Overview

- Introduction
- Noncooperative and cooperative games
- One ISP with a single class of users
- Conclusions
Introduction

• Why we want to do pricing
  1. To increase competition among different service providers
  2. To reduce congestion or to control the traffic intensity

• How to do pricing using game-theoretical approach
  1. Leader-follower game (non-cooperative game)
  2. Bargain problem (cooperative game)
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Noncooperative game (leader-follower problem)

- Two players of the game: the ISP (leader) and the user (follower)
- Outcome of the game: the utilities of each player (profit of the ISP and user)
- $c$ is the price that the ISP announces
- With price $c$, user determines its demand $r$
- ISP’s utility is defined as $U(c, r)$
- User’s utility is defined as $V(c, r)$
- User chooses $r(c) = \arg\{\max_{r \in R} [U(c, r(c))]\}$
- ISP chooses a price $c^* = \arg\{\max_{c \in C} [V(c, r(c))])\}$
Cooperative game (bargaining problem)

- Two players (bargainers) of the game: the ISP and the user
- Each pair of policies \((c, r)\) corresponds to one point \((u = U(c, r), v = V(c, r))\) in the utility space
- The set of all policies \((c, r)\) is defined as the bargaining set \(S\).
- \(S\) is a convex set with random policies.
- Starting point \(s_0 = (u_0, v_0) \in S\)
- Only points in \(S\) with \(u \geq u_0\) and \(v \geq v_0\) should be considered.
- Solution of the game \((S, s_0)\): the fair point chosen by the two players.
Cooperative game (fairness criteria)

- Three basic axioms
  1. Symmetry: If $S$ is symmetric with respect to the axis $u = v$ and the starting point is on this axis, then the solution is also on this axis.
  2. Pareto optimality: The solution is on the Pareto boundary.
  3. Invariance with respect to utility transformations: The solution to bargaining problem $(f(S), f(s_0), f)$ is any positive affine transformation, and $f(S) = \{ \text{all } f(s) : s \in S \}$. 
Cooperative game (fairness criteria)

- Independence of irrelevant (Nash’s solution)
  1. If $s^*$ is the solution of $(S, s_0)$, and $S' \subseteq S, s^* \in S'$, then $s^*$ is also the solution of $(S', s_0)$.
  2. Maximize the value of the product of the two utilities $u \times v$

- Normalize the problem
  1. With the third axiom, any problem can be normalize into a problem with starting point $(0, 0)$ and
     \[ \max\{u : s = (u, v) \in S\} = \max\{v : s = (u, v) \in S\} = 1 \]
  2. Define preference functions
     \[ w_1 = u + \beta(1 - v), \ w_2 = v + \beta(1 - u), \ -1 \leq \beta \leq 1 \]
3. Replace utility function by preference function, we obtain a set of solutions (parameterized by $\beta$)

$$u^*, v^* = \arg \{ \max_{u,v} [w_1 \times w_2] \}$$

4. Nash’s solution is the special case when $\beta = 0$. The bargaining problem becomes to determine a value $\beta$ that is acceptable for both players. Once $\beta$ is agreed upon, a fair solution can be uniquely determined.
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Define a QoS model

- QoS requirement is measured by response time
- One ISP with link capacity $\mu$
- User’s requests come from a Poisson process with arrival rate $\lambda$
- Each request is charged a fee $c$ by the ISP. It requires a unit bandwidth to serve for an exponentially distributed time with a unit mean.
- We assume each request has a maximal acceptable response time, denoted as $s$ which has a density function $f(s)$. If the real response time is greater than $s$, it is considered as a failure.
- When a request with maximal response time $s$ arrives, the user submit it to the Internet with probability $\alpha(s)$. And the user earns $g(s) \geq 0$ for a successful service.
Express the utility functions

- The arrival rate to the Internet is $a\lambda$, where
  \[ a = \int_0^\infty a(s)f(s)ds. \]

- The link can be modeled as M/M/1 queue with arrival rate $a\lambda$ and service rate $\mu$.

- Let $r$ denote the response times in the queue. Then for any given $s$, the probability of $r > s$ is
  \[ p(r > s) = e^{-(1-\rho)\mu s} = e^{-(\mu-a\lambda)s}. \]
Express the utility functions

- The user’s utility is

\[ U(c, \alpha) = \lambda \int_0^\infty g(s)\alpha(s)f(s)[1 - e^{-(\mu - \alpha \lambda)s}] ds - \alpha \lambda c. \]

- The ISP’s utility is

\[ V(c, \alpha) = \alpha \lambda c. \]
• Assume $\alpha$ is fixed.
• We define

$$I_\alpha = \{I_\alpha \subset [0, \infty) : h(s) \geq h(s') \text{ if } s \in I_\alpha \text{ and } s' \notin I_\alpha; \int_{I_\alpha} f(s)ds = \alpha\}$$

where

$$h(s) = g(s)[1 - e^{-(\mu-\alpha\lambda)s}].$$

• $h(s)$ is the expected gain for a request with maximal response time $s$. 

**Eliminate $\alpha(s)$**
Internet Pricing With a Game Theoretical Approach: Concepts and Examples

• So when \( h(s) \) is continuous and has only one peak, \( \alpha(s) \) is

\[
\alpha(s) = \begin{cases} 
1 & \text{if } s_1 \leq s \leq s_2 \\
0 & \text{otherwise}
\end{cases}
\]

where \( s_1 \) and \( s_2 \) satisfy

\[
\alpha = \int_{s_1}^{s_2} f(s) \, ds
\]

and

\[
h(s_1) = h(s_2), \text{ if } s_1 > 0; \ h(s_1) \geq h(s_2), \text{ if } s_1 = 0
\]

• Then utility function becomes

\[
U(c, s_1, s_2) = \lambda \int_{s_1}^{s_2} g(s) f(s) [1 - e^{-(\mu - \alpha \lambda)s}] \, ds - \alpha \lambda c.
\]
Apply to Leader-Follower Game

• Fix for price $c$ first.
• The use select for their best utility for certain $c$.
• The choices from the user, the ISP choose the final price $c^*$.
Apply to Cooperative Game

- On the Pareto boundary, the social welfare defined as
  \[ S(\alpha) = U(c, \alpha) + V(c, \alpha) \]
  is maximized.
- Find the fixed traffic intensity \( \alpha \) for Pareto boundary, since it is only depend on \( \alpha \).
- Determine \( \beta \) the policy of the Pareto boundary.
- Arbitrate a starting point.
- Find the solution on the Pareto boundary.
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Conclusion

• Lead-Follower solutions may not be on the Pareto boundary
  1. \( (\partial/\partial \alpha)U(c, \alpha) = 0 \) for a fixed \( c \)
  2. Therefore on this point, \( dS(\alpha)/d\alpha = \lambda c \)

• Arbitration is necessary
  1. User want to change the plan
  2. ISP changes the price versus the user’s change
  3. Final stable point goes back to the solution of the lead-follower problem