Internet Pricing With a Game Theoretical Approach: Concepts and Examples

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***Abstract*—The basic concepts of three branches of game theory, leader–follower, cooperative, and two-person nonzero sum games, are reviewed and applied to the study of the Internet pricing issue. In particular, we emphasize that the cooperative game (also called the bargaining problem) provides an overall picture for the issue. With a simple model for Internet quality of service (QoS), we demonstrate that the leader–follower game may lead to a solution that is not Pareto optimal and in some cases may be "unfair," and that the cooperative game may provide a better solution for both the Internet service provider (ISP) and the user. The practical implication of the results is that government regulation or arbitration may be helpful. The QoS model is also applied to study the competition between two ISPs, and we find a Nash equilibrium point from which the two ISPs would not move out without cooperation. The proposed approaches can be applied to other Internet pricing problems such as the Paris Metro pricing scheme.**

***Index Terms*—Bargaining problems, cooperative games, leader–follower games, quality of services, Paris Metro pricing, two-person nonzero sum games.**

I. Introduction

In recent years, substantial progress has been made to understand Internet economics by both the engineering and economic research communities [10], [29]–[31]. The central issue of Internet economics is pricing, which can be used as an effective means to recover cost, to increase competition among different service providers, and to reduce congestion or to control the traffic intensity. There are many approaches in determining a pricing strategy, e.g., the cost-based approach, the optimization-based approach, and *edge pricing* [16].

The optimization-based approach may provide insight about market value of the services (how much others are willing to pay). This approach is referred to as *yield management* in operation research literature. It has been widely practiced in capacity-constrained service industries (e.g., airline and hotel services) to match prices to demand, and has begun to be employed in computer networks [19], [20]. The fundamentals of the multiparty (ISPs and users) optimization problems can be captured by game theory (see, e.g., [1], [10], [21], [23], [24], [27]). Another closely related area is the application of game theory to the resource allocation problem (e.g., routing) in networks [4]–[6], [8], [18], [22], [25], [26].

Most of the existing works applying game theory to Internet pricing adopt a leader–follower game framework in which the ISP sets up a price as a leader and the users respond with a demand (see, e.g., [1], [28]). The ISP's task is to set up the right price to induce a desirable demand from the users to achieve a profit (or a welfare function) as large as possible. This belongs to the domain of noncooperative game. In this paper, we propose to study the Internet pricing problem by using another branch of game theory, i.e., the cooperative game, or the bargaining problems. With this approach, we study all the possible outcomes in a utility space, and the players (ISP and user) determine, through negotiation or arbitration, a particular outcome as their *fair* solution. The solution of a bargaining problem depends on the concept of fairness, which can be specified clearly by a set of axioms. The study confirms our original concern about the solution of the leader–follower game approach: it may not be on the Pareto boundary and may not be fair. Thus, we show that by cooperation a "fair" solution can be obtained at which both the ISP and the user are better off than the leader–follower solution. This result seems to be consistent with the current industry trend toward cooperation between corporations [11], and indicates that government regulations or arbitration may be helpful. The cooperative game approach was used in [5] to study the fairness issue of the admission control of broadband networks.

One engineering feature distinguishes Internet pricing from other pricing problems: users pay for quality of service (QoS), which deteriorates as the demands increase if the bandwidth is shared. Therefore, determining QoS is an important part of Internet pricing. Different applications require different QoS; analytical results do not always exist in most cases. To facilitate analysis, in this paper, we propose a simple model for QoS; with this model, we are able to analyze the pricing issue by numerical methods. This QoS model is also used in the two ISP competition study with the two-person nonzero sum game theory; we find a Nash equilibrium for the two ISPs. A few special cases based on the model are studied. For problems where analytical formulas for numerical methods do not exist (e.g., priority queueing), we use simulation to evaluate QoS. The main insight obtained by these examples are the same and do not depend on the particular model.

In Section II, we briefly review the fundamental concepts of game theory that will be used in this paper. In Sections III and IV,

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we study the ISP versus user case. We propose a simple model for QoS, and based on it some analytical formulas are developed. The numerical examples show that the leader–follower game does not lead to the Pareto optimum and the cooperative game solution is better for both the ISP and the user; these examples also indicate that in some sense the leader–follower solution may not be "fair." In Section V, we study the two-ISP case and illustrate, by a numerical example, that Nash equilibriums exist (no rigorous proof is provided, nor is uniqueness established). The example also shows that cooperation is better for both ISPs at the cost of a reduced profit for the user. The paper concludes with some discussion in Section VI.

II. Noncooperative and Cooperative Games

We start with reviewing some basic concepts of game theory [14], [9], [15], [17], [2] by using the Internet pricing framework. We will discuss both cooperative (bargaining problems) and noncooperative games (leader–follower games and two-person games). Two players of the game are the ISP and the user (representing a group of users having the same characteristics). The ISP sets up a price for its service; based on the price and other considerations, the user determines the amount of request that it wants to submit to the Internet. The outcomes of a game are the *utilities* of each player (i.e., profits for the ISP and the user). In the game, the ISP and the user choose respectively their best *strategies* (price for the ISP and demand for the user) to get their desired outcomes.

The leader–follower game model has been widely used in studying the pricing issue. Let c be the price that the ISP announces. With this price, the user determines its demand r. The utilities for the user and the ISP are denoted as $U(c,r)$ and $V(c,r)$. Given any price c, the user chooses $r(c) = \arg\{\max_{r\in R}[U(c,r)]\}$, where R is the set of all r's, to maximize its utility. Knowing this reaction of the user, the ISP chooses a price $c^* = \arg\{\max_{c\in C}[V(c,r(c))]\}$, where C is the set for all c's, to maximize its utility.

In a cooperative game [9], [15], [17], [2], the two players are called *bargainers*. They work on the utility space (u,v). Each pair of policies (c,r) corresponds to one point $(u=U(c,r), v=V(c,r))$ in the utility space. The set of all the points in the utility space corresponding to all the feasible policies, S, is called a *bargaining set*. With random policies if necessary, S is a convex set. The problem that the two bargainers face is to negotiate for a "fair" point on this convex set as the outcome. If no agreement can be reached by the two bargainers, one particular point $s_0 = (u_0, v_0) \in S$, called the *starting point*, will be the outcome of the game. Thus, only those points in S with $u \geq u_0$ and $v \geq v_0$ should be considered. Such a bargaining problem is denoted as (S, s_0). The fair point chosen by the two players is called the *solution* to the problem.

The outcome is chosen based on certain *fairness* criteria that both bargainers agree upon. The fairness criteria are clearly expressed by a set of axioms, which usually uniquely determines a points on the bargaining set [15]. The first three axioms are very simple and have clear meanings:

- *Symmetry:* If S is symmetric with respect to the axis $u = v$ and the starting point is on this axis, then the solution is also on this axis (two players are treated equally).
- *Pareto optimality:* The solution is on the Pareto boundary (no outcome is better for both players).
- *Invariance with respect to utility transformations:* The solution to bargaining problem $(f(S), f(s_0))$ is $f(s^*)$, where s^* is the solution to problem (S, s_0), f is any positive affine transformation, and $f(S) = \{\text{all } f(s) : s \in S\}$. (the solution is the same if different currencies are used).

Three arbitration schemes are of particular interest: Nash, Raiffa–Kailai–Smorodinsky (henceforth called Raiffa), and the modified Thomson solutions. The Nash bargaining solution is uniquely determined by the above three axioms and the following one [15]:

- *Independence of irrelevant alternatives (IIA):* If the solution for (S, s_0) is s^*, and $S' \subseteq S, s^* \in S'$, then the solution for (S', s_0) is also s^*. (Since s^* is fairer than all the other points in S, and $S' \subseteq S$, then s^* must be fairer than all the other points in S'.)

The Nash solution maximizes the value of the product of the two utilities, $u \times v$. It is the tangent point of the hyperbola $u \times v = \text{constant}$ with the Pareto boundary.

The Raiffa solution [9] can be uniquely determined by the first three axioms and the following axiom.

- *Monotonicity:* If $S' \subseteq S$ and $\max\{u : s = (u,v) \in S\} = \max\{u : s = (u,v) \in S'\}$ and $\max\{v : s = (u,v) \in S\} \geq \max\{v : s = (u,v) \in S'\}$, then $v^* \geq v'^*$, with $s^* = (u^*, v^*)$ and $s'^* = (u'^*, v'^*)$ being the solutions to problems $(S, (0,0))$ and $(S', (0,0))$, respectively.

The modified Thomson solution was defined in [2] by modifying the utilitarian rule [17] that maximizes the sum of $u + v$ in the normalized problem defined below.

One of the drawbacks of Nash's fairness criteria is that it implies that each player does not care about how much the other side has given up. With this observation, [2] proposed a set of solutions that represent the concerns of each player for how much it gets as well as how much the other side gives up, with a parameter whose value indicates the tradeoff between those two concerns.

First, with the third axiom, any game problem can be normalized into a problem with starting point $(0,0)$ and $\max\{u : s = (u,v) \in S\} = \max\{v : s = (u,v) \in S\} = 1$. With the normalized problem, the *preference functions* [2] for both players are defined as

$$w_1 = u + \beta(1-v), \quad w_2 = v + \beta(1-u), \quad -1 \leq \beta \leq 1.$$

Since the maximum utility is 1, $1-u$ or $1-v$ represents how much the player has given up.

With the utility function replaced by the preference function and applying the first three axioms and the axiom IIA, we obtain a set of solutions (parameterized by β)

$$u^*, v^* = \arg\{\max_{u,v}[w_1 \times w_2]\}.$$

It is shown in [2] that the Nash, Raiffa, and modified Thomson solutions are special cases corresponding to $\beta = 0$, 1, and -1, respectively, and when β changes continuously from -1

to 1, the solution moves monotonically and continuously on the Pareto boundary of S from the modified Thomson solution to the Nash solution and then to the Raiffa solution. The bargaining problem now becomes to determine a value for β that is acceptable for both players. Once β is agreed upon, a fair solution can be uniquely determined.

Finally, the two-person (nonzero sum) game model can be applied to study the competition between two ISPs. Each ISP can set up different prices, denoted as (c_1, c_2). Based on the prices, the user chooses to submit its request to either one or both ISPs to maximize its utility. Each ISP may reduce its price to attract more requests to make more profits. However, in many cases there exist one or more pairs of prices such that if one changes its price without the cooperation of the other, one cannot improve its utility. Such points are called *Nash equilibriums*. See Section V for more details.

III. One ISP With a Single Class of Users

In this section, we study the case where there is only one service provider with link capacity μ unit. Users generate s (say, packets). Since we assume that the behavior of all users are identical, we can view all the requests as being from the same user and simply use the singular word "user." We will apply both the leader–follower game and cooperative game approaches to the problem and compare the results obtained.

Each request is associated with a QoS requirement. QoS may take many different forms, such as response time, bit-error rate, or both; we choose response time as QoS in our examples (other QoS criteria can also be studied by the same principle with more complicated analytical or simulation techniques). It is well known that the analytical solutions to response time distributions are only available for some simple systems. Thus, we will make further assumptions, which are for the purpose of facilitating analysis; when an analytical solution does not exist, we use simulation to evaluate QoS. We hope that with the simplified assumptions, we can clearly illustrate the main ideas and insights.

We first assume that the requests come from a Poisson process with arrival rate λ, and each request requires a unit bandwidth to serve for an exponentially distributed time with a unit mean. Then we propose a simple model for the QoS requirement. We assume that each request has a maximal acceptable response time, denoted as s; if the real service response time (the transmission time plus the waiting time) is smaller than s, the service is successful; otherwise, it is considered a failure. We assume that s has a distribution density function $f(s)$. (This function can be discretized by using Dirac delta functions to represent a finite set of QoS requirements corresponding to different applications, such as voice over IP (VoIP), and email; see Examples 2 and 3.) A user earns $g(s) \geq 0$ for a successful service with a maximal acceptable response time s ($g(s)$ can also be discretized), and earns 0 for a failed service.

We first study the case where the ISP provides only one type of service (best effort); thus, for every request, the ISP charges a fee c. (More realistically, the charge should depend on the length of the request. However, since in our model the service discipline does not depend on the request length, each request will have the same probability distribution for its waiting time, regardless of its length. Thus, the results will be the same if we use a constant charge which equals the average.) Later, we will study the case where the ISP provides two types of services with different priorities and prices. Next, we assume that the user only employs static policies; i.e., its policy depends only on statistics, not on the state of the system. When a request with maximal acceptable response time s arrives, the user submits it to the Internet with a probability $\alpha(s)$ and discards it with a probability $1-\alpha(s)$. We will see that if $f(s)$ is a continuous function, $\alpha(s)$ is either 0 or 1. When $f(s)$ is discretized, $\alpha(s)$ specifies the portion of a particular application that is submitted by the user. Overall, the arrival rate to the Internet is $\alpha\lambda$, where

$$\alpha = \int_0^\infty \alpha(s) f(s)\, ds. \tag{1}$$

With the above setting, the link can be modeled simply by an M/M/1 queue with arrival rate $\alpha\lambda$ and service rate μ. Let $\rho = \alpha\lambda/\mu$ be the traffic intensity, and $p_n, n = 0, 1, 2, \ldots$, be the steady-state probability that there are n requests in the system. Then $p_n = (1-\rho)\rho^n, n = 0, 1, \ldots$. From this, it is easy to verify that the response time of an M/M/1 queue is exponentially distributed with mean $1/[(1-\rho)\mu]$. Let r denote the response times in the queue. Then for any given s, the probability of $r > s$ is

$$p(r > s) = e^{-(1-\rho)\mu s} = e^{-(\mu - \alpha\lambda)s}. \tag{2}$$

The user's utility is

$$U(c,\alpha) = \lambda \int_0^\infty g(s)\alpha(s) f(s)[1 - e^{-(\mu-\alpha\lambda)s}]\, ds - \alpha\lambda c. \tag{3}$$

The ISP's utility is

$$V(c,\alpha) = \alpha\lambda c. \tag{4}$$

First, let us determine $\alpha(s)$ for a fixed traffic intensity α, i.e., to maximize $U(c,\alpha)$ under the constraint of (1) with α being a constant. For any $\alpha, 0 < \alpha < 1$, we define

$$I_\alpha = \left\{ I_\alpha \subset [0,\infty) : h(s) \geq h(s') \text{ if } s \in I_\alpha \text{ and } s' \notin I_\alpha; \int_{I_\alpha} f(s)\, ds = \alpha \right\}$$

where

$$h(s) = g(s)[1 - e^{-(\mu-\alpha\lambda)s}].$$

It is easy to see that for a fixed $\alpha, U(c,\alpha)$ reaches its maximum when we choose $\alpha(s)$ as the indicator function of I_α:

$$\alpha(s) = \begin{cases} 1 & \text{if } s \in I_\alpha \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Note that $h(s)$ is the expected gain for a request with maximal accepted response time s; thus, the user should submit all the requests which have the largest expected gains. I_α is an interval if $h(s)$ is continuous and has only one peak. (In fact, if I_α consists of two intervals, then $h(s)$ has at least two peaks. Therefore, concavity is not required.) In this case, $\alpha(s)$ is

$$\alpha(s) = \begin{cases} 1 & \text{if } s_1 \le s \le s_2 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where s_1 and s_2 satisfy

$$\alpha = \int_{s_1}^{s_2} f(s)\, ds \tag{7}$$

and

$$h(s_1) = h(s_2), \text{ if } s_1 > 0;\ h(s_1) \ge h(s_2), \quad \text{if } s_1 = 0. \tag{8}$$

(3) becomes

$$U(c, s_1, s_2) = \lambda \int_{s_1}^{s_2} g(s) f(s) [1 - e^{-(\mu - \alpha\lambda)s}]\, ds - \alpha\lambda c. \tag{9}$$

A. The Leader–Follower Game

In the leader–follower game approach, the problem now becomes: for a given price set by the ISP, the user determines s_1 and s_2 such that $U(c, s_1, s_2)$ in (9) is maximized under the constraints (7) and (8). The ISP's task is to choose a c such that its utility (4) is maximized. To continue the analysis, we further assume that the value of the service decreases exponentially as the maximal acceptable response time increases, i.e., we let $g(s) = de^{-\chi s}$; we also assume that $f(s)$ is exponentially distributed, i.e., $f(s) = \eta e^{-\eta s}$. It is easy to verify that for any α, $h(s)$ has only one peak. Putting these into (9), we get

$$\begin{aligned} &U(c, s_1, s_2) \\ &= \frac{\lambda d\eta}{\chi + \eta} \left[e^{-(\chi+\eta)s_1} - e^{-(\chi+\eta)s_2} \right] - \frac{\lambda d\eta}{\chi + \eta + \mu - \alpha\lambda} \\ &\quad \cdot \left[e^{-(\chi+\eta+\mu-\alpha\lambda)s_1} - e^{-(\chi+\eta+\mu-\alpha\lambda)s_2} \right] - \alpha\lambda c \end{aligned} \tag{10}$$

$$V(c, s_1, s_2) = \lambda c [e^{-\eta s_1} - e^{-\eta s_2}] \tag{11}$$

and

$$\alpha = e^{-\eta s_1} - e^{-\eta s_2}. \tag{12}$$

Furthermore, the optimal solution satisfies

$$e^{-\chi s_1} \left[1 - e^{-(\mu - \alpha\lambda)s_1} \right] = e^{-\chi s_2} \left[1 - e^{-(\mu - \alpha\lambda)s_2} \right]. \tag{13}$$

To illustrate the idea, we provide a numerical example.

Example 1: In this example, the bandwidth of the Internet is $\mu = 1000$ packet/s; the arrival rate is $\lambda = 1200$ packet/s; $g(s) = 10e^{-s}$; and $f(s) = e^{-s}$. For any given price c and traffic intensity α (or equivalently $\rho = \alpha\lambda/\mu = 1.2\alpha$), we first calculate s_1 and s_2 using (12) and (13), and then we obtain the values of utilities corresponding to these c and ρ by using (10) and (11). For every price c, the user can respond with different ρ, leading to a curve on the utility space (u, v). We plot seven such curves in Fig. 1, corresponding to $c = 1, 2, 3, 4, 5, 6,$ and 7, respectively. For each c, the user chooses an intensity ρ to maximize its utility, such a point corresponds to the tangent point of the vertical lines with the curve corresponding to c. Connecting these points yields the dashed curve in Fig. 1 representing the

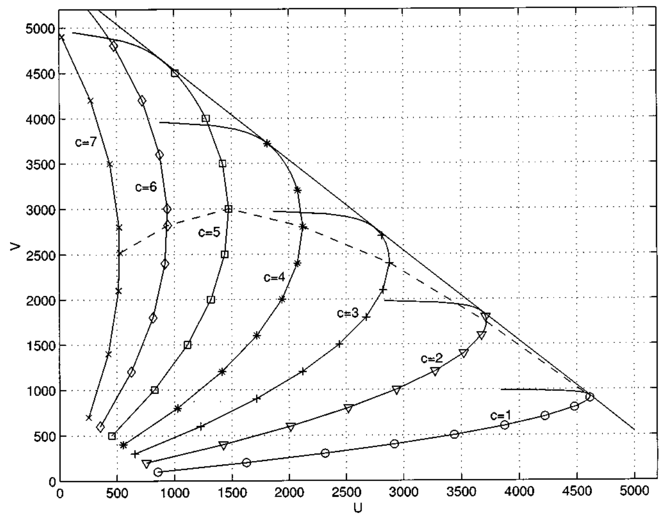

Fig. 1. The leader–follower game solution.

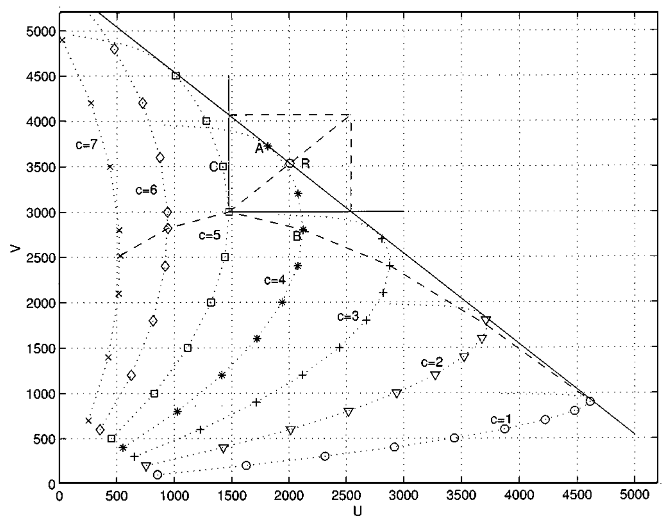

Fig. 2. The cooperative game solution.

outcomes of the game for different prices. From this curve, it is clear that the maximum utility is reached if the ISP sets up the price as $c = 5$, for which the user responds with $\rho = 0.6$, the maximum utility for the ISP is about 3000 and that for the user is 1440. □

To examine how good this leader–follower game solution is, we plot out all the possible outcomes in the utility space. All these points, including the ones corresponding to random policies, form a convex set, the bargaining set. The boundary of this set is shown by the dark line in Fig. 2. This example clearly shows that the solution of the leader–follower game approach is not on the Pareto boundary. Thus, the utilities for both the ISP and the user can be improved. However, such improvement cannot be achieved without cooperation between both parties; such cooperation may be realized by negotiation between the two parties or by a third party (e.g., government) arbitration.

B. The Cooperative Game

The cooperative game theory provides guidance to the negotiation or arbitration process. In this approach, the two players, the ISP and the user, will choose a point on the Pareto boundary as their solution. On the Pareto boundary, the social welfare defined as

$$S(\alpha) = U(c,\alpha) + V(c,\alpha) \\ = \lambda \int_0^\infty g(s)\alpha(s)f(s)\left[1 - e^{-(\mu-\alpha\lambda)s}\right] ds$$

is maximized. Note that $S(\alpha)$ does not depend on c. Thus, $(dS(\alpha)/d\alpha) = 0$, on the Pareto boundary. However, the leader–follower game solution must satisfy $(\partial/\partial\alpha)U(c,\alpha) = 0$ for a fixed c. Therefore, on this point

$$\frac{dS(\alpha)}{d\alpha} = \frac{d}{d\alpha}[U(c,\alpha) + \lambda\alpha c] = \lambda c$$

which is not zero. Thus, the leader–follower game solution cannot be on the Pareto boundary.

The two players are faced with two main issues: what point should be chosen as the starting point, and what fairness criteria should be used. Recall that if the negotiation fails, the starting point will be picked up as the outcome. Thus, there seem to be two natural ways of choosing the starting point. The first one is to pick up the solution of the leader–follower game as the starting point of the bargaining problem. This implies that if they fail in negotiation, the ISP will determine a price. In this case, with the cooperative game, both the ISP and the user are better off. Another way is to choose the origin $(0,0)$ as the starting point. This may happen when either ISP or the user thinks that the leader–follower game gives an unacceptable solution and therefore decides not to have any business if negotiation fails. In this case, the cooperative game solution is fairer than the leader–follower one. After the starting point is determined, we only need to consider the upper-right quadrant with the starting point as the origin. The problem that remains is simply how to choose a parameter β to determine the fair point on the Pareto boundary.

For the shape of the Pareto boundary shown in this example, the solutions for different $\beta, -1 \leq \beta \leq 1$, are very close. Thus, we can choose either Nash or Raiffa solution as the solution for the problem. With the solution to the leader–follower game as the starting point, the Nash solution R is shown in Fig. 2. To show that arbitration is needed to maintain the fair solution, we consider the following scenario. Suppose point A is the desirable solution (with $c = 4$ and $\rho = 0.9$). Assume that one day, the user thinks that by reducing its demands to $\rho = 0.7$ and moving the solution point to B, it can get a larger utility and therefore it indeed reduces the demand. Noticing this change, the ISP responds by increasing the price to $c = 5$, the solution point then moves to C. The user then will reduce further its demands to increase its utility. The procedure continues until it reaches the solution to the leader–follower game, i.e., the starting point of the cooperative game.

We have shown, by a numerical example, that the cooperative game approach, which considers all the possible outcomes of a game, provides a clear picture for Internet pricing, and that cooperation (or arbitration) is needed in order to achieve a better and fairer (in a sense) solution.

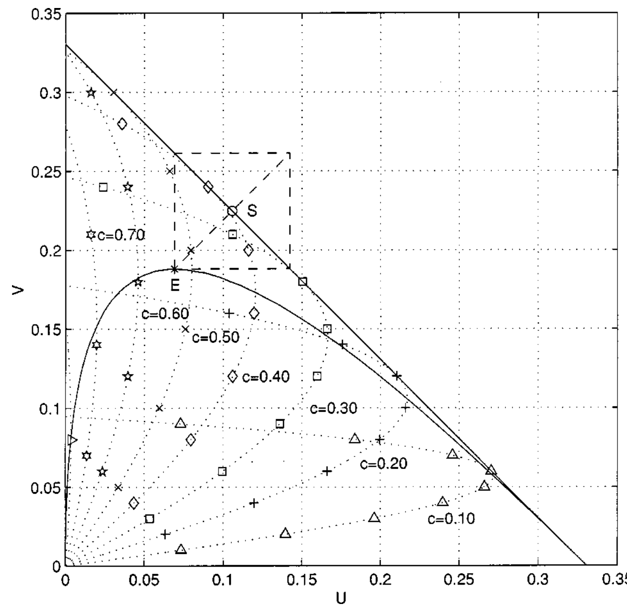

Fig. 3. VoIP1.

IV. More Examples

In this section, we provide a few examples for different discretized functions $f(s)$ representing some possible applications. We will show that the basic insight remains the same. We begin with only one application type. In other words, all the requests have the same maximal acceptable response time s and a user earns g (with $g > c$) for a successful service. The possible applications for this case may be VoIP, which usually has a strict delay requirement, and email, which is delay-insensitive. Requests have arrival rate λ but the user only submits $\alpha\lambda$ to the Internet and thus $\rho = \alpha\lambda/\mu$. By (3) and (4), the user's utility is

$$U(c,\alpha) = \alpha\lambda g\left[1 - e^{-(1-\rho)\mu s}\right] - \alpha\lambda c. \tag{14}$$

The ISP's utility is

$$V(c,\alpha) = \alpha\lambda c. \tag{15}$$

Example 2: In this example, we study three cases with $s = 2\cdot(1/\mu), g = 1; s = 4\cdot(1/\mu), g = 0.5$; and $s = \infty$ and $g = 0.1$. The first two may be viewed as voice applications and the last one, email. Since the real value of μ will not affect the shape of the curves; here, we just normalize it to be 1. For different prices c, the user responds with different traffic intensity ρ, leading to a curve in the utility space. The curves for the above three cases are shown in Figs. 3 to 5, respectively, with the solution for the leader–follower game indicated by E and the Nash solution by S.

The exact values of the solutions are shown in Table I. For the first two cases, both the ISP and the user are better off by negotiation. The last case is very illustrative: in the leader–follower game, the ISP can deduct the best profit by charging a high price, leaving the user almost no profit at all. This extreme case reflects the unfairness of the leader–follower rule. Apparently, no business can be conducted in that way. Therefore, a reasonable approach is to use the origin $(0,0)$ as the starting point of the cooperative game. The Nash solution is simply dividing the profit between the two parties.

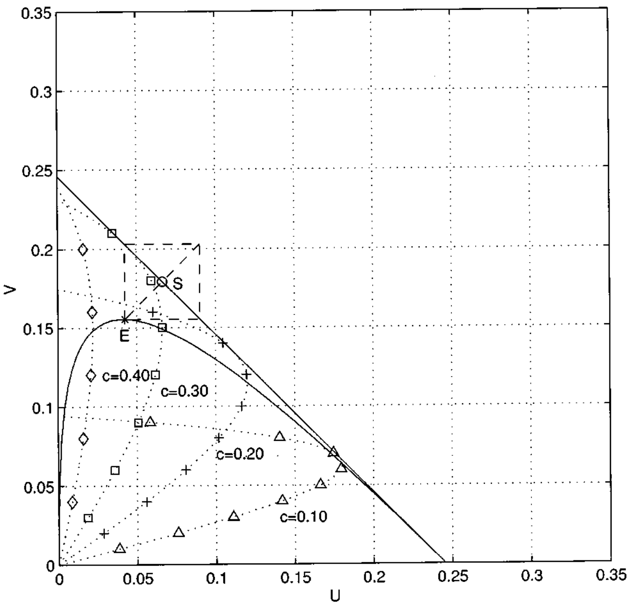

Fig. 4. VoIP2.

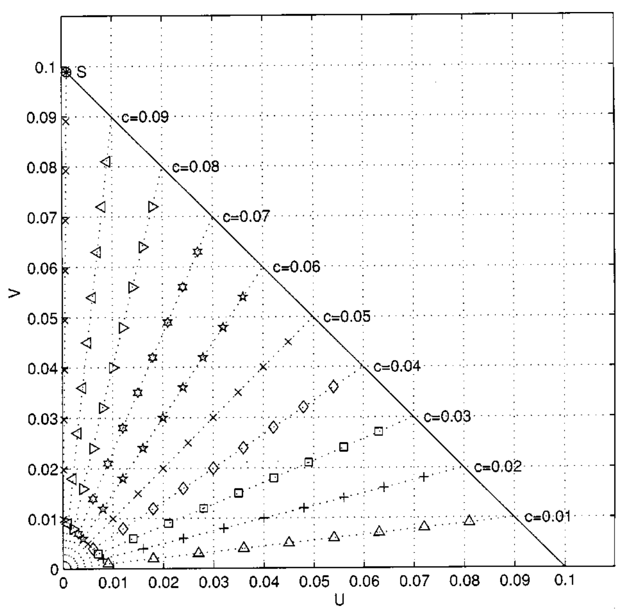

Fig. 5. Email.

In the previous analysis, we simply adopted the first-in-first-out (FIFO) model. This is the "best effort" service model of the current Internet. To model other service disciplines such as those in DiffServ and IntServ, we investigate *priority queueing* in this section. The ISP provides two types of services with two different priorities and prices. Since there are no formulae for the waiting time distributions for priority queueing, the results are obtained by simulation.

Example 3: The user has two types of applications with $s_1 = 2 \cdot (1/\mu), g_1 = 1$, and $s_2 = 4 \cdot (1/\mu), g_2 = 0.5$, with type 1 having a higher priority to be served than type 2 in a nonpreemptive way. Given the traffic intensities ρ_H and ρ_L of these two priorities, we find out the distribution of response time r for these two types of applications by simulation. The service

TABLE I
RESULTS FOR EXAMPLE 2

			the leader-follower game solution				the Nash solution	
$\mu = 1$	s	g	c	ρ	U	V	U	V
VoIP1	$2 \cdot \frac{1}{\mu}$	1	0.53	0.355	0.069129	0.188150	0.105702	0.224723
VoIP2	$4 \cdot \frac{1}{\mu}$	0.5	0.35	0.444	0.042585	0.155400	0.066354	0.179169
Email	∞	0.1	0.1*	1*	0*	0.1*	0*	0.1*

* : very close to

TABLE II
USER'S STRATEGIES (ρ_H, ρ_L) GIVEN THE ISP'S PRICES

c_H c_L	0.1	0.2	0.3	0.4
0.1	0.55 0.05	-	-	-
0.2	0.40 0.20	0.50 0.05	-	-
0.3	0.25 0.40	0.45 0.05	0.45 0.05	-
0.4	0.05 0.55	0.25 0.30	0.40 0.05	0.40 0.05
0.5	0.05 0.55	0.05 0.55	0.35 0.05	0.35 0.05
0.6	0.05 0.55	0.05 0.55	0.05 0.45	0.30 0.05
0.7	0.05 0.55	0.05 0.55	0.05 0.45	0.20 0.05

TABLE III
USER'S AND THE ISP'S UTILITIES (U, V)

c_H c_L	0.1	0.2	0.3	0.4
0.1	0.281 0.060	-	-	-
0.2	0.228 0.100	0.222 0.110	-	-
0.3	0.193 0.115	0.173 0.145	0.168 0.150	-
0.4	0.181 0.075	0.135 0.160	0.124 0.175	0.119 0.180
0.5	0.176 0.080	0.121 0.135	0.086 0.190	0.081 0.195
0.6	0.171 0.085	0.116 0.140	0.068 0.165	0.048 0.200
0.7	0.166 0.090	0.111 0.145	0.063 0.170	0.024 0.160

rate μ is normalized to be 1. Each simulation runs for 100 000 requests. The user's and the ISP's utilities are

$$U(c_H, c_L, \rho_H, \rho_L) = \mu\rho_H g_1 p_H(r \leq s_1) + \mu\rho_L g_2 p_L(r \leq s_2) - \mu\rho_H c_H - \mu\rho_L c_L, \tag{16}$$

$$V(c_H, c_L, \rho_H, \rho_L) = \mu\rho_H c_H + \mu\rho_L c_L. \tag{17}$$

Tables II and III show the user's strategies to maximize its own utility given the ISP's prices and the resulted utilities of the two parties. Table IV shows the leader–follower game solution and Nash solution. Both parties are better off by cooperation.

V. TWO COMPETITIVE ISPs

In this section, we will apply the noncooperative game approach to study the competition between two ISPs, ISP1 and ISP2. This study is based on the simple QoS model proposed in the previous section. ISP1 provides services with bandwidth μ_1 and charges a price c_1, ISP2 with μ_2 and service charge c_2. Let λ be user's request arrival rate. As discussed in the previous section, the requests are characterized by two functions $g(s)$ and

TABLE IV
RESULTS FOR EXAMPLE 3

the leader-follower game solution						the Nash solution			
c_H	c_L	ρ_H	ρ_L	U	V	ρ_H	ρ_L	U	V
0.6	0.4	0.30	0.05	0.048	0.200	0.60	0.05	0.095	0.246

$f(s)$. When a request with maximal acceptable response time s arrives, a user submits it to μ_1 with probability $\alpha(s)$, to μ_2 with probability $\beta(s)$, and discards a request with probability $1-\alpha(s)-\beta(s)$. The arrival rates to μ_1 and μ_2 are $\alpha\lambda$ and $\beta\lambda$, respectively, where

$$\alpha = \int_0^\infty \alpha(s)f(s)\,ds, \quad \beta = \int_0^\infty \beta(s)f(s)\,ds. \tag{18}$$

Let $\rho_1 = \alpha\lambda/\mu_1$ and $\rho_2 = \beta\lambda/\mu_2$ be the traffic intensities of the two links. We use $n_i, i=1,2$, to denote the numbers of requests in the two queues and $p_{i,k}, i=1,2, k=0,1,2,\ldots$, to denote the probabilities that there are $n_i = k$, requests at these two queues, respectively. Then $p_{i,k} = (1-\rho_i)\rho_i^k, i = 1,2, k=0,1,\ldots$. Let $r_i, i=1,2$, be the response times in the two queues. The probability of $r_i > s$ is

$$p(r_i > s) = e^{-(1-\rho_i)\mu_i s}, \qquad i = 1,2. \tag{19}$$

The user's utility is

$$\begin{aligned} U(c_1,c_2,\alpha,\beta) = \lambda \int_0^\infty g(s)\alpha(s)f(s)[1-e^{-(\mu_1-\alpha\lambda)s}]\,ds \\ + \lambda\int_0^\infty g(s)\beta(s)f(s)[1-e^{-(\mu_2-\beta\lambda)s}]\,ds \\ - \alpha\lambda c_1 - \beta\lambda c_2. \end{aligned} \tag{20}$$

The ISPs' utilities are

$$V_1(c_1,c_2,\alpha,\beta) = \alpha\lambda c_1, \quad V_2(c_1,c_2,\alpha,\beta) = \beta\lambda c_2 \tag{21}$$

Let

$$\begin{aligned} h_1(s) &= g(s)\left[1-e^{-(\mu_1-\alpha\lambda)s}\right] \\ h_2(s) &= g(s)\left[1-e^{-(\mu_2-\beta\lambda)s}\right]. \end{aligned}$$

For the sake of discussion, we assume that $\mu_1 - \alpha\lambda \geq \mu_2 - \beta\lambda$. Thus, $h_1(s) \geq h_2(s)$ for all $s \geq 0$. Let

$$h(s) = h_1(s) - h_2(s) = g(s)[e^{-(\mu_2-\beta\lambda)s} - e^{-(\mu_1-\alpha\lambda)s}].$$

For any fixed α and β, we define I_α and I_β as the two subsets of $[0,\infty)$ satisfying

$$I_\alpha \cap I_\beta = \emptyset; \quad \int_{I_\alpha} f(s)\,ds = \alpha; \quad \int_{I_\beta} f(s)\,ds = \beta;$$
$$h(s) \geq h(s'), \text{ if } s \in I_\alpha \text{ and } s' \in I_\beta$$

and

$$h_1(s) \geq h_1(s'), \qquad \text{if } s \in I_\beta \text{ and } s' \notin I_\alpha \cup I_\beta.$$

We can verify that for fixed α and β, $U(c_1,c_2,\alpha,\beta)$ reaches its maximum if

$$\alpha(s) = \begin{cases} 1 & \text{if } s \in I_\alpha \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

$$\beta(s) = \begin{cases} 1 & \text{if } s \in I_\beta \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

The idea is, for fixed α and β, the user should submit to ISP1, who provides services with short response times, those requests that have the largest difference of the expected gains between the two ISPs. Now (20) becomes

$$\begin{aligned} U(c_1,c_2,\alpha,\beta) = \lambda\int_{I_\alpha} g(s)f(s)\left[1-e^{-(\mu_1-\alpha\lambda)s}\right]\,ds \\ + \lambda\int_{I_\beta} g(s)f(s)\left[1-e^{-(\mu_2-\beta\lambda)s}\right]\,ds \\ - \alpha\lambda c_1 - \beta\lambda c_2. \end{aligned} \tag{24}$$

With the above formulation, the problem for this two-ISP case becomes: Given prices c_1 and c_2, the user chooses α and β such that (24) is maximized; the two ISPs set their own prices independently, with the hope that they can improve their own utilities. The goal of our study is to find the Nash equilibrium points of the game, if any, at which both ISPs cannot change their prices without cooperation to obtain more utilities. (The problem is obviously a two-person nonzero sum game, for which Nash equilibrium points may or may not exist. In the following, we will show, by numerical method, that such an equilibrium does exist for our example.)

Even with the exponential functions $g(s)$ and $f(s)$, it is still difficult to find out the optimal sets defined above. Thus, we need to resort to numerical approaches. The first step is to find the functions $\alpha(s)$ and $\beta(s)$ for a fixed pair of α and β that yield the best utility (24) for the user. It is obvious that with fixed α and β, we only need to maximize the sum of the two integrals in (20). Next, we observe that because the expected gain is always positive, the equality in constraints (18) can be changed to "less than or equal to." The problem becomes to optimize

$$\int_0^\infty \alpha(s)g(s)f(s)\left[1-e^{-(\mu_1-\alpha\lambda)s}\right]\,ds + \int_0^\infty \beta(s)g(s)f(s)\left[1-e^{-(\mu_2-\beta\lambda)s}\right]\,ds$$

subject to

$$\int_0^\infty \alpha(s)f(s)\,ds \leq \alpha, \quad \int_0^\infty \beta(s)f(s)\,ds \leq \beta.$$

To analyze numerically, we divide $[0,\infty)$ into small intervals and approximate the integrals by summations. Let Δ be a small positive number and N be a large integer such that the expected gains for $s > N\Delta$ are negligible. For $i = 1,2,\ldots,N$, define $s_i = i\Delta, x_i = \alpha(s_i), y_i = \beta(s_i)$, and

$$\begin{aligned} a_i &= g(s_i)f(s_i)\left[1-e^{-(\mu_1-\alpha\lambda)s_i}\right] \\ b_i &= g(s_i)f(s_i)\left[1-e^{-(\mu_2-\beta\lambda)s_i}\right], \quad c_i = f(s_i). \end{aligned}$$

TABLE V
ISP1'S UTILITIES IN EXAMPLE 4

c_2 c_1	4.00	5.00	6.00	7.00	8.00
4.00	3600	4000	3600	2100	800
5.00	3600	4250	3600	2100	800
6.00	3600	4500	4500	2100	800
7.00	3600	4500	5400	4200	800
8.00	3600	4500	5400	6300	3600

TABLE VI
ISP2'S UTILITIES IN EXAMPLE 4

c_2 c_1	4.00	5.00	6.00	7.00	8.00
4.00	3600	3600	3600	3600	3600
5.00	4000	4250	4500	4500	4500
6.00	3600	3600	4500	5400	5400
7.00	2100	2100	2100	4200	6300
8.00	800	800	800	800	3600

Then for any fixed α and β, the optimization problem becomes a standard linear programming (LP) problem: To maximize

$$\sum_{i=1}^{N}(a_i x_i + b_i y_i) \tag{25}$$

subject to

$$x_i \geq 0, \quad y_i \geq 0, \quad x_i + y_i \leq 1 \tag{26}$$

$$\sum_{i=1}^{N} c_i x_i \leq \frac{\alpha}{\Delta}, \quad \sum_{i=1}^{N} c_i y_i \leq \frac{\beta}{\Delta} \tag{27}$$

x_i and y_i are the probabilities that the user submits the request with maximum acceptable response time s_i to ISP1 and ISP2, respectively. We illustrate the idea with an example.

Example 4: We study the two-ISP case. Both ISPs provide Internet services with the same bandwidth of 1000 packet/s; and $g(s) = 10e^{-0.5s}$ and $f(s) = e^{-s}$. We apply the linear programming (26)–(27) and choose $\Delta = 0.0046$ and $N = 1000$. With these values, the gains for $s > N\Delta$ are less than 0.1% of the maximum gain. Tables V and VI list the two ISPs' utilities under different prices, and Table VII lists the user's utilities.

From Tables V and VI, with $c_1 = 5.00$ and $c_2 = 5.00$, the utilities for both ISPs are 4250. Furthermore, if each ISP changes its price individually, its utility will be reduced. That is, no ISP has any incentive to move away from this price without the cooperation of the other. Therefore, $c_1 = c_2 = 5.00$ is a Nash equilibrium.

It is interesting to note that the point $c_1 = c_2 = 6.00$, with utilities for both ISPs being 4500, has a similar property. However, there is one difference: the utility for ISP1 with $c_1 = 5.00$ and $c_2 = 6.00$ and that for ISP2 with $c_1 = 6.00$ and $c_2 = 5.00$ have the same value of 4500. Therefore, if ISP1 cuts its price to $c_1 = 5.00$ because it thinks that it can still maintain the same utility 4500, then the point will move to $c_1 = 5.00$ and $c_2 = 6.00$, at which ISP2 only gets 3600. This will force ISP2 also reduce its price to $c_2 = 5.00$, i.e., the Nash equilibrium. We can see that $c_1 = c_2 = 6.00$ is not a stable point. However, if the two ISPs work in cooperation, they may try to maximize the sum of their utilities and then divide the total utility among themselves. Then the maximum is obtained at $c_1 = c_2 = 6.00$. This shows that in this two-person game case, cooperation is also better for both ISPs. Of course, the utility of the user decreases.

TABLE VII
USER'S UTILITIES IN EXAMPLE 4

c_2 c_1	4.00	5.00	6.00	7.00	8.00
4.00	6601	5782	5084	4622	4454
5.00	5782	4882	4184	3722	3554
6.00	5084	4184	3284	2822	2654
7.00	4622	3722	2822	1922	1754
8.00	4454	3554	2654	1754	881

The numerical results show that the probabilities x_i and y_i are indeed either 0 or 1. □

Finally, there are $2N$ variables in the LP problem. If N is large, the problem may be computationally complicated. Fortunately, the functions $f(s)$ and $g(s)$ are usually smooth, and in many cases even may take only a few values. The size of Δ thus depends largely on $1-e^{-(\mu_1-\alpha\lambda)s}$ and $1-e^{-(\mu_2-\beta\lambda)s}$. As shown in the example, $N = 1000$ already leads to a good solution. Therefore, the LP approach is computationally feasible for practical implementation.

VI. DISCUSSION AND CONCLUSION

In this paper, we proposed a cooperative game approach to Internet pricing. With a simple QoS model, we demonstrated that the leader–follower game may lead to a solution that is not Pareto optimal and in some sense may be unfair, and the cooperative approach can provide a better solution. The cooperative game approach between the ISP and the user is usually difficult to maintain. The practical implication is that some regulations or arbitration will be helpful for reaching a fairer and more efficient solution. We also applied the QoS model to study the competition between two ISPs in a numerical example and found the Nash equilibrium point from which the two ISPs would not move without cooperation (a rigorous study is needed). The proposed approaches can be applied to other Internet pricing problems, such as the PMP scheme; see [13] for problem description and [3] for a game-theory-based study.

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