|\Delta C\| \le \sqrt{\rho_C}, \quad \|\Delta A\| \le \sqrt{\rho_A}, \quad \|\Delta B\| \le \sqrt{\rho_B}, \tag{6}$$

where ρ_C, ρ_A and ρ_B are positive constants.

A_5 : $\tau(t)$ is a bounded function of time t, and the following condition is satisfied:

$$r = \max_{t\in\mathbb{R}}\{\tau(t)\},$$

where r is a positive constant.

Clearly, the origin is the equilibrium point of neural network (1) without the controller. In this paper, we try to design a controller u to control the state to converge to the origin.

Let $\|\cdot\|$ denote the Euclidean norm $\|\cdot\|_2$ in the Euclidean space $\mathbb{R}^n$. A symmetric matrix $A>0$ means that A is a positive definite matrix, and $A>B$ means that $A-B$ is a positive definite matrix. $F = diag(F_1, F_2, \ldots, F_n) \in \mathbb{R}^{n\times n}$ is a positive diagonal matrix. In addition, I denotes the identity matrix.

In many real applications, we are interested in designing a memoryless state-feed-back controller

$$u(t) = Kx(t), \tag{7}$$

where $K \in \mathbb{R}^{n\times n}$ is a constant gain matrix.

For a special case where the information on the size of time-varying delay $\tau(t)$ is available, consider a delayed feedback controller of the following form:

$$u(t) = Kx(t) + K_1x(t-\tau(t)). \tag{8}$$

Although a memoryless controller (7) has an advantage of easy implementation, its performance can not be better than a delayed feedback controller which utilizes the available information of the size of time-varying delay. A more general form of a delayed feedback controller is:

$$u(t) = Kx(t) + \int_{t-\tau(t)}^{t} K_2x(s)ds. \tag{9}$$

However, the task of storing all the previous state $x(\cdot)$ is difficult. In this respect, the controller (8) could be considered as a compromise between the performance improvement and the implementation simplicity. In this paper, we will consider the feed back controller (7) because of the complexity of the uncertain delayed stochastic neural network. The same results can be extended when using the delayed feedback controller (8).

Assume that $\phi(t)$ is the initial function of (1), where $\phi(t) \in L^2_{\mathcal{F}_0}([-r, 0]; \mathbb{R}^n)$, here $L^2_{\mathcal{F}_0}([-r, 0]; \mathbb{R}^n)$ denotes the family of $\mathbb{R}^n$-valued stochastic processes $\xi(s)$, $-r \le s \le 0$ such that $\xi(s)$ is $\mathcal{F}_0$-measurable and $\int_{-r}^{0} \mathbf{E}\|\xi(s)\|^2 ds < \infty$. It is well known that system (1) has a unique solution [30,31].

Also, the following definition is needed:

Definition 1 System (1) is said to be globally asymptotically stable in mean square if there exists a controller u and for any given condition such that

$$\lim_{t\longrightarrow\infty} \mathbf{E}\|x(t)\|^2 \longrightarrow 0, \tag{10}$$

where $\mathbf{E}\{\cdot\}$ is the mathematical expectation.

Before starting the main results, some lemmas are given in the following:

Lemma 1 [32] *For any vectors $x, y \in \mathbb{R}^n$ and positive definite matrix $G \in \mathbb{R}^{n\times n}$, the following matrix inequality holds:*

$$2x^T y \leq x^T G x + y^T G^{-1} y.$$

Lemma 2 (Schur complement [33]) *The following linear matrix inequality (LMI)*

$$\begin{pmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{pmatrix} > 0,$$

where $Q(x) = Q(x)^T$, $R(x) = R(x)^T$, is equivalent to one of the following conditions:

(i) $Q(x) > 0$, $R(x) - S(x)^T Q(x)^{-1} S(x) > 0$,
(ii) $R(x) > 0$, $Q(x) - S(x) R(x)^{-1} S(x)^T > 0$.

Lemma 3 (Jensen inequality [34]) *For any constant matrix $W \in \mathbb{R}^{m\times m}$, $W = W^T$, scalar $r > 0$, vector function $\omega : [0, r] \in \mathbb{R}^{m\times m}$ such that the integrations concerned are well defined, then*

$$r\int_0^r \omega(s) W \omega(s) ds \geq \left(\int_0^r \omega(s) ds\right)^T W \left(\int_0^r \omega(s) ds\right).$$

Lemma 4 [32] *If $V, W \in \mathbb{R}^{n\times n}$ are two matrices with property that $|V| = (|v_{ij}|_{n\times n}) \leq W = (w_{ij})_{n\times n}$, i.e., $|v_{ij}| \leq w_{ij}$, then $\|V\|_2 \leq \|W\|_2$.*

3 Criteria of Global Asymptotical Stability

In this section, new criteria are presented for the global asymptotical stability of the equilibrium point of the neural network defined by (1), and thus the designed controllers are sufficient to ensure the global asymptotical stability of the uncertain delayed stochastic neural network in the mean square. Its proof is based on a new Lyapunov functional method and LMI approach.

In this paper, we add the memoryless state-feed-back controller (7) $u = Kx(t)$ to the uncertain stochastic delayed neural network (1). To ensure the global asymptotical stability in mean square, the following lemma is established.

Lemma 5 *Under the assumptions $A_1 - A_4$, the equilibrium point of model (1) is globally asymptotically stable in mean square if there are positive definite diagonal matrix $D = diag(d_1, d_2, \ldots, d_n) > 0$, positive definite matrices $Q = (h_{ij})_{n\times n}$, $P = (p_{ij})_{n\times n}$, $R = (r_{ij})_{n\times n}$, and positive constants $\varepsilon_A, \varepsilon_B, \varepsilon_C, \rho$, such that*

$$N = \begin{pmatrix} \Phi_{11} & 0 & PA + FD & PB & P & P & P \\ 0 & \Phi_{22} & 0 & 0 & 0 & 0 & 0 \\ A^T P + DF & 0 & \rho_A \varepsilon_A I + R - 2D & 0 & 0 & 0 & 0 \\ B^T P & 0 & 0 & \rho_B \varepsilon_B I - (1-h)R & 0 & 0 & 0 \\ P & 0 & 0 & 0 & -\varepsilon_C I & 0 & 0 \\ P & 0 & 0 & 0 & 0 & -\varepsilon_A I & 0 \\ P & 0 & 0 & 0 & 0 & 0 & -\varepsilon_B I \end{pmatrix} < 0, \tag{11}$$

$$P \le \rho I, \tag{12}$$

where

$$\Phi_{11} = -2PC + PK + K^T P + \rho_C \varepsilon_C I + Q + \rho M^T M,\ \Phi_{22} = \rho M_1^T M_1 - (1-h)Q,$$

and I is the identity matrix.

Proof Consider the Lyapunov candidate

$$V(t) = \sum_{i=1}^{i=3} V_i(t), \tag{13}$$

where

$$V_1(t) = x^T(t)Px(t),\ V_2(t) = \int_{t-\tau(t)}^{t} x^T(s)Qx(s)ds,\ V_3(t) = \int_{t-\tau(t)}^{t} f^T(x(s))Rf(s)ds,$$

$P = (p_{ij})_{n\times n}$, $Q = (q_{ij})_{n\times n}$ and $R = (r_{ij})_{n\times n}$ are positive definite matrices.

The weak infinitesimal operator $\mathcal{L}$ of the stochastic process $\{x_t = x(t+s), t \ge 0, -r \le s \le 0\}$ is given by [21,22]

$$\begin{aligned}\mathcal{L}V_1(t) = 2x^T(t)P[(-C - \Delta C + K)x(t) + (A + \Delta A)f(x(t))\\ +(B + \Delta B)f(x(t-\tau(t)))]\\ +trace[\sigma^T(t, x(t), x(t-\tau(t))P\sigma(t, x(t), x(t-\tau(t))].\end{aligned} \tag{14}$$

By Assumption A_2 and (12),

$$\begin{aligned}&trace[\sigma^T(t, x(t), x(t-\tau(t))P\sigma(t, x(t), x(t-\tau(t))]\\ &\le \rho trace[\sigma^T(t, x(t), x(t-\tau(t))\sigma(t, x(t), x(t-\tau(t))]\\ &= \rho[x^T(t)M^T Mx(t) + x^T(t-\tau(t))M_1^T M_1 x(t-\tau(t))].\end{aligned} \tag{15}$$

$$\mathcal{L}V_2(t) = x^T(t)Qx(t) - (1-\dot{\tau}(t))x^T(t-\tau(t))Qx(t-\tau(t)). \tag{16}$$

$$\mathcal{L}V_3(t) = f^T(x(t))Rf(x(t)) - (1-\dot{\tau}(t))f^T(x(t-\tau(t)))Rf(x(t-\tau(t))). \tag{17}$$

From Assumption A_1, it is obvious that

$$\begin{aligned}f^T(x(t))DFx(t) &= \sum_{i=1}^{n} f_i(x_i(t))d_i F_i x_i(t) \ge \sum_{i=1}^{n} d_i f_i^2(x_i(t))\\ &= f^T(x(t))Df(x(t)),\end{aligned} \tag{18}$$

where $D = diag(d_1, d_2, \ldots, d_n)$ and $F = diag(f_1, f_2, \ldots, f_n)$ are positive definite diagonal matrices.

From Lemma 1 and Assumption A_4, we obtain

$$\begin{aligned}2x^T(t)P\Delta Cx(t) &\le \varepsilon_C^{-1}x^T(t)P^2x(t) + \varepsilon_C x^T(t)\Delta C^T\Delta Cx(t)\\ &\le \varepsilon_C^{-1}x^T(t)P^2x(t) + \rho_C\varepsilon_C x^T(t)x(t),\end{aligned} \tag{19}$$

$$\begin{aligned}2x^T(t)P\Delta Af(x(t)) &\le \varepsilon_A^{-1}x^T(t)P^2x(t) + \varepsilon_A f^T(x(t))\Delta A^T\Delta Af(x(t))\\ &\le \varepsilon_A^{-1}x^T(t)P^2x(t) + \rho_A\varepsilon_A f^T(x(t))f(x(t)),\end{aligned} \tag{20}$$

and

$$2x^T(t)P\Delta Bf(x(t-\tau)) \le \varepsilon_B^{-1}x^T(t)P^2x(t) + \varepsilon_B f^T(x(t-\tau))\Delta B^T\Delta Bf(x(t-\tau))$$
$$\le \varepsilon_B^{-1}x^T(t)P^2x(t) + \rho_B\varepsilon_B f^T(x(t-\tau))f(x(t-\tau)). \quad (21)$$

Therefore, combining (14)–(21) we have

$$\begin{aligned}
\mathcal{L}V(t) \le\ & x^T(t)[-2PC+2PK+\rho_C\varepsilon_C I+Q+\rho M^T M]x(t)+2x^T(t)(PA+FD)f(x(t)) \\
&+2x^T(t)PBf(x(t-\tau(t)))+x^T(t-\tau(t))[\rho M_1^T M_1-(1-h)Q]x(t-\tau(t)) \\
&+f^T(x(t))[\rho_A\varepsilon_A I+R-2D]f(x(t)) \\
&+f^T(x(t-\tau(t)))[\rho_B\varepsilon_B I-(1-h)R]f(x(t-\tau(t))) \\
&+(\varepsilon_C^{-1}+\varepsilon_A^{-1}+\varepsilon_B^{-1})x^T(t)P^2x(t) \qquad (22)\\
=\ & \left(x^T(t)\ x^T(t-\tau(t))\ f^T(x(t))\ f^T(x(t-\tau(t)))\right) N_1 \begin{pmatrix} x(t) \\ x(t-\tau(t)) \\ f(x(t)) \\ f(x(t-\tau(t))) \end{pmatrix},
\end{aligned}$$

where

$$N_1 = \begin{pmatrix} \Phi_{11}+(\varepsilon_C^{-1}+\varepsilon_A^{-1}+\varepsilon_B^{-1})P^2 & 0 & PA+FD & PB \\ 0 & \rho M_1^T M_1-(1-h)Q & 0 & 0 \\ A^TP+DF & 0 & \rho_A\varepsilon_A I+R-2D & 0 \\ B^TP & 0 & 0 & \rho_B\varepsilon_B I-(1-h)R \end{pmatrix}.$$

From Lemma 2, it is easy to see that $N<0$ is equivalent to $N_1<0$. From It$\widehat{o}$ formula, it is obvious to see that

$$\mathbf{E}V(t)-\mathbf{E}V(t_0)=\mathbf{E}\int_{t_0}^{t}\mathcal{L}V(s)ds. \quad (23)$$

From the definition of $V(t)$ in (13), there exists positive constant λ_1 such that

$$\begin{aligned}
\lambda_1\mathbf{E}\|x(t)\|^2 \le \mathbf{E}V(t) &\le \mathbf{E}V(t_0)+\mathbf{E}\int_{t_0}^{t}\mathcal{L}V(s)ds \\
&\le \mathbf{E}V(t_0)+\lambda_{max}\mathbf{E}\int_{t_0}^{t}\|x(s)\|^2ds, \qquad (24)
\end{aligned}$$

where λ_{max} is the maximal eigenvalue of N and it is negative.

Therefore, from (24) and the discussion in [31], we know that the equilibrium of (1) is globally asymptotically stable in mean square. This completes the proof.

Note that if the gain matrix K is unknown, then (11) is not LMI. Next, we give a theorem to solve this problem. It is easy to see that if K is unknown, sufficient conditions to ensure the global asymptotical stability of uncertain delayed stochastic neural network can still be obtained. Therefore, the gain matrix K can be attained.

Theorem 1 *Under the assumptions A_1–A_4, the equilibrium point of model (1) is globally asymptotically stable in mean square if there are positive definite diagonal matrix* $D=$

$diag(d_1, d_2, \ldots, d_n) > 0$, positive definite matrices $Q = (h_{ij})_{n\times n}$, $P = (p_{ij})_{n\times n}$, $R = (r_{ij})_{n\times n}$, matrix $K' \in \mathbb{R}^{n\times n}$ and positive constants $\varepsilon_A, \varepsilon_B, \varepsilon_C, \rho$, such that

$$\begin{pmatrix} \Phi_{11} & 0 & PA+FD & PB & P & P & P \\ 0 & \Phi_{22} & 0 & 0 & 0 & 0 & 0 \\ A^TP+DF & 0 & \rho_A\varepsilon_A I + R - 2D & 0 & 0 & 0 & 0 \\ B^TP & 0 & 0 & \rho_B\varepsilon_B I - (1-h)R & 0 & 0 & 0 \\ P & 0 & 0 & 0 & -\varepsilon_C I & 0 & 0 \\ P & 0 & 0 & 0 & 0 & -\varepsilon_A I & 0 \\ P & 0 & 0 & 0 & 0 & 0 & -\varepsilon_B I \end{pmatrix} < 0, \tag{25}$$

$$P \le \rho I, \tag{26}$$

where

$$\Phi_{11} = -2PC + K' + K'^T + \rho_C\varepsilon_C I + Q + \rho M^T M, \Phi_{22} = \rho M_1^T M_1 - (1-h)Q,$$

and I is the identity matrix. Moreover, the estimation gain $K = P^{-1}K'$.

Proof Let $K = P^{-1}K'$ in Lemma 5, it is obvious to see.

Corollary 1 *Under the assumptions A_1–A_4, the equilibrium point of model (1) is globally asymptotically stable in mean square if there are positive definite matrices $Q = (h_{ij})_{n\times n}$, $P = (p_{ij})_{n\times n}$, $R = (r_{ij})_{n\times n}$, and positive constants $\varepsilon_A, \varepsilon_B, \varepsilon_C, \varepsilon_1, \varepsilon_2, \rho$, such that*

$$\begin{pmatrix} \Phi_{11} & 0 & PA & PB & P & P & P \\ 0 & \Phi_{22} & 0 & 0 & 0 & 0 & 0 \\ A^TP & 0 & \rho_A\varepsilon_A I + R - \varepsilon_1 I & 0 & 0 & 0 & 0 \\ B^TP & 0 & 0 & \rho_B\varepsilon_B I - (1-h)R - \varepsilon_2 I & 0 & 0 & 0 \\ P & 0 & 0 & 0 & -\varepsilon_C I & 0 & 0 \\ P & 0 & 0 & 0 & 0 & -\varepsilon_A I & 0 \\ P & 0 & 0 & 0 & 0 & 0 & -\varepsilon_B I \end{pmatrix} < 0, \tag{27}$$

$$P \le \rho I, \tag{28}$$

where

$$\Psi_{11} = -2PC + PK + K^TP + \rho_C\varepsilon_C I + Q + \rho M^T M + \varepsilon_1 F^2, \Psi_{22} = \rho M_1^T M_1 - (1-h)Q + \varepsilon_2 F^2,$$

and I is the identity matrix.

Proof Instead of using (18), we use the following equalities:

$$0 \le \varepsilon_1[x^T(t)F^2x(t) - f^T(x(t))f(x(t))], \tag{29}$$

and

$$0 \le \varepsilon_2[x^T(t-\tau(t))F^2x(t-\tau(t)) - f^T(x(t-\tau(t)))f(x(t-\tau(t)))]. \tag{30}$$

It is easy to obtain the condition (27) and (28), this completes the proof.

Corollary 2 *Under the assumptions A_1–A_4, the equilibrium point of model (1) is globally asymptotically stable in mean square if there are positive definite matrices $Q = (h_{ij})_{n\times n}$, $P = (p_{ij})_{n\times n}$, $R = (r_{ij})_{n\times n}$, matrix $K' \in R^{n\times n}$ and positive constants $\varepsilon_A, \varepsilon_B, \varepsilon_C, \varepsilon_1, \varepsilon_2, \rho$, such that*

$$\begin{pmatrix} \Phi_{11} & 0 & PA & PB & P & P & P \\ 0 & \Phi_{22} & 0 & 0 & 0 & 0 & 0 \\ A^T P & 0 & \rho_A\varepsilon_A I + R - \varepsilon_1 I & 0 & 0 & 0 & 0 \\ B^T P & 0 & 0 & \rho_B\varepsilon_B I - (1-h)R - \varepsilon_2 I & 0 & 0 & 0 \\ P & 0 & 0 & 0 & -\varepsilon_C I & 0 & 0 \\ P & 0 & 0 & 0 & 0 & -\varepsilon_A I & 0 \\ P & 0 & 0 & 0 & 0 & 0 & -\varepsilon_B I \end{pmatrix} < 0, \tag{31}$$

$$P \le \rho I, \tag{32}$$

where

$$\begin{aligned}\Psi_{11} &= -2PC + K' + K'^T + \rho_C\varepsilon_C I + Q + \rho M^T M + \varepsilon_1 F^2, \Psi_{22} \\ &= \rho M_1^T M_1 - (1-h)Q + \varepsilon_2 F^2,\end{aligned}$$

and I is the identity matrix. Moreover, the estimation gain $K = P^{-1}K'$.

Next, we give some delay-dependent criteria to ensure the global asymptotical stability of the uncertain delayed stochastic neural network (1).

Lemma 6 *Under the assumptions A_1–A_2 and A_4–A_5, the equilibrium point of model (1) is globally asymptotically stable in mean square if there are positive definite matrices $Q = (h_{ij})_{n\times n}$, $P = (p_{ij})_{n\times n}$, $R = (r_{ij})_{n\times n}$, $H = (h_{ij})_{n\times n}$, $T_i \in \mathbb{R}^{n\times n} (i = 1, 2, 3)$, and positive constants $\varepsilon_A, \varepsilon_B, \varepsilon_C, \varepsilon_i (i = 1, 2, 3, 4, 5)$, ρ, ρ', such that (33)–(35) are satisfied:*

$$\begin{pmatrix} \Omega_{11} & P & PA & PB & 0 & 0 & 0 & 0 & P & P & P & 0 & 0 & 0 \\ P & \Omega_{22} & 0 & 0 & \Omega_{25} & -PA & -PB & -P & 0 & 0 & 0 & P & P & P \\ A^T P & 0 & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B^T P & 0 & 0 & \Omega_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Omega_{25}^T & 0 & 0 & \Omega_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -A^T P & 0 & 0 & 0 & \Omega_{66} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -B^T P & 0 & 0 & 0 & 0 & \Omega_{77} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -P^T & 0 & 0 & 0 & 0 & 0 & -\frac{H}{r} & 0 & 0 & 0 & 0 & 0 & 0 \\ P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_C I & 0 & 0 & 0 & 0 & 0 \\ P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_A I & 0 & 0 & 0 & 0 \\ P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_B I & 0 & 0 & 0 \\ 0 & P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I & 0 & 0 \\ 0 & P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I & 0 \\ 0 & P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_5 I \end{pmatrix} < 0, \tag{33}$$

$$P \le \rho I, \tag{34}$$

$$H \le \rho' I, \tag{35}$$

where

$$\Omega_{11} = -2PC + PK + K^T P + \rho_C \varepsilon_C I + Q + (\rho + r\rho')M^T M + rT_1 + \varepsilon_1 F^2,$$
$$\Omega_{25} = -PK + PC,$$

$$\Omega_{22} = (\rho + r\rho')M_1^T M_1 - (1-h)Q + \varepsilon_2 F^2 - 2P,$$
$$\Omega_{33} = \rho_A \varepsilon_A I + R + rT_2 + rT_3 - \varepsilon_1 I,$$
$$\Omega_{44} = \rho_B \varepsilon_B I - (1-h)R - \varepsilon_2 I, \quad \Omega_{55} = \varepsilon_3 \rho_C I - \frac{T_1}{r}, \quad \Omega_{66} = \varepsilon_4 \rho_A I - \frac{T_2}{r},$$
$$\Omega_{77} = \varepsilon_5 \rho_B I - \frac{T_3}{r},$$

and I is the identity matrix.

Proof Consider the Lyapunov functional

$$V(t) = \sum_{i=1}^{i=7} V_i(t), \tag{36}$$

where

$$V_1(t) = x^T(t)Px(t),\ V_2(t) = \int_{t-\tau(t)}^{t} x^T(s)Qx(s)ds,\ V_3(t) = \int_{t-\tau(t)}^{t} f^T(x(s))Rf(s)ds,$$

$$V_4(t) = \int_{-r}^{0} d\theta \int_{t+\theta}^{t} x^T(s)T_1x(s)ds,\ V_5(t) = \int_{-r}^{0} d\theta \int_{t+\theta}^{t} f^T(x(s))T_2 f(x(s))ds,$$

$$V_6(t) = \int_{-r}^{0} d\theta \int_{t+\theta-\tau(\theta)}^{t} f^T(x(s))T_3 f(x(s))ds,$$

$$V_7(t) = \int_{-r}^{0} d\theta \int_{t+\theta}^{t} trace[\sigma^T(s, x(s), x(s-\tau(s))H\sigma(s, x(s), x(s-\tau(s))]d\omega(s),$$

where $P = (p_{ij})_{n\times n}$, $Q = (q_{ij})_{n\times n}$, $R = (r_{ij})_{n\times n}$, $H = (h_{ij})_{n\times n}$ and $T_i \in \mathbb{R}^{n\times n}(i = 1, 2, 3)$ are positive definite matrices.

Using the same inequalities as (14)–(17) and (19)–(21) together with Lemma 3, we obtain

$$\begin{aligned}
\mathcal{L}V_4(t) &= rx^T(t)T_1x(t) - \int_{t-r}^{t} x^T(\theta)T_1x(\theta)d\theta \\
&\le rx^T(t)T_1x(t) - \int_{t-\tau(t)}^{t} x^T(\theta)T_1x(\theta)d\theta \\
&\le rx^T(t)T_1x(t) - \frac{1}{r}\left(\int_{t-\tau(t)}^{t} x(\theta)d\theta\right)^T T_1 \left(\int_{t-\tau(t)}^{t} x(\theta)d\theta\right).
\end{aligned} \tag{37}$$

$$
\begin{aligned}
\mathcal{L}V_5(t) &= rf^T(x(t))T_2 f(x(t)) - \int_{t-r}^{t} f^T(x(\theta))T_2 f(x(\theta))d\theta \\
&\le rf^T(x(t))T_2 f(x(t)) - \int_{t-\tau(t)}^{t} f^T(x(\theta))T_2 f(x(\theta))d\theta \\
&\le rf^T(x(t))T_2 f(x(t)) - \frac{1}{r}\left(\int_{t-\tau(t)}^{t} f(x(\theta))d\theta\right)^T T_2 \left(\int_{t-\tau(t)}^{t} f(x(\theta))d\theta\right). \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}V_6(t) &= rf^T(x(t))T_3 f(x(t)) - \int_{t-r}^{t} f^T(x(\theta-\tau(\theta)))T_3 f(x(\theta-\tau(\theta)))d\theta \\
&\le rf^T(x(t))T_3 f(x(t)) - \int_{t-\tau(t)}^{t} f^T(x(\theta-\tau(\theta)))T_3 f(x(\theta-\tau(\theta)))d\theta \\
&\le rf^T(x(t))T_3 f(x(t)) - \frac{1}{r}\left(\int_{t-\tau(t)}^{t} f(x(\theta-\tau(\theta)))d\theta\right)^T \\
&\quad T_3 \left(\int_{t-\tau(t)}^{t} f(x(\theta-\tau(\theta)))d\theta\right). \qquad (39)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}V_7(t) &= rtrace[\sigma^T(t, x(t), x(t-\tau(t))H\sigma(t, x(t), x(t-\tau(t))] \\
&\quad - \int_{t-r}^{t} trace[\sigma^T(s, x(s), x(s-\tau(s))H\sigma(s, x(s), x(s-\tau(s))]d\omega(s) \\
&\le rtrace[\sigma^T(t, x(t), x(t-\tau(t))H\sigma(t, x(t), x(t-\tau(t))] \\
&\quad - \int_{t-\tau(t)}^{t} trace[\sigma^T(s, x(s), x(s-\tau(s))H\sigma(s, x(s), x(s-\tau(s))]d\omega(s) \\
&\le rtrace[\sigma^T(t, x(t), x(t-\tau(t))H\sigma(t, x(t), x(t-\tau(t))] \\
&\quad - \frac{1}{r}\left(\int_{t-\tau(t)}^{t} trace\sigma(s, x(s), x(s-\tau(s))d\omega(s)\right)^T \\
&\quad H\left(\int_{t-\tau(t)}^{t} trace\sigma(s, x(s), x(s-\tau(s))d\omega(s)\right). \qquad (40)
\end{aligned}
$$

It follows from (1) that

$$
\begin{aligned}
x(t) &= x(t-\tau(t)) + \int_{t-\tau(t)}^{t} \dot{x}(s)ds \\
&= x(t-\tau(t)) + \int_{t-\tau(t)}^{t} [-(C+\Delta C)x(s) + (A+\Delta A)f(x(s)) \\
&\quad +(B+\Delta B)f(x(s-\tau(s))) \\
&\quad +Kx(s)]ds + \int_{t-\tau(t)}^{t} \sigma(s, x(s), x(s-\tau(s)))d\omega(s). \qquad (41)
\end{aligned}
$$

For matrix $P \in R^{n\times n}$, we have

$$
\begin{aligned}
&2x^T(t-\tau(t))P[x(t) - x(t-\tau(t) - \int_{t-\tau(t)}^{t} [-(C+\Delta C)x(s) + (A+\Delta A)f(x(s)) \\
&+(B+\Delta B)f(x(s-\tau(s))) + Kx(s)]ds \\
&+\int_{t-\tau(t)}^{t} \sigma(s, x(s), x(s-\tau(s)))d\omega(s)] = 0. \qquad (42)
\end{aligned}
$$

Also, it is obvious to see that

$$
\begin{aligned}
&2x^T(t-\tau(t))P\Delta C\int_{t-\tau(t)}^{t}x(s)ds\\
&\leq \varepsilon_3^{-1}x^T(t-\tau(t))P^2x(t-\tau(t))+\varepsilon_3\left(\int_{t-\tau(t)}^{t}x(s)ds\right)^T\Delta C^T\Delta C\left(\int_{t-\tau(t)}^{t}x(s)ds\right)\\
&\leq \varepsilon_3^{-1}x^T(t-\tau(t))P^2x(t-\tau(t))+\rho_C\varepsilon_3\left(\int_{t-\tau(t)}^{t}x(s)ds\right)^T\left(\int_{t-\tau(t)}^{t}x(s)ds\right),
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
&2x^T(t-\tau(t))P\Delta A\int_{t-\tau(t)}^{t}f(x(s))ds\\
&\leq \varepsilon_4^{-1}x^T(t-\tau(t))P^2x(t-\tau(t))+\varepsilon_4\left(\int_{t-\tau(t)}^{t}f(x(s))ds\right)^T\\
&\quad \Delta A^T\Delta A\left(\int_{t-\tau(t)}^{t}f(x(s))ds\right)\\
&\leq \varepsilon_4^{-1}x^T(t-\tau(t))P^2x(t-\tau(t))+\rho_A\varepsilon_4\left(\int_{t-\tau(t)}^{t}f(x(s))ds\right)^T\\
&\quad \left(\int_{t-\tau(t)}^{t}f(x(s))ds\right),
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
&2x^T(t-\tau(t))P\Delta B\int_{t-\tau(t)}^{t}f(x(s-\tau(s)))ds\\
&\leq \varepsilon_5^{-1}x^T(t-\tau(t))P^2x(t-\tau(t))+\varepsilon_5\left(\int_{t-\tau(t)}^{t}f(x(s-\tau(s)))ds\right)^T\\
&\times\Delta B^T\Delta B\left(\int_{t-\tau(t)}^{t}f(x(s-\tau(s)))ds\right)\\
&\leq \varepsilon_5^{-1}x^T(t-\tau(t))P^2x(t-\tau(t))\\
&+\rho_B\varepsilon_5\left(\int_{t-\tau(t)}^{t}f(x(s-\tau(s)))ds\right)^T\left(\int_{t-\tau(t)}^{t}f(x(s-\tau(s)))ds\right),
\end{aligned}
\tag{45}
$$

and

$$
\begin{aligned}
&trace[\sigma^T(t,x(t),x(t-\tau(t))H\sigma(t,x(t),x(t-\tau(t))]\\
&\leq \rho' trace[\sigma^T(t,x(t),x(t-\tau(t))\sigma(t,x(t),x(t-\tau(t))]\\
&=\rho'[x^T(t)M^TMx(t)+x^T(t-\tau(t))M_1^TM_1x(t-\tau(t))].
\end{aligned}
\tag{46}
$$

Therefore, combining (37)–(46) together with (14)–(17), (19)–(21) and (29)–(30)we have

$$
\begin{aligned}
\mathcal{L}V(t) \le\ & x^T(t)[-2PC + PK + K^TP + \rho_C\varepsilon_C I + Q + (\rho + r\rho')M^TM + rT_1 + \varepsilon_1 F^2 \\
& + \left(\varepsilon_C^{-1} + \varepsilon_A^{-1} + \varepsilon_B^{-1}\right)P^2]x(t) + 2x^T(t)Px(t-\tau(t)) + 2x^T(t)PAf(x(t)) \\
& + 2x^T(t)PBf(x(t-\tau(t))) + x^T(t-\tau(t))[(\rho + r\rho')M_1^TM_1 - (1-h)Q + \varepsilon_2 F^2 \\
& + \left(\varepsilon_3^{-1} + \varepsilon_4^{-1} + \varepsilon_5^{-1}\right)P^2 - 2P]x(t-\tau(t)) \\
& + f^T(x(t))[\rho_A\varepsilon_A I + R + rT_2 + rT_3 - \varepsilon_1 I]f(x(t)) \\
& + f^T(x(t-\tau(t)))[\rho_B\varepsilon_B I - (1-h)R - \varepsilon_2 I]f(x(t-\tau(t))) \\
& + 2x^T(t-\tau(t))P(C-K) \\
& \times \int_{t-\tau(t)}^t x(s)ds - 2x^T(t-\tau(t))PA\int_{t-\tau(t)}^t f(x(s))ds \\
& - 2x^T(t-\tau(t))PB\int_{t-\tau(t)}^t f(x(s-\tau(s)))ds \\
& - 2x^T(t-\tau(t))P\int_{t-\tau(t)}^t \sigma(s, x(s), x(s-\tau(s))d\omega(s) \\
& + \left(\int_{t-\tau(t)}^t x(s)ds\right)^T \left(\varepsilon_3\rho_C - \frac{T_1}{r}\right)\left(\int_{t-\tau(t)}^t x(s)ds\right) \\
& + \left(\int_{t-\tau(t)}^t f(x(s))ds\right)^T \left(\varepsilon_4\rho_A - \frac{T_2}{r}\right)\left(\int_{t-\tau(t)}^t f(x(s))ds\right) \\
& + \left(\int_{t-\tau(t)}^t f(x(s-\tau(s)))ds\right)^T \\
& \times \left(\varepsilon_5\rho_B - \frac{T_3}{r}\right)\left(\int_{t-\tau(t)}^t f(x(s-\tau(s)))ds\right) \\
& - \frac{1}{r}\left(\int_{t-\tau(t)}^t trace\sigma(s, x(s), x(s-\tau(s))d\omega(s)\right)^T H \\
& \times \left(\int_{t-\tau(t)}^t trace\sigma(s, x(s), x(s-\tau(s))d\omega(s)\right) \\
=\ & \Big(x^T(t)\ x^T(t-\tau(t))\ f^T(x(t))\ f^T(x(t-\tau(t)))\ \textstyle\int_{t-\tau(t)}^t x^T(s)ds\ \int_{t-\tau(t)}^t f^T(x(s))ds \\
& \textstyle\int_{t-\tau(t)}^t f^T(x(s-\tau(s)))ds\ \int_{t-\tau(t)}^t trace\sigma^T(s, x(s), x(s-\tau(s))d\omega(s)\Big) J_1 \\
& \times \Big(x^T(t)\ x^T(t-\tau(t))\ f^T(x(t))\ f^T(x(t-\tau(t)))\ \textstyle\int_{t-\tau(t)}^t x^T(s)ds\ \int_{t-\tau(t)}^t f^T(x(s))ds \\
& \textstyle\int_{t-\tau(t)}^t f^T(x(s-\tau(s)))ds\ \int_{t-\tau(t)}^t trace\sigma^T(s, x(s), x(s-\tau(s))d\omega(s)\Big)^T,
\end{aligned}
\tag{47}
$$

where

$$J_1=\begin{pmatrix} \Omega_{11}+\Gamma_{11} & P & PA & PB & 0 & 0 & 0 & 0 \\ P & \Omega_{22}+\Gamma_{22} & 0 & 0 & -PK+PC & -PA & -PB & -P \\ A^TP & 0 & \Omega_{33} & 0 & 0 & 0 & 0 & 0 \\ B^TP & 0 & 0 & \Omega_{44} & 0 & 0 & 0 & 0 \\ 0 & -K^TP+C^TP & 0 & 0 & \varepsilon_3\rho_C I-\frac{T_1}{r} & 0 & 0 & 0 \\ 0 & -A^TP & 0 & 0 & 0 & \varepsilon_4\rho_A I-\frac{T_2}{r} & 0 & 0 \\ 0 & -B^TP & 0 & 0 & 0 & 0 & \varepsilon_5\rho_B I-\frac{T_3}{r} & 0 \\ 0 & -P & 0 & 0 & 0 & 0 & 0 & -\frac{H}{r} \end{pmatrix}, \tag{48}$$

$$\Omega_{11}=-2PC+PK+K^TP+\rho_C\varepsilon_C I+Q+(\rho+r\rho')M^TM+rT_1+\varepsilon_1F^2,$$

$$\Gamma_{11}=(\varepsilon_C^{-1}+\varepsilon_A^{-1}+\varepsilon_B^{-1})P^2,\ \Gamma_{22}=(\varepsilon_3^{-1}+\varepsilon_4^{-1}+\varepsilon_5^{-1})P^2,\ \Omega_{22}$$
$$=(\rho+r\rho')M_1^TM_1-(1-h)Q+\varepsilon_2F^2-2P,$$

$$\Omega_{33}=\rho_A\varepsilon_A I+R+rT_2+rT_3-\varepsilon_1 I,\ \Omega_{44}=\rho_B\varepsilon_B I-(1-h)R-\varepsilon_2 I.$$

From Lemma 2, it is easy to see that the condition (33) is equivalent to $J_1<0$. From (48) and It$\widehat{o}$ formula, the proof is completed the same as Theorem 1.

Note that if the gain matrix K is still not solved. Next, a theorem is deduced to solve the feedback gain matrix K.

Theorem 2 *Under the assumptions A_1–A_2 and A_4–A_5, the equilibrium point of model (1) is globally asymptotically stable in mean square if there are positive definite matrices $Q=(h_{ij})_{n\times n}$, $P=(p_{ij})_{n\times n}$, $R=(r_{ij})_{n\times n}$, $H=(h_{ij})_{n\times n}$, $T_i\in\mathbb{R}^{n\times n}(i=1,2,3)$, matrix $K'=\in\mathbb{R}^{n\times n}$, and positive constants $\varepsilon_A,\varepsilon_B,\varepsilon_C,\varepsilon_i(i=1,2,3,4,5)$, ρ, ρ', such that*

$$\begin{pmatrix} \Omega_{11} & P & PA & PB & 0 & 0 & 0 & 0 & P & P & P & 0 & 0 & 0 \\ P & \Omega_{22} & 0 & 0 & \Omega_{25} & -PA & -PB & -P & 0 & 0 & 0 & P & P & P \\ A^TP & 0 & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B^TP & 0 & 0 & \Omega_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Omega_{25}^T & 0 & 0 & \Omega_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -A^TP & 0 & 0 & 0 & \Omega_{66} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -B^TP & 0 & 0 & 0 & 0 & \Omega_{77} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -P^T & 0 & 0 & 0 & 0 & 0 & -\frac{H}{r} & 0 & 0 & 0 & 0 & 0 & 0 \\ P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_C I & 0 & 0 & 0 & 0 & 0 \\ P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_A I & 0 & 0 & 0 & 0 \\ P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_B I & 0 & 0 & 0 \\ 0 & P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I & 0 & 0 \\ 0 & P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I & 0 \\ 0 & P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_5 I \end{pmatrix}<0, \tag{49}$$

$$P\le\rho I, \tag{50}$$

$$H\le\rho' I, \tag{51}$$

where

$$\Omega_{11}=-2PC+K'+K'^T+\rho_C\varepsilon_C I+Q+(\rho+r\rho')M^TM+rT_1$$
$$+\varepsilon_1F^2,\ \Omega_{25}=-K'+PC,$$

$$\Omega_{22} = (\rho + r\rho')M_1^T M_1 - (1-h)Q + \varepsilon_2 F^2 - 2P, \quad \Omega_{33} = \rho_A \varepsilon_A I + R + rT_2 + rT_3 - \varepsilon_1 I,$$

$$\Omega_{44} = \rho_B \varepsilon_B I - (1-h)R - \varepsilon_2 I, \quad \Omega_{55} = \varepsilon_3 \rho_C I - \frac{T_1}{r}, \quad \Omega_{66} = \varepsilon_4 \rho_A I - \frac{T_2}{r}, \Omega_{77} = \varepsilon_5 \rho_B I - \frac{T_3}{r},$$

and I is the identity matrix. Moreover, the estimation gain $K = P^{-1}K'$.

Note that if Assumption A_3 is not satisfied, that is $\dot{\tau}(t) \geq 1$ for some t. But Assumption A_5 is satisfied, and Theorem 2 can still work. Assumption A_3 is satisfied in many research papers, however, we improved this assumption in this paper.

Remark 1 Note that if $\sigma(t, x(t), x(t-\tau(t))) = 0$ and $\Delta A = \Delta B = \Delta C$, then the stability analysis is ordinary global asymptotical stability that has been intensively discussed. Also, there are many research works [1–17,27–29] about robust stability of delayed system. Some delay-dependent sufficient conditions are derived to ensure the robust stability of delayed system. However, due to the random uncertainties in the system, we consider the robust stability of delayed stochastic system.

Remark 2 Recently, the discussion about the delayed stochastic system [21–26] becomes a hot topic, which are mainly about the stability analysis of linear system. However, in many cases we want to stabilize the nonlinear system. Thus, in this paper, we study the robust control of delayed stochastic neural network. In particular, the gain matrix K is solved. Some sufficient delay-independent (Theorem 1) and delay-dependent (Theorem 2) conditions are derived.

Remark 3 Studies about the delayed stochastic system is mainly linear system with constant time delay, however, we consider the nonlinear system with time-varying delay. Also, we extend the Assumption A_3 to A_5, and we do not need $\dot{\tau}(t) < 1$. Thus in this paper we generalize the derivative of the time varying delay to any given value although most of the former results are based on Assumption A_3.

4 Numerical Example

In this section, an example is constructed to justify the Theorem 2 obtained above.

Example Consider the system (1) of a typical delayed Hopfield neural network as follows:

$$\begin{aligned} dx(t) = &[-(C + \Delta C)x(t) + (A + \Delta A)f(x(t)) + (B + \Delta B)f(x(t-\tau(t))) + u(t)]dt \\ &+\sigma(t, x(t), x(t-\tau(t)))d\omega(t), \end{aligned}$$

where

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, A = \begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 2.8 \end{pmatrix}, B = \begin{pmatrix} -1.6 & -0.1 \\ -0.3 & -2.5 \end{pmatrix}, \tau(t) = \frac{1 + 0.5\sin(30t)}{10},$$

$$f(x) = \begin{pmatrix} \tanh x_1 \\ \tanh x_2 \end{pmatrix}, \sigma(t, x(t), x(t-\tau(t))) = \begin{pmatrix} 0.1\|x(t)\| & 0 \\ 0 & 0.1\|x(t-\tau(t))\| \end{pmatrix}.$$

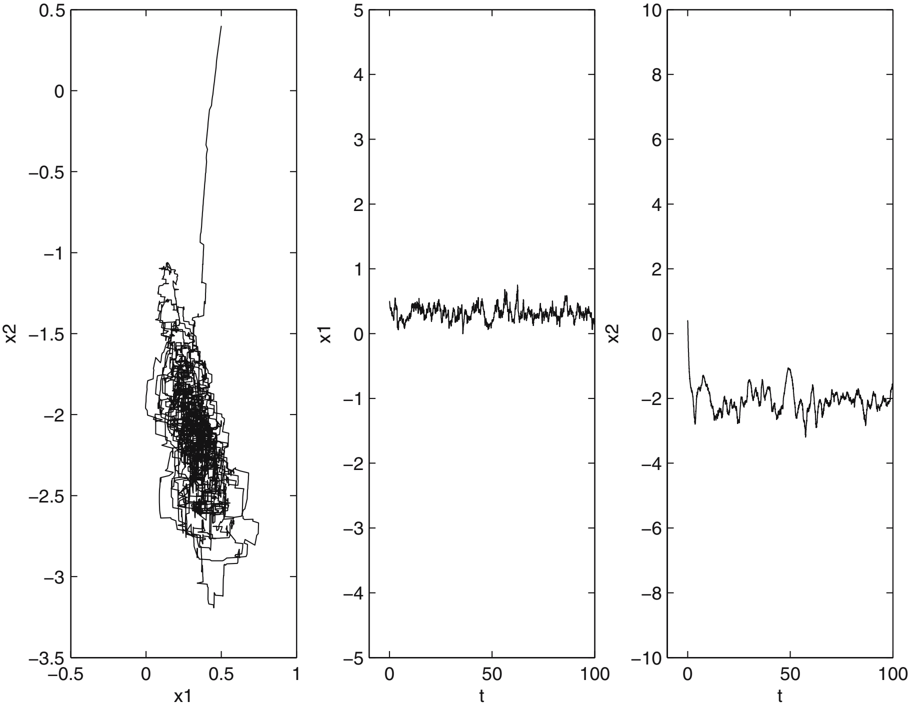

Fig. 1 Trajectories of state variables $x_1(t)$ and $x_2(t)$ without controller

It is easy to see that the assumptions A_1–A_2 and A_4–A_5 are satisfied. Suppose each element of uncertainty matrices ΔC, ΔA and ΔB are random in $[-0.02, 0.02]$ every time, since the parameters of the system (1) are perturbed by many factors every time. From Lemma 4, we know that

$$\|\Delta C\| \leq \left\| \begin{pmatrix} 0.02 & 0.02 \\ 0.02 & 0.02 \end{pmatrix} \right\| = 0.04, \ \|\Delta A\| \leq 0.04, \ \|\Delta B\| \leq 0.04.$$

It is easy to see that $\rho_C = \rho_A = \rho_B = 0.04$.

It is obvious that $F = I, h = 1.5, r = 0.15, M = M_1 = 0.1I$, where I is the identity matrix. Because $\dot{\tau}(t) > 1$ for some t, so we solve this problem by Theorem 2. From Theorem 2 and using LMI toolbox in Matlab, we can obtain

$$P = \begin{pmatrix} 7.5850 & 0.1315 \\ 0.1315 & 5.5585 \end{pmatrix}, \ Q = \begin{pmatrix} 1.6664 & 0.1569 \\ 0.1569 & 0.1472 \end{pmatrix},$$

$$R = \begin{pmatrix} 0.6607 & -0.0252 \\ -0.0252 & 0.1550 \end{pmatrix}, \ H = \begin{pmatrix} 65.2540 & -1.5344 \\ -1.5344 & 78.9859 \end{pmatrix},$$

$$T_1 = \begin{pmatrix} 317.5086 & -41.4986 \\ -41.4986 & 980.8271 \end{pmatrix}, \ T_2 = \begin{pmatrix} 162.2870 & -41.8527 \\ -41.8527 & 102.3825 \end{pmatrix},$$
$$T_3 = \begin{pmatrix} 50.8932 & 33.9789 \\ 33.9789 & 144.6886 \end{pmatrix},$$

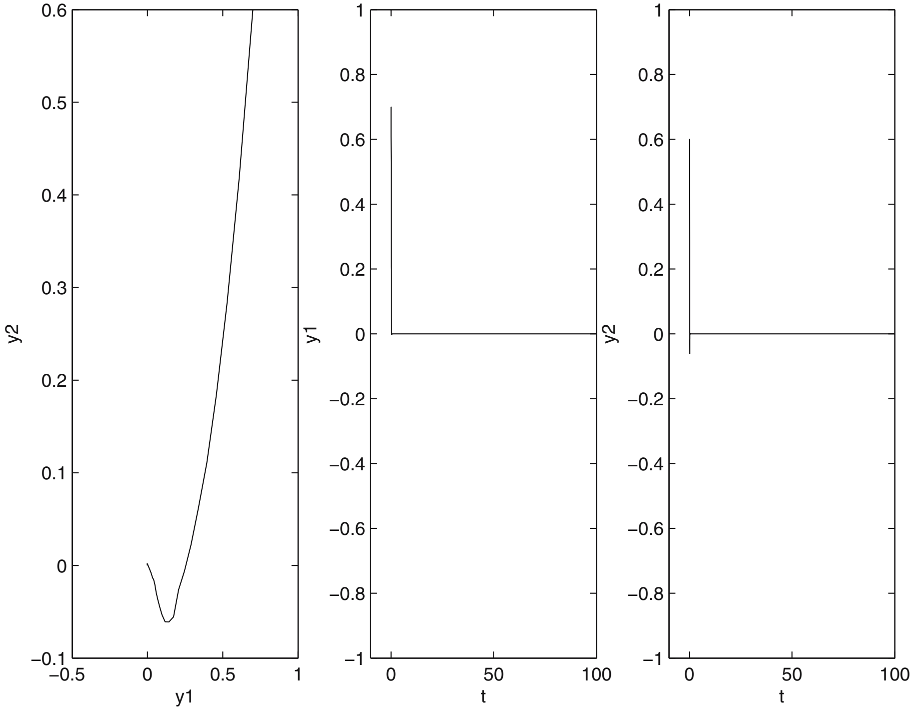

Fig. 2 Trajectories of state variables $x_1(t)$ and $x_2(t)$ with controller

$$\varepsilon_1 = 74.0653, \quad \varepsilon_2 = 2.9476, \quad \varepsilon_3 = 440.6692, \quad \varepsilon_4 = 443.7623, \quad \varepsilon_5 = 446.3644,$$

$$\varepsilon_C = 129.2733, \quad \varepsilon_A = 132.7056, \quad \varepsilon_B = 8.8944, \quad \rho = 18.0541, \quad \rho_1 = 136.9841,$$

$$K' = \begin{pmatrix} -116.0874 & 0.9174 \\ 0.9174 & -181.3261 \end{pmatrix}, \quad K = P^{-1}K' = \begin{pmatrix} -15.3140 & 0.6868 \\ 0.5274 & -32.6374 \end{pmatrix}.$$

The trajectories of system without controller and with controller are shown in Figs. 1, 2, respectively. It is easy to see that without controller, it is chaotic in system, but it is stable by adding a controller. The designed controller is effective and can stabilize the delayed stochastic system.

5 Conclusions

In this paper, we have considered robust control of uncertain stochastic recurrent neural networks with time-varying delay. Lyapunov functional method and LMI technique are used to solve this problem. Several sufficient conditions have been derived to ensure the global asymptotical stability for the delayed stochastic neural network, and the design of the feedback controller which is used to stabilize the system can be achieved. The obtained results are novel since there are few works about the robust control of uncertain delayed stochastic neural network. It is easy to apply these sufficient conditions to the real networks. Finally, a numerical simulation is constructed to verify the theoretical results.

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References

1. Niu Y, Wang X (2004) Delay-dependent robust control for a class of nonlinear state-delayed systems. Proceedings of the 5th World Congress on Intelligent Control and Automation pp 907–910
2. Kwon OM, Park JH (2004) On improved delay-dependent robust control for uncertain time-delay systems. IEEE Trans Automat Contr 49(11):1991–1995
3. Chen J (2004) Robust control for uncertain neutral systems with time-delays in state and control input via LMI and GAs. Appl Math and Comput 157:535–548
4. Liao XF, Wong KW, Wu Z, Chen G (2001) Novel robust stability criteria for interval-delayed Hopfield neural networks. IEEE Trans Circuits Syst I 48(11):1355–1359
5. Singh V (2005) Global robust stability of delayed neural networks: an LMI approach. IEEE Trans Circuits Syst II 52(1):33–36
6. Singh V (2005) A novel global robust stability criterion for neural networks with delay. Phys Lett A 337:369–373
7. Liao XF, Yu JB (1998) Robust stability for interval Hopfield neural networks with time delay. IEEE Trans Neural Netw 9(5):1042–1045
8. Cao J, Huang D-S, Qu Y (2005) Global robust stability of delayed recurrent neural networks. Chaos, Solitons & Fractals 23(1):221–229
9. Arik S (2003) Global robust stability of delayed neural networks. IEEE Trans Circuits Syst I 50(1):156–160
10. Rong L (2005) LMI-based criteria for robust stability of Cohen-Grossberg neural networks with delay. Phys Lett A 339:63–73
11. Xu S, Lam J, Ho DWC (2005) Novel global robust stability criteria for interval neural networks with multiple time-varying delays. Phys Lett A 342:322–330
12. Singh V (2004) Robust stability of cellular neural networks with delay: linear matrix inequality approach. IEE Proc Control Theory Appl 151(1):125–129
13. Ye H, Michel AN (1996) Robust stability of nonlinear time-delay systems with applications to neural networks. IEEE Trans Circuits Syst I 43(7):532–543
14. Cao J, Wang J (2004) Delay-dependent robust stability of uncertain nonlinear systems with time delay. Appl Math Comput 154:289–297
15. Huang H, Cao J, Qu Y (2004) Global robust stability of delayed recurrent neural networks. J Comput Sys Sci 69:688–700
16. Chen A, Cao J, Huang L (2005) Global robust stability of interval cellular neural networks with time-varying delays. Chaos, Solitons & Fractals 23:787–799
17. Liao X, Wong K (2004) Robust stability of interval bidirectional associative memory neural network with time delays. IEEE Trans Syst Man Cyb 34(2):1142–1154
18. Yu W, Cao J (2007) Synchronization control of stochastic delayed neural networks. Physica A 373:252–260
19. Yu W (2007) A LMI-based approach to global asymptotic stability of neural networks with time varying delays. Nonlinear Dynam 48 (1-2):165–174, DOI: 10.1007/s11071-006-9080-6
20. Yu W, Yao L (2007) Global robust stability of neural networks with time varying delays. J Comput Appl Math 206(2):679–687, DOI: 10.1016/j.cam.2006.08.017
21. Blythe S, Mao X, Liao X (2001) Stability of stochastic delay neural networks. J Franklin Inst 338:481–495
22. Liao X, Mao X (2000) Exponential stability of stochastic delay interval systems. Syst Contr Lett 40:171–181
23. Chen W, Guan Z, Liu X (2005) Delay-dependent exponential stability of uncertain stochastic systems with multiple delays: an LMI approach. Sys Contr Lett 54:547–555
24. Wan L, Sun J (2005) Mean square exponential stability of stochastic delayed Hopfield neural networks. Phy Lett A 343:306–318
25. Yue D, Won S (2001) Delay-dependent robust stability of stochasitc systems with time delay and nonlinear uncertainties. IEE Electron Lett 37(15):992–993
26. Chen W, Xu J, Guan Z (2003) Guaranteed cost control for uncertain Markovian jump systems with mode-dependent time-delays. IEEE Trans Automa Contr 48(12):2270–2276

27. Wang Z, Ho DWC, Liu X (2005) State estimation for delayed neural networks. IEEE Trans Neural Netw 16(1):279–284
28. Moon Y, Park P, Kwon W, Lee Y (2001) Delay-dependent robust stabilization of uncertain state-delayed systems. Int J Contr 74(14):1447–1455
29. Wang Z, Goodall DP, Burnham KJ (2002) On designing observers for time-delay systems with non-linear disturbances. Int J Contr 75(11):803–811
30. Kolmanovskii VB, Myskis AD (1999) Introduction to the theory and applications of functional differential equations. Kluwer, Academic Publishers, Dordrecht
31. Schuss Z (1980) Theory and applications of stochastic differential equations, Wiley
32. Cao J, Wang J (2005) Global asymptotic and robust stability of recurrent neural networks with time delays. IEEE Trans Circuits Syst I 52(2):417–426
33. Boyd S, Ghaoui LE, Feron E, Balakrishnan V (1994) Linear matrix inequalities in system and control theory. SIAM, Philadelphia, PA
34. Gu KQ, Kharitonov VL, Chen J (2003) Stability of time-delay systems. Birkhauser, Boston