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Distributed Beamforming and Rate Allocation in Multi-Antenna Cognitive Radio Networks

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***Abstract*—We consider decentralized multi-antenna cognitive radio networks where secondary (cognitive) users are granted simultaneous spectrum access along with license-holding (primary) users. We investigate the problem of designing beamformers for the secondary users by maximizing the minimum rate, subject to a limited sum-power budget and constraints on the interference level imposed on each primary receiver. We consider two scenarios: the first one allows only single-user decoding at each secondary receiver whereas in the second case each secondary receiver is allowed to employ advanced multi-user decoding and is free to decode any subset of secondary users. We provide an optimal distributed algorithm for the first scenario and an explicit formulation of the optimization problem corresponding to the second scenario. This problem however is non-convex and hence cannot be efficiently solved even in a centralized setup. As a remedy, we suggest a two-step approach. In particular, the beamformers are first designed assuming single user decoding at each secondary receiver. An optimal distributed low-complexity algorithm is then proposed to allocate excess rates to the secondary users, which are made possible due to the use of advanced decoders at the secondary receivers. Simulation results demonstrate the gains yielded by the optimal beamformers as well as the rate allocation algorithms.**

I. Introduction

In classical cognitive radio systems the secondary users can only transmit in "white spaces" which denote the frequency bands (or time intervals) where the primary (or licensed) users are silent [1]. On the other hand, in generalized cognitive radio systems, the secondary users can also transmit simultaneously with primary users, as long as certain co-existence constraints are satisfied [2]. Clearly the latter systems can achieve higher spectral efficiencies but at the expense of additional side-information at the secondary users and increased signaling overhead. We consider decentralized multi-antenna cognitive radio networks where secondary transceivers can co-exist with primary ones. However, in our setup no secondary transmitter has access to any primary user's message. Instead, each secondary transmitter employs beamforming to communicate with its desired receiver while ensuring that the interference seen by each primary receiver does not exceed a specified level (interference margin).

Our goal is to design optimal beamformers for the secondary users in a distributed fashion in order to maximize the smallest weighted rate among secondary users, subject to sum-power and interference margin constraints. We consider two scenarios. In the first one each secondary receiver employs a minimum mean-square error (MMSE) decoder and attempts to decode only the signal transmitted by its designated transmitter after suppressing the remaining signals via linear filtering (a.k.a. single user decoding). We propose an efficient *distributed* algorithm that yields a globally optimal set of beamformers. The resulting beamformers are optimal for practical networks where the exchange of codebooks among secondary transceivers is not possible or where advanced decoding at the secondary receivers is not feasible due to complexity constraints.

In the second scenario, we assume that advanced multi-user decoding is used at the secondary receivers. There is no restriction on the decoding complexity, i.e., each receiver is allowed to decode any arbitrary subset of other secondary users, if such decoding is deemed beneficial for decoding its respective user. We provide an explicit formulation of the optimization problem in this scenario. However, the problem is non-convex and hence cannot be efficiently solved even in a centralized setup. As a remedy, we adopt a two-step approach. In the first step, we obtain a set of beamformers which is optimal under MMSE decoding at each receiver. In the second step, we consider the allocation of excess rates to the secondary users beyond their minimum rates (achieved via MMSE decoding), such that some notion of fairness is maintained and all the users remain decodable at their respective receivers. In particular, we propose a distributed approach for allocating such excess rates based on the notion of *weighted max-min* fairness, which yields rate increments for all secondary users beyond their minimum specified rates, such that the minimum normalized rate increment among all secondary users is maximized. A key feature of our distributed rate allocation algorithm is that the complexity at each secondary receiver is only polynomial in the number of secondary users.

Finally, we remark that our restriction of allowing each secondary user to use one codebook is in contrast to the optimal two-codebooks-per-user strategy proposed for the interference channel in [3] and [4]. While users with single-codebooks can achieve lower rates, they are more practical and consequently such Gaussian interference channels have been recently investigated in [5].

II. System Descriptions

A. System Model

We consider a decentralized cognitive network comprising of M_s secondary transmitter-receiver pairs co-existing with

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M_p primary transceiver pairs via *underlaid* [6] spectrum access, wherein the interference seen by each primary receiver is kept below a certain level. The secondary transceivers form a multi-antenna Gaussian interference channel (GIC) in which M_s transmitters, each equipped with N_s transmit antennas, communicate with their designated single-antenna receivers. Each one of the primary transmitters has N_p transmit antennas and the primary receivers have one receive antenna each. We assume quasi-static flat fading channels and denote the channels from the j^{th} secondary transmitter to the i^{th} secondary receiver and the i^{th} primary receiver by $\boldsymbol{h}_{i,j}^{s,s} \in \mathbb{C}^{1\times N_s}$, and $\boldsymbol{h}_{i,j}^{p,s} \in \mathbb{C}^{1\times N_s}$, respectively. Further, we denote the channels from the j^{th} primary transmitter to the i^{th} secondary receiver and the i^{th} primary receiver by $\boldsymbol{h}_{i,j}^{s,p} \in \mathbb{C}^{1\times N_p}$ and $\boldsymbol{h}_{i,j}^{p,p} \in \mathbb{C}^{1\times N_p}$, respectively.

Let x_i^s and x_i^p be scalar complex valued random variables with unit power representing the information symbols of the i^{th} secondary and primary transmitters, respectively and let $\boldsymbol{w}_i^s \in \mathbb{C}^{N_s\times 1}$ and $\boldsymbol{w}_i^p \in \mathbb{C}^{N_p\times 1}$ denote their respective beamforming vectors. The received signal at the i^{th} secondary receiver is

$$y_i^s = \sum_{j=1}^{M_s} \boldsymbol{h}_{i,j}^{s,s}\boldsymbol{w}_j^s x_j^s + \sum_{j=1}^{M_p} \boldsymbol{h}_{i,j}^{s,p}\boldsymbol{w}_j^p x_j^p + z_i^s, \qquad (1)$$

where $z_i^s \in \mathbb{C}$ and $z_i^p \in \mathbb{C}$ are the additive white Gaussian noise terms with variances σ_i^s and σ_i^p, respectively. No primary (secondary) receiver tries to decode the signal intended for any secondary (primary) user.

B. Problem Statement

We denote the rate assigned to the i^{th} secondary user by R_i and and we will say that the rate vector $\boldsymbol{R} \triangleq [R_1, \dots, R_{M_s}]^T$ is decodable for the given channel coefficients, choice of transmit beamformers and decoders employed by the secondary receivers, if for any rate vector $\tilde{\boldsymbol{R}} \prec \boldsymbol{R}$ and any arbitrarily fixed $\epsilon > 0$, there exists a set of M_s codes such that each secondary receiver can decode its desired user (secondary transmitter) with a probability of error no greater than ϵ. The interference level seen by the i^{th} primary receiver due to the secondary transmissions is denoted by J_i and note that we have $J_i = \sum_{j=1}^{M_s} |\boldsymbol{h}_{i,j}^{p,s}\boldsymbol{w}_j^s|^2$. The i^{th} primary receiver specifies a parameter ϵ_i which is the maximum interference it can tolerate from the secondary transmissions. We are interested in maximizing the worst-case secondary weighted rate subject to secondary weighted sum-power constraint and the constraint that the interference seen by the i^{th} primary receiver must not exceed ϵ_i, i.e.,

$$\mathcal{S}(P_0) = \begin{cases} \max_{\{\boldsymbol{R},\boldsymbol{w}_i^s\}} & \min_i \frac{R_i}{\rho_i} \\ \text{s.t.} & \sum_{i=1}^{M_s} \alpha_i ||\boldsymbol{w}_i^s||^2 \leq P_0 \\ & J_i \leq \epsilon_i \quad \text{for } i = 1, \dots, M_p, \\ & \boldsymbol{R} \text{ is decodable} \end{cases} \qquad (2)$$

$\{\rho_i\}$ and $\{\alpha_i\}$ are all positive real values that account for weighting the individual rates and powers, respectively. Let $\boldsymbol{\rho} = [\rho_1, \cdots, \rho_{M_s}], \boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_{M_s}]$ and $\boldsymbol{\epsilon} = [\epsilon_1, \cdots, \epsilon_{M_p}]$. In our formulations, solving $\mathcal{S}(P_0)$ is facilitated by solving another optimization problem given below.

$$\mathcal{P}(\boldsymbol{\rho}) = \begin{cases} \min_{\{\boldsymbol{w}_i^s\}} & \sum_{i=1}^{M_s} \alpha_i ||\boldsymbol{w}_i^s||^2 \\ \text{s.t.} & R_i \geq \rho_i \quad \text{for } i = 1, \dots, M_s \\ & J_i \leq \epsilon_i \quad \text{for } i = 1, \dots, M_p, \\ & \boldsymbol{R} \text{ is decodable} \end{cases} \qquad (3)$$

Note that the decodability of $\boldsymbol{R}$ clearly depends on the choice of decoders employed at the secondary receivers. For the decoders considered in this paper, we have the following useful result, where we assume that $\mathcal{P}(\boldsymbol{\rho})$ is feasible.

Theorem 1: For given $\boldsymbol{\rho}, \boldsymbol{\alpha}, \boldsymbol{\epsilon}$, the optimization problems $\mathcal{S}(P_0)$ and $\mathcal{P}(\boldsymbol{\rho})$ are related as

$$\mathcal{S}\Big(\mathcal{P}(\boldsymbol{\rho})\Big) = 1 \quad \text{and} \quad \mathcal{P}\Big(\mathcal{S}(P_0)\cdot\boldsymbol{\rho}\Big) \leq P_0.$$

The proofs of Theorem 1 and Theorem 2 given in Section III, follow from the tools developed in [7] which considers precoder design in MIMO systems.

III. MMSE Receivers: Beamformer Optimization

In this section, we assume that each secondary receiver uses the MMSE single-user decoder which only decodes the desired user and treats the other users as Gaussian interferers. Then, for a given choice of beamformers, the rate that can be achieved for the i^{th} user is $R_i = \log(1+\mathsf{SINR}_i)$, where SINR_i denotes the signal-to-interference-plus-noise-ratio at the i^{th} secondary receiver and is given by

$$\mathsf{SINR}_i = \frac{|\boldsymbol{h}_{i,i}^{s,s}\boldsymbol{w}_i^s|^2}{\sum_{j\neq i}|\boldsymbol{h}_{i,j}^{s,s}\boldsymbol{w}_j^s|^2 + \sum_j |\boldsymbol{h}_{i,j}^{s,p}\boldsymbol{w}_j^p|^2 + \sigma_i^s}. \qquad (4)$$

We first provide a distributed algorithm for obtaining $\mathcal{P}(\boldsymbol{\rho})$. Then, by exploiting the connection between the problems $\mathcal{S}(P_0)$ and $\mathcal{P}(\boldsymbol{\rho})$ as established in Theorem 1, we use this algorithm to obtain another distributed algorithm for determining $\mathcal{S}(P_0)$. By defining $\gamma_i = 2^{\rho_i} - 1$ and $\boldsymbol{\gamma} = [\gamma_1, \cdots, \gamma_{M_s}]$, the optimization problem $\mathcal{P}(\boldsymbol{\rho})$ is equivalent to

$$\mathcal{P}(\boldsymbol{\gamma}) = \begin{cases} \min_{\{\boldsymbol{w}_i^s\}} & \sum_{i=1}^{M_s} \alpha_i ||\boldsymbol{w}_i^s||^2 \\ \text{s.t.} & \mathsf{SINR}_i \geq \gamma_i, \quad i = 1, \dots, M_s \\ & J_i \leq \epsilon_i, \quad i = 1, \dots, M_p \end{cases} \qquad (5)$$

Theorem 2: Strong duality holds for the problem $\mathcal{P}(\boldsymbol{\gamma})$ and therefore it can be solved via its Lagrangian dual.

Next, for any set of channel realizations $\{\boldsymbol{h}_{i,j}^{s,s},\ \boldsymbol{h}_{i,j}^{s,p},\ \boldsymbol{h}_{i,j}^{p,s}\ \&\ \boldsymbol{h}_{i,j}^{p,p}\}$ and given sets of $\{\alpha_i\}_{i=1}^{M_s}$, $\{\gamma_i\}_{i=1}^{M_s}$ and $\{\epsilon_i\}_{i=1}^{M_p}$ we define $\tilde{\boldsymbol{w}}_i^s \triangleq \sqrt{\alpha_i}\boldsymbol{w}_i^s$ and

$$\tilde{\boldsymbol{h}}_{i,j}^{p,s} \triangleq \frac{\boldsymbol{h}_{i,j}^{p,s}}{\sqrt{\alpha_i}}, \quad \tilde{\boldsymbol{h}}_{i,i}^{s,s} \triangleq \frac{\boldsymbol{h}_{i,i}^{s,s}}{\sqrt{\alpha_i\gamma_i}}, \quad \tilde{\boldsymbol{h}}_{i,j}^{s,s} \triangleq \frac{\boldsymbol{h}_{i,j}^{s,s}}{\sqrt{\alpha_i}} \text{ for } i \neq j. \qquad (6)$$

By introducing $a_i^s \triangleq \sum_j |\boldsymbol{h}_{i,j}^{s,p}\boldsymbol{w}_j^p|^2 + \sigma_i^s$ for $i = 1, \dots, M_s$ the *partial* Lagrangian function of (5) for the non-negative set of multipliers $\boldsymbol{\lambda} \triangleq [\lambda_1 \dots \lambda_{M_p}]$, associated with the primary interference margins, is given by

$$L(\{\tilde{\boldsymbol{w}}_i^s\}, \boldsymbol{\lambda}) = \sum_{i=1}^{M_s} ||\tilde{\boldsymbol{w}}_i^s||^2 + \sum_{j=1}^{M_p} \lambda_j \Bigg[\sum_{i=1}^{M_s} |\tilde{\boldsymbol{h}}_{j,i}^{p,s}\tilde{\boldsymbol{w}}_i^s|^2 - \epsilon_j\Bigg]$$

$$= \sum_{i=1}^{M_s} (\tilde{\boldsymbol{w}}_i^s)^H \left[\boldsymbol{I} + \sum_{j=1}^{M_p} (\tilde{\boldsymbol{h}}_{j,i}^{p,s})^H \tilde{\boldsymbol{h}}_{j,i}^{p,s} \lambda_j \right] \tilde{\boldsymbol{w}}_i^s - \sum_{j=1}^{M_p} \lambda_j \epsilon_j.$$

Next, we consider the following Cholesky decomposition

$$\boldsymbol{U}_i^H \boldsymbol{U}_i = \boldsymbol{I} + \sum_{j=1}^{M_p} (\tilde{\boldsymbol{h}}_{j,i}^{p,s})^H \tilde{\boldsymbol{h}}_{j,i}^{p,s} \lambda_j, \quad (7)$$

and define $\hat{\boldsymbol{w}}_i^s \triangleq \boldsymbol{U}_i \tilde{\boldsymbol{w}}_i^s$ and $\hat{\boldsymbol{h}}_{j,i}^{s,s} \triangleq \tilde{\boldsymbol{h}}_{j,i}^{s,s} \boldsymbol{U}_i^{-1}$. As a result, the Lagrange dual function is given by

$$g(\boldsymbol{\lambda}) = \begin{cases} \min_{\{\hat{\boldsymbol{w}}_i^s\}} & \sum_{i=1}^{M_s} (\hat{\boldsymbol{w}}_i^s)^H \hat{\boldsymbol{w}}_i^s - \sum_{j=1}^{M_p} \lambda_j \epsilon_j \\ \text{s.t.} & \forall i \;\; \frac{|\hat{\boldsymbol{h}}_{i,i}^{s,s} \hat{\boldsymbol{w}}_i^s|^2}{\sum_{j \neq i} |\hat{\boldsymbol{h}}_{i,j}^{s,s} \hat{\boldsymbol{w}}_j^s|^2 + a_i^s} \geq 1 \end{cases} \quad (8)$$

By invoking the result of Theorem 2, we conclude that $\max_{\boldsymbol{\lambda} \succeq \boldsymbol{0}} g(\boldsymbol{\lambda}) = \mathcal{P}(\boldsymbol{\gamma})$. Now, note that for any given set of $\{\lambda_i\}$, the term $\sum_{j=1}^{M_p} \lambda_j \epsilon_j$ is constant. Therefore the problem in (8) is equivalent to the optimization problem in [8] which examines downlink (MISO) transmission in a multi-cell system, and considers optimizing the beamformers jointly in order to minimize the weighted sum-power under the constraint that for all the specified users an $\mathsf{SINR} \geq 1$ is ensured. By following the same lines as in [8], which exploits uplink-downlink duality, we can show that for any given set of $\{\lambda_i\}$, the problem in (8) can be solved in a distributed way. Consequently, we are able to find the optimal set of $\{\hat{\boldsymbol{w}}_i^s\}$ in (8) for any given $\{\lambda_i\}$, in a distributed fashion. In order to maximize $g(\boldsymbol{\lambda})$ over all $(\lambda_1, \ldots, \lambda_{M_p}) \in \mathbb{R}_+^{M_p}$, we first note that it is concave in $\boldsymbol{\lambda}$. Moreover, since $g(\boldsymbol{\lambda})$ is a non-explicit function of $\boldsymbol{\lambda}$, we utilize the *subgradient method* which provides a simple algorithm for minimizing non-differentiable convex functions [9] and apply it on $\hat{g}(\boldsymbol{\lambda}) \triangleq -g(\boldsymbol{\lambda})$ (which is convex). According to this method, in order to minimize $g(\boldsymbol{\lambda}) : \mathbb{R}^{M_s} \to \mathbb{R}$, the following iterative method is deployed

$$\boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k - \frac{1}{k} \boldsymbol{s}^{(k)},$$

where $\boldsymbol{s}^{(k)}$ is *any* subgradient of $\hat{g}$ at $\boldsymbol{\lambda}^k$. Recall that the subgradient of $\hat{g}$ at $\boldsymbol{\lambda}^k$ is any vector $\boldsymbol{s}^{(k)}$ that satisfies [9]

$$\hat{g}(\boldsymbol{\lambda}') \geq \hat{g}(\boldsymbol{\lambda}^k) + (\boldsymbol{s}^{(k)})^T (\boldsymbol{\lambda}' - \boldsymbol{\lambda}^k) \quad \forall \boldsymbol{\lambda}'.$$

It can be readily shown that $\boldsymbol{s}^{(k)} \triangleq [s_1^{(k)}, \ldots, s_{M_p}^{(k)}]^T$ where

$$s_j^{(k)} = \epsilon_j - \sum_{i=1}^{M_s} |\tilde{\boldsymbol{h}}_{j,i}^{p,s} \boldsymbol{U}_i^{-1} \hat{\boldsymbol{\omega}}_i^s|^2, \text{ for } j = 1, \ldots, M_p, \quad (9)$$

and $\{\hat{\boldsymbol{\omega}}_i^s\}$ is an optimal solution of (8) at $\boldsymbol{\lambda}^k$, is a valid subgradient. Thus, we can find the minimum of $-g(\boldsymbol{\lambda})$ or the maximum of $g(\boldsymbol{\lambda})$ which is the optimal value of $\mathcal{P}(\boldsymbol{\gamma})$. The following distributed algorithm summarizes the steps involved in obtaining $\mathcal{P}(\boldsymbol{\gamma})$.

Algorithm 1- Solving $\mathcal{P}(\boldsymbol{\gamma})$

```
Input α, γ, ε, and {h^{s,s}_{i,j}}, {h^{s,p}_{i,j}}, {h^{p,s}_{i,j}}, {h^{p,p}_{i,j}}
Define {h̃^{s,s}_{i,j}}, {h̃^{p,s}_{i,j}} as specified in (6)
Initialize λ and k = 1
repeat
  Construct U_i as in (7); obtain ĥ^{s,s}_{j,i} = h̃^{s,s}_{j,i} U_i^{-1}
  Solve g(λ) using the distributed algorithm of
  [8] and find {ŵ^s_i}
  Obtain {w̃^s_i} using transformation w̃^s_i = U_i^{-1} ŵ^s_i
  Calculate the subgradient s^{(k)} as in (9)
  Update λ^{(k+1)} = λ^{(k)} − (1/k) s^{(k)} and k ← k + 1
until convergence
Output {w^s_i} = {(1/√α_i) w̃^s_i} and   P(γ) = Σ_i α_i ||w^s_i||^2
```

Finally, using the relationship established in Theorem 1, a bi-section search can now be utilized to determine $\mathcal{S}(P_0)$ in a distributed fashion, as proposed in Algorithm 2.

Algorithm 2- Solving $\mathcal{S}(P_0)$

```
Input α, ρ, ε, δ and {h^{s,s}_{i,j}}, {h^{s,p}_{i,j}}, {h^{p,s}_{i,j}}, {h^{p,p}_{i,j}}
Initialize ρ_min, ρ_max
  ρ_0 ← ρ_min,  γ ← 2^{ρ_0 ρ} − 1
repeat
  Solve P(γ) using Algorithm 1
  if P_0 ≥ P(γ)
    ρ_min ← ρ_0; update {w^s_i}
  else
    ρ_max ← ρ_0
  end if
  ρ_0 ← (ρ_min + ρ_max)/2 and γ ← 2^{ρ_0 ρ} − 1
until ρ_max − ρ_min ≤ δ
Output S(P_0) = ρ_min and {w^s_i}
```

IV. Unconstrained Group Decoders

In this section we assume that each secondary receiver is equipped with advanced multi-user decoders. Note that each secondary receiver is interested in recovering only the codeword transmitted by its designated secondary transmitter. On the other hand, the maximum likelihood (ML) decoder jointly decodes all transmitted codewords and is optimal only with respect to the joint error probability. As a better alternative to the ML decoder, we consider the unconstrained group decoder (UGD) at each secondary receiver [10]. The UGD may decode the designated user jointly with any arbitrary subset of other users, while treating the rest as Gaussian interferers. Let us consider the optimization problem in (2) and for the sake of simplicity, let us assume identical rate weighting for all secondary users, i.e., $\rho = \rho_i, \ \forall \ i$. Then, without loss of optimality we can restrict our attention to symmetric rate allocation $R_i = R, \forall \ i$.

Remark 1: The i^{th} secondary user is decodable at the i^{th} secondary receiver using the UGD, if and only if there exists a subset of users $\mathcal{U} \subseteq \{1, \ldots, M_s\}$ such that $i \in \mathcal{U}$ and for all $\mathcal{A} \subseteq \mathcal{U}$

$$\log \left(1 + \frac{\sum_{j \in \mathcal{A}} |\boldsymbol{h}_{i,j}^{s,s} \boldsymbol{w}_j^s|^2}{\sum_{j \notin \mathcal{U}} |\boldsymbol{h}_{i,j}^{s,s} \boldsymbol{w}_j^s|^2 + \sum_j |\boldsymbol{h}_{i,j}^{s,p} \boldsymbol{w}_j^p|^2 + \sigma_i^s} \right) \geq |\mathcal{A}| R.$$

We offer the following lemma.

Lemma 1: If the i^{th} secondary user is decodable by the i^{th} secondary receiver, then all users $k \in \{1,\ldots,M_s\}$ such that $|\boldsymbol{h}^{s,s}_{i,k}\boldsymbol{w}^s_k| \geq |\boldsymbol{h}^{s,s}_{i,i}\boldsymbol{w}^s_i|$ are also decodable at the i^{th} secondary receiver.

The lemma above conveys that for finding a decodable subset containing the i^{th} user, we do not have to exhaustively search all possible subsets and instead can only consider those that contain i as well as all k such that $|\boldsymbol{h}^{s,s}_{i,k}\boldsymbol{w}^s_k| \geq |\boldsymbol{h}^{s,s}_{i,i}\boldsymbol{w}^s_i|$.

Let $\pi_i(\cdot)$ be a permutation operator on the indices of the users such that

$$|\boldsymbol{h}^{s,s}_{i,\pi_i(1)}\boldsymbol{w}^s_{\pi_i(1)}| \geq \cdots \geq |\boldsymbol{h}^{s,s}_{i,\pi_i(M_s)}\boldsymbol{w}^s_{\pi_i(M_s)}|. \qquad (10)$$

Remark 2: Any subset of users $\{\pi_i(1),\ldots,\pi_i(p)\}$, where $\pi_i^{-1}(i) \leq p \leq M_s$, is decodable at the i^{th} secondary receiver if and only if $f(i,p) \geq R$ where $f(i,p)$ is defined in (12). Consequently, the i^{th} secondary user is decodable if and only if $\max_{\pi_i^{-1}(i)\leq p\leq M_s} f(i,p) \geq R$. Therefore, the optimization problem $\mathcal{S}(P_0)$ is equivalent to

$$\begin{cases} \max_{\{\boldsymbol{w}^s_i\}} & R \\ \text{s.t.} & \sum_{i=1}^{M_s} \alpha_i ||\boldsymbol{w}^s_i||^2 \leq P_0 \\ & \max_{\pi_i^{-1}(i)\leq p\leq M_s} f(i,p) \geq R \quad \forall i \\ & J_i \leq \epsilon_i, \quad i=1,\ldots,M_p. \end{cases}$$

This beamforming optimization problem for UGDs is a non-linear non-convex problem and hence an optimal solution cannot be guaranteed even in a centralized setup. Note that in the case that ρ_i are not identical, the problem is even more involved. Therefore, in the following sections we propose an alternative two-stage approach.

V. UGDs: Fair Rate Allocation

Let us denote the optimal beamforming vectors obtained by using Algorithm 2 by $\{\boldsymbol{w}^*_i\}$. Further, let $\boldsymbol{R}^{\min} = [R_1^{\min},\ldots,R_{M_s}^{\min}]$ be the vector of rates achieved using MMSE decoders, henceforth referred to as the minimum rate vector. In this section, we consider increasing the rates of all users based on some pre-determined priorities reflected by the factors $\{\rho_i\}$.

We use $h_{i,j} \in \mathbb{C}$ to denote the combined effect of beamforming vectors ($\boldsymbol{w}^*_j$) and fading ($\boldsymbol{h}^{s,s}_{i,j}$). Thus,

$$h_{i,j} \triangleq \boldsymbol{h}^{s,s}_{i,j}\boldsymbol{w}^*_j \quad \text{for } i,j=1,\ldots,M_s.$$

We also define $\boldsymbol{x} \triangleq [x^s_1,\ldots,x^s_{M_s}]^T$ and $\boldsymbol{h}^i \triangleq [h_{i,1},\ldots,h_{i,M_s}]$. Therefore, the signal received by the i^{th} secondary receiver is

$$y_i = \boldsymbol{h}^i\boldsymbol{x} + z_i$$

where $z_i \in \mathbb{C}$ accounts for the Gaussian noise as well as the interference seen from the primary users. Without loss of generality, it is assumed to be $\mathcal{CN}(0,1)$. As before we assume that x^s_i, the information symbol of the i^{th} user, has unit power and is drawn from a Gaussian alphabet.

We use $\mathcal{K} = \{1,\ldots,M_s\}$ to refer to the set of all secondary users and construct the vector $\boldsymbol{h}^i_{\mathcal{A}} \triangleq [h_{i,j}]_{j\in\mathcal{A}}$. $\boldsymbol{R}_{\mathcal{A}}$ denotes the rate vector of the users with indices in $\mathcal{A} \subseteq \mathcal{K}$. For any two disjoint subsets $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{K}$, let $\mathcal{C}(\boldsymbol{h}^i,\mathcal{A},\mathcal{B})$ denote the instantaneous achievable rate region for the users in $\mathcal{A}$ jointly decoded using maximum likelihood (ML) decoder while treating the users in $\mathcal{B}$ as Gaussian interferers. Therefore,

$$\mathcal{C}(\boldsymbol{h}^i,\mathcal{A},\mathcal{B}) = \left\{\boldsymbol{r} \in \mathbb{R}_+^{|\mathcal{A}|} \;\middle|\; \sum_{j\in\mathcal{D}} r_j \leq \mathbb{I}(\boldsymbol{x}_{\mathcal{D}}; y_i \mid \boldsymbol{x}_{\mathcal{A}\setminus\mathcal{D}}),\ \forall\mathcal{D}\subseteq\mathcal{A}\right\},$$

where

$$\mathbb{I}(\boldsymbol{x}_{\mathcal{D}}; y_i \mid \boldsymbol{x}_{\mathcal{A}\setminus\mathcal{D}}) = \log\left[1 + \boldsymbol{h}^i_{\mathcal{D}}\left(1 + \boldsymbol{h}^i_{\mathcal{B}}{\boldsymbol{h}^i_{\mathcal{B}}}^H\right)^{-1}{\boldsymbol{h}^i_{\mathcal{D}}}^H\right].$$

Let also $\{\mathcal{G}_i,\mathcal{G}_i^c\}$ denote a partition of $\mathcal{K}$ such that $i \in \mathcal{G}_i$ and the users in $\mathcal{G}_i$ are jointly decoded after treating those in $\mathcal{G}_i^c$ as noise. Therefore, the minimum rate vector $\boldsymbol{R}^{\min}$ is decodable if and only if there exist sets $\{\mathcal{G}_1,\ldots,\mathcal{G}_{M_s}\}$ such that

$$\boldsymbol{R}^{\min}_{\mathcal{G}_i} \in \mathcal{C}(\boldsymbol{h}^i,\mathcal{G}_i,\mathcal{G}_i^c) \qquad \text{for } i=1,\ldots,M_s.$$

A. Distributed Weighted Max-Min Fair Rate Allocation

For any receiver i and any two disjoint subsets $\mathcal{A},\mathcal{B}$ of $\mathcal{K}$ we define the metric

$$\theta(\boldsymbol{h}^i,\mathcal{A},\mathcal{B},\boldsymbol{R}^{\min},\boldsymbol{\rho}) \triangleq \begin{cases} \max & \min_k \frac{r_k}{\rho_k} \\ \text{s.t.} & \boldsymbol{r}_{\mathcal{A}} + \boldsymbol{R}^{\min}_{\mathcal{A}} \in \mathcal{C}(\boldsymbol{h}^i,\mathcal{A},\mathcal{B}) \end{cases},$$

which picks the rate-vector within the rate region $\mathcal{C}(\boldsymbol{h}^i,\mathcal{A},\mathcal{B})$ that satisfies weighted max-min fairness for the users in $\mathcal{A}$.

Lemma 2: $\theta(\boldsymbol{h}^i,\mathcal{A},\mathcal{B},\boldsymbol{R}^{\min},\boldsymbol{\rho})$ can be computed using

$$\theta(\boldsymbol{h}^i,\mathcal{A},\mathcal{B},\boldsymbol{R}^{\min},\boldsymbol{\rho}) = \min_{S\neq\emptyset, S\subseteq\mathcal{A}} \frac{\Delta(\boldsymbol{h}^i,S,\mathcal{B},\boldsymbol{R}^{\min})}{\sum_{j\in S}\rho_j},$$

where

$$\Delta(\boldsymbol{h}^i,S,\mathcal{B},\boldsymbol{R}^{\min}) = \log\det\left[\boldsymbol{I} + {\boldsymbol{h}^i_S}^H\left(\boldsymbol{I} + \boldsymbol{h}^i_{\mathcal{B}}{\boldsymbol{h}^i_{\mathcal{B}}}^H\right)^{-1}\boldsymbol{h}^i_S\right] - \sum_{j\in S} R^{\min}_j.$$

By using the results on submodular function minimization, it can be shown that $\theta(\boldsymbol{h}^i,\mathcal{A},\mathcal{B},\boldsymbol{R}^{\min},\boldsymbol{\rho})$ can be determined with a complexity that is polynomial in $|\mathcal{A}|$. Now consider the i^{th} receiver and let $\{r^i_1,\ldots,r^i_{M_s}\}$ denote an optimal set of excess rates where r^i_k is the rate increment for the k^{th} user such that the i^{th} user is decodable at the i^{th} receiver and max-min fairness is sustained. Thus, we have

$$\min_k \frac{r^i_k}{\rho_k} = \max_{\mathcal{G}\subseteq\mathcal{K}, i\in\mathcal{G}} \theta(\boldsymbol{h}^i,\mathcal{G},\mathcal{K}\setminus\mathcal{G},\boldsymbol{R}^{\min},\boldsymbol{\rho}). \qquad (11)$$

$$f(i,p) \triangleq \min_{1\leq q\leq p}\left\{\frac{1}{p-q+1}\log\left(1 + \frac{\sum_{m=q}^{p}|\boldsymbol{h}^{s,s}_{i,\pi_i(m)}\boldsymbol{w}^s_{\pi_i(m)}|^2}{\sum_{m=p+1}^{M_s}|\boldsymbol{h}^{s,s}_{i,\pi_i(m)}\boldsymbol{w}^s_{\pi_i(m)}|^2 + \sum_j|\boldsymbol{h}^{s,p}_{i,j}\boldsymbol{w}^p_j|^2 + \sigma^s_i}\right)\right\} \quad \forall i. \qquad (12)$$

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We are ready to offer our distributed max-min fair rate allocation. The underlying idea is that each receiver, individually and based on its own perception of the network, proposes rate increments for all users beyond their minimum rates. Therefore, each secondary user receives M_s rate increment suggestions and picks the smallest one as the rate increment for itself. Algorithm 3, provided in the sequel, is a computationally efficient scheme (with a complexity polynomial in M_s) for finding an optimal set of rate increments $\{r_1^i, \ldots, r_{M_s}^i\}$ at any given receiver i.

Algorithm 3- Rate increment recommendations by individual receivers

Initialize $S = \mathcal{K}$, $\mathcal{G} = \emptyset$, $\mathcal{G}^i = \emptyset$, $k = 1$ and $\boldsymbol{R}^{\min}$
repeat
  Find $\delta^k = \min_{B \neq \emptyset, B \subseteq S} \frac{\Delta(\boldsymbol{h}^i, B, \mathcal{G}, \boldsymbol{R}^{\min})}{\sum_{j \in B} \rho_j}$
  $B^k = \arg\min_{B \neq \emptyset, B \subseteq S} \frac{\Delta(\boldsymbol{h}^i, B, \mathcal{G}, \boldsymbol{R}^{\min})}{\sum_{j \in B} \rho_j}$
  If there are multiple choices for B^k pick any one such that $i \notin B^k$.
  if $i \in B^k$ or $i \in \mathcal{G}$
    $r_j^i = \delta^k \rho_j$ for all $j \in B^k$
    $S \leftarrow S \backslash B^k$, $\mathcal{G} \leftarrow \mathcal{G} \cup B^k$, $\mathcal{G}^i \leftarrow B^k \cup \mathcal{G}^i$, $k \leftarrow k+1$
  else
    $r_j^i = +\infty$ for all $j \in B^k$, $S \leftarrow S \backslash B^k$, $\mathcal{G} \leftarrow \mathcal{G} \cup B^k$, $k \leftarrow k+1$
  end if
until $S = \emptyset$
Output $\{r_k^i\}$ and $\mathcal{G}^i$

Theorem 3: For a given decodable rate vector $\boldsymbol{R}^{\min}$, the i^{th} user is decodable under the rate assignment $\{R_k^{\min} + r_k^i\}_{k=1}^{M_s}$, where $\{r_k^i\}$ is yielded by Algorithm 3. Furthermore,

$$\min_{k \in \mathcal{K}} \frac{r_k^i}{\rho_k} \geq \min_{k \in \mathcal{K}} \frac{\tilde{r}_k^i}{\rho_k}$$

where $\{\tilde{r}_1^i, \ldots, \tilde{r}_{M_s}^i\}$ is any other rate increment vector for which the i^{th} user is decodable under the rates $\{R_k^{\min} + \tilde{r}_k^i\}_{k=1}^{M_s}$.

Proof: Proved in the Appendix. ■

By using Algorithm 3, we now propose our distributed weighted max-min rate allocation scheme.

Algorithm 4- Weighted max-min fair rate allocation

Initialize $\boldsymbol{d} = \boldsymbol{R}^{\min}$ and $q = 0$
repeat
  for $i = 1, \ldots, M_s$ **do**
    Run Algorithm 3 with $\boldsymbol{d}$ as the minimum rate vector
  end for
  Update $q \leftarrow q + 1$ and $R_k^{(q)} = d_k + \min_{1 \leq i \leq M_s} \{r_k^i\}$ and $\boldsymbol{d} \leftarrow \boldsymbol{R}^{(q)}$
until $\boldsymbol{R}^{(q)}$ converges
Output $\hat{\boldsymbol{R}} = \boldsymbol{R}^{(q)}$ and $\{\mathcal{G}^i\}_{i=1}^{M_s}$

Theorem 4: The iterative rate allocation algorithm (Algorithm 4) has the following properties:

1) It is monotonic in the sense that $\boldsymbol{R}^{(q+1)} \succeq \boldsymbol{R}^{(q)}$ and is convergent.

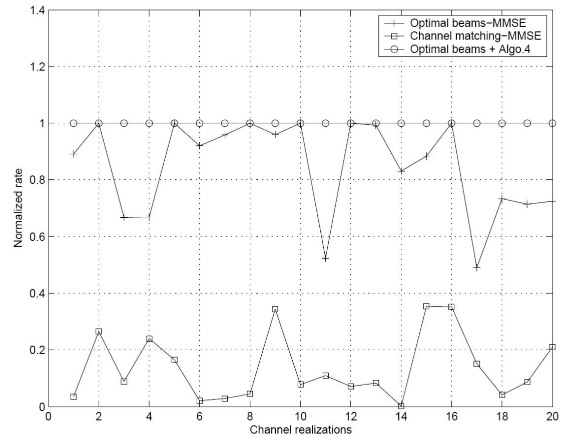

Fig. 1. Normalized minimum rates.

2) At each iteration the vector $\boldsymbol{R}^{(q)} \succeq \boldsymbol{R}^{\min}$ is decodable and max-min optimal, i.e., for any other arbitrary decodable rate vector $\tilde{\boldsymbol{R}} \succeq \boldsymbol{R}^{\min}$ we have

$$\min_{k \in \mathcal{K}} \frac{R_k^{(q)} - R_k^{\min}}{\rho_k} \geq \min_{k \in \mathcal{K}} \frac{\tilde{R}_k - R_k^{\min}}{\rho_k}.$$

3) The rate allocation $\hat{\boldsymbol{R}}$ yielded by Algorithm 4 is also pareto-optimal, i.e., for any arbitrary decodable rate vector $\tilde{\boldsymbol{R}} \succeq \boldsymbol{R}^{\min}$ such that $\tilde{R}_k > \hat{R}_k$ for some $k \in \mathcal{K}$, we must have that $\exists\, j \neq k : \tilde{R}_j < \hat{R}_j$.

VI. Simulation Results

We consider a cognitive radio network with two primary transceivers and three secondary transceivers., each (primary and secondary) with three transmit and one receive antenna. The sum power available for all secondary transmitters is taken to be 20 dB. We first obtain the optimal beamformer design using Algorithm 2, where assume that all γ_i and all α_i are unity. We then implement Algorithm 4 (with identical priorities for all users) to obtain the optimal rate increments. In Fig. 1, we consider 20 channel realizations and for each realization, we plot the normalized minimum rate obtained using the optimal beam vectors and the MMSE decoder at each secondary receiver, where the minimum rate is normalized by the minimum rate obtained after Algorithm 4. We also plot the normalized minimum rate obtained using the MMSE decoder at each secondary receiver, when each secondary transmitter employs a beam that is matched to the forward channel vector to its intended receiver. From the figure it is seen that as opposed to a naive channel matching scheme, optimal beamforming can provide substantial gains. Further rate gains are achievable via advanced decoding at each receiver.

VII. Conclusions

We considered decentralized multi-antenna cognitive radio networks where secondary transceivers co-exist with primary ones. We designed optimal beamformers for the secondary (or cognitive) users by maximizing the minimum rate in a distributed fashion, subject to a sum-power budget, interference margin constraints and single-user decoding at each secondary receiver. We also proposed an optimal distributed fair rate allocation algorithm for the scenario in which each secondary receiver is allowed to decode any subset of secondary users.

VIII. APPENDIX

Assume that the set $\mathcal{K}$ is partitioned to the disjoint sets $\{B^1,\ldots,B^p\}$ with corresponding parameters $\{\delta^1,\ldots,\delta^p\}$ such that $i\in B^{m+1}$ for some $m\le p-1$. Using the techniques developed in [11], we can readily show that

$$\delta^1\le\cdots\le\delta^{m+1}\le\cdots\le\delta^p.$$

As proposed by the algorithm, $r_k^i=+\infty$ for $k\in\cup_{j=1}^m B^j$ and for $k\in B^j$, where $j\ge m+1$, $\frac{r_k^i}{\rho_k}=\delta^j$. Therefore,

$$\min_{k\in\mathcal{K}}\frac{r_k^i}{\rho_k}=\min\{+\infty,\delta^{m+1},\ldots,\delta^p\}=\delta^{m+1}. \quad (13)$$

Now consider any valid partitioning of $\mathcal{K}=\{\mathcal{G}_i^*,\mathcal{K}\backslash\mathcal{G}_i^*\}$ that supports the rate increments $\{\tilde{r}_1^i,\ldots,\tilde{r}_{M_s}^i\}$ which also satisfy the max-min optimality. Based on the definition in (11) we have

$$\delta_i^*=\min_{k\in\mathcal{K}}\frac{\tilde{r}_k^i}{\rho_k}=\theta(\boldsymbol{h}^i,\mathcal{G}_i^*,\mathcal{K}\backslash\mathcal{G}_i^*.\boldsymbol{R}^{\min},\boldsymbol{\rho}).$$

Letting $\mathcal{G}_i=\mathcal{K}\backslash\cup_{j=1}^m B^j$, we have to show that $\delta_i^*=\delta^{m+1}$. We establish the proof by contradiction. Suppose that we have $\delta_i^*>\delta^{m+1}$. Then:

1) $\mathcal{G}_i^*\subseteq\mathcal{G}_i$:

We show that $\mathcal{G}_i^*\cap B^j=\emptyset$ for $j=1,\ldots,m$. By contradiction, let us assume that $\mathcal{G}_i^*$ has non-empty intersection with some sets $\{B^j\}_{j=1}^m$ and denote k as the smallest value such that $\mathcal{G}_i^*\cap B^k\neq\emptyset$, while for $j=1,\ldots,k-1$, we have $\mathcal{G}_i^*\cap B^j=\emptyset$. By expanding $B^k=\{\mathcal{G}_i^*\cap B^k\}\cup\{\{\mathcal{K}\backslash\mathcal{G}_i^*\}\cap B^k\}$ we get

$$\begin{aligned}\delta^k\sum_{j\in B^k}\rho_j &= \Delta(\boldsymbol{h}^i,B^k,\cup_{j=1}^{k-1}B^j,\boldsymbol{R}^{\min})\\ &=\Delta(\boldsymbol{h}^i,\underbrace{\{\mathcal{K}\backslash\mathcal{G}_i^*\}\cap B^k}_{\subseteq B^k},\cup_{j=1}^{k-1}B^j,\boldsymbol{R}^{\min})\\ &\quad+\Delta(\boldsymbol{h}^i,\mathcal{G}_i^*\cap B^k,\underbrace{\{\{\mathcal{K}\backslash\mathcal{G}_i^*\}\cap B^k\}\cup\{\cup_{j=1}^{k-1}B^j\}}_{\subseteq\mathcal{K}\backslash\mathcal{G}_i^*},\boldsymbol{R}^{\min})\\ &\ge\delta^k\sum_{j\in\{\mathcal{K}\backslash\mathcal{G}_i^*\}\cap B^k}\rho_j+\Delta(\boldsymbol{h}^i,\underbrace{\mathcal{G}_i^*\cap B^k}_{\subseteq\mathcal{G}_i^*},\mathcal{K}\backslash\mathcal{G}_i^*,\boldsymbol{R}^{\min})\\ &\ge\delta^k\sum_{j\in\{\mathcal{K}\backslash\mathcal{G}_i^*\}\cap B^k}\rho_j+\underbrace{\delta_i^*}_{>\delta^k}\sum_{j\in\mathcal{G}_i^*\cap B^k}\rho_j>\delta^k\sum_{j\in B^k}\rho_j,\end{aligned}$$

which is a contradiction. Therefore, for all $j\in\{1,\ldots,m\}$, $\mathcal{G}_i^*\cap B^j=\emptyset$ and as a result $\mathcal{G}_i^*\subseteq\{\mathcal{K}\backslash\cup_{j=1}^m B^j\}=\mathcal{G}_i$

2) $B^{m+1}\subseteq\mathcal{G}_i^*$:

If $B^{m+1}\nsubseteq\mathcal{G}_i^*$ then $B^{m+1}\cap\{\mathcal{G}_i\backslash\mathcal{G}_i^*\}\neq\emptyset$. By expanding $B^{m+1}=\{B^{m+1}\cap\mathcal{G}_i^*\}\cup\{B^{m+1}\cap\{\mathcal{G}_i\backslash\mathcal{G}_i^*\}\}$ we have

$$\begin{aligned}&\delta^{m+1}\sum_{j\in B^{m+1}}\rho_j=\Delta(\boldsymbol{h}^i,B^{m+1},\cup_{j=1}^m B^j,\boldsymbol{R}^{\min})\\ &=\Delta(\boldsymbol{h}^i,B^{m+1}\cap\mathcal{G}_i^*,\underbrace{\{B^{m+1}\cap\{\mathcal{G}_i\backslash\mathcal{G}_i^*\}\}\cup\{\cup_{j=1}^m B^j\}}_{\subseteq\mathcal{K}\backslash\mathcal{G}_i^*},\boldsymbol{R}^{\min})\\ &+\Delta(\boldsymbol{h}^i,B^{m+1}\cap\{\mathcal{G}_i\backslash\mathcal{G}_i^*\},\cup_{j=1}^m B^j,\boldsymbol{R}^{\min})\\ &\ge\Delta(\boldsymbol{h}^i,\underbrace{B^{m+1}\cap\mathcal{G}_i^*}_{\subseteq\mathcal{G}_i^*},\mathcal{K}\backslash\mathcal{G}_i^*,\boldsymbol{R}^{\min})+\delta^{m+1}\sum_{j\in\{B^{m+1}\cap\{\mathcal{G}_i\backslash\mathcal{G}_i^*\}\}}\rho_j\\ &\ge\underbrace{\delta_i^*}_{>\delta^{m+1}}\sum_{j\in\{B^{m+1}\cap\mathcal{G}_i^*\}}\rho_j+\delta^{m+1}\sum_{j\in\{B^{m+1}\cap\{\mathcal{G}_i\backslash\mathcal{G}_i^*\}\}}\rho_j\\ &>\delta^{m+1}\sum_{j\in B^{m+1}}\rho_j,\end{aligned}$$

which is a contradiction and therefore $B^{m+1}\subseteq\mathcal{G}_i^*$.

Now, let us define $\mathcal{C}\triangleq\mathcal{G}_i\backslash\mathcal{G}_i^*$. Due to having $\mathcal{G}_i^*\subseteq\mathcal{G}_i$ and $B^{m+1}\subseteq\mathcal{G}_i^*$ we get

$$\begin{aligned}\delta^{m+1}&=\min_{B\neq\emptyset,B\subseteq\mathcal{G}_i}\frac{\Delta(\boldsymbol{h}^i,B,\mathcal{K}\backslash\mathcal{G}_i,\boldsymbol{R}^{\min})}{\sum_{j\in B}\rho_j}\\ &=\frac{\Delta(\boldsymbol{h}^i,B^{m+1},\cup_{j=1}^m B^j,\boldsymbol{R}^{\min})}{\sum_{j\in B^{m+1}}\rho_j}\\ &=\min_{B\neq\emptyset,B\subseteq\mathcal{G}_i\backslash\mathcal{C}}\frac{\Delta(\boldsymbol{h}^i,B,\mathcal{K}\backslash\mathcal{G}_i,\boldsymbol{R}^{\min})}{\sum_{j\in B}\rho_j}\\ &\ge\min_{B\neq\emptyset,B\subseteq\mathcal{G}_i\backslash\mathcal{C}}\frac{\Delta(\boldsymbol{h}^i,B,\{\mathcal{K}\backslash\mathcal{G}_i\}\cup\mathcal{C},\boldsymbol{R}^{\min})}{\sum_{j\in B}\rho_j}\\ &=\min_{B\neq\emptyset,B\subseteq\mathcal{G}_i^*}\frac{\Delta(\boldsymbol{h}^i,B,\mathcal{K}\backslash\mathcal{G}_i^*,\boldsymbol{R}^{\min})}{\sum_{j\in B}\rho_j}=\delta_i^*,\end{aligned}$$

which contradicts the assumption that $\delta_i^*>\delta^{m+1}$. Thus $\delta_i^*=\delta^{m+1}$ which establishes the desired result.

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