BEACON-ASSISTED SPECTRUM ACCESS WITH COOPERATIVE COGNITIVE TRANSMITTER AND RECEIVER

Ali Tajer and Xiaodong Wang

Columbia University, New York, NY 10027

ABSTRACT

We propose a novel cooperative communication protocol for multicasting a common message from one source to two destinations and based on that offer a spectrum access scheme for the cognitive radios that seek to utilizing the spectrum holes within the bands licensed to the legacy systems. The proposed cooperation model has two major traits; first, by opportunistically and dynamically assigning one of the destination nodes as relay for the other one, via a single-time relaying both destinations achieves a second order diversity gain. Secondly, it guarantees performance improvement over all SNR regimes, which is not the case in most cooperation protocols as diversity gain is a high SNR measure and yielding higher diversity orders ensures improvement only over high enough SNRs. Next, we consider cognitive users, to whom the codebook of the primary users is known as side information, and offer a beacon-assisted mechanism for spectrum access. We assume that a primary user multicasts a beacon message upon releasing a spectrum band and adopt the proposed cooperation model to strengthen a cognitive transmitter-receiver pair in decoding the beacon message. Finally, we quantify the effect of such cooperation on the capacity of the channel between the cognitive transmitter-receiver pair, as a meaningful measure to assess how the proposed cooperation model assists the secondary users in exploiting communication opportunities.

Index Terms— Cognitive radio, cooperative communication, spectrum access, channel capacity.

1. INTRODUCTION

Under-utilizing the licensed spectrum bands as well as the increasing demand for frequency bands motivates opportunistic access to licensed bands by unlicensed (secondary) users as a potential solution for alleviating scarcity of frequency spectrum in overly crowded environments [1, 2]. The notion of having secondary users, enables accommodating ad-hoc links within currently established wireless communication infrastructures. For this purpose, secondary users continuously monitor the spectrum in order to efficiently uncover and exploit the spectrum holes.

We propose a beacon-assisted channel access where we assume that a codeword from the codebook of the primary users is reserved as a beacon, and each time a primary user releases a channel, it broadcasts this beacon message. The codeword of the primary users as well as the beacon codeword are a priori known to the secondary users which can be justified by noting that it is widely assumed the secondary users have some some side information about the primary users (see for example [3, 4]). The merits of deploying beacon-based spectrum access over conventional energy detectors are also discussed in [5].

Motivated by enhancing the quality of detecting the beacon by the secondary users, we propose a multicast cooperative protocol for the communication between the beacon-emitting primary user and a secondary transmitter-receiver pair intending to detect the beacon.

Finally, as a measure to assess how the proposed cooperation protocol is effective in assisting the secondary users identify communication opportunities, we find the effect of successful spectrum hole detection on the capacity of the channel between the cognitive transmitter and receiver.

2. SYSTEM DESCRIPTIONS

We denote the primary transmitter by $T_p$ and the pair of secondary transmitter and receiver by $T_t$ and $T_r$, respectively. The secondary users continuously monitor the channel used by the primary user seeking opportunity for taking it over when it is unused by the primary user. We denote the flat fading channels between the primary transmitter and the secondary transmitter and receiver by $\gamma_{p,t}$ and $\gamma_{p,r}$ respectively. The channel between the secondary nodes are assumed to be reciprocal and are denoted by $\gamma_{t,r}$ and $\gamma_{r,t}$. The physical channel between nodes $i \in \{p, t, r\}$ and $j \in \{t, r\}$ has the instantaneous realization

$$\gamma_{i,j} = \sqrt{\lambda_{i,j}} \cdot h_{i,j},$$

where fading coefficients $h_{i,j}$ are assumed to be distributed as $CN(0, 1)$, and the term $\lambda_{i,j}$ accounts for path loss and shadowing.

We consider $N$ consecutive uses of channel for the transmission of the beacon message. The beacon message sent by the primary user is denoted by $x_b = [x_b[1], \ldots, x_b[N]]^T$ and the received signals by the secondary transmitter and receivers nodes are referred to by $y_t = [y_t[1], \ldots, y_t[N]]^T$ and $y_r = [y_r[1], \ldots, y_r[N]]^T$, respectively. For the non-cooperative transmission from the primary user to the secondary nodes, as the baseline in our comparisons, we have

$$y_t[n] = \gamma_{p,t} x_b[n] + z_t[n],$$

and

$$y_r[n] = \gamma_{p,r} x_b[n] + z_r[n], \text{ for } n = 1 \ldots, N.$$
where we assume that all transmissions are contaminated with zero mean additive white complex Gaussian noise with variance $N_0$ and denoted by $z_t$ and $z_r$. Also we assume average transmission power constraint, $E[|x|^2] \leq P_p$, and denote $\rho \triangleq \frac{P_p}{N_0}$ as the SNR without fading, pathloss and shadowing. Therefore, the instantaneous SNR between nodes is given by

$$\text{SNR}_{i,j} = \rho |h_{i,j}|^2.$$  \hfill (4)

Throughout the paper we say that two functions $f(\rho)$ and $g(\rho)$ are as exponentially equal, denoted by $f(\rho) \doteq g(\rho)$ if

$$\lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho} = \lim_{\rho \to \infty} \frac{\log g(\rho)}{\log \rho}$$

3. COOPERATIVE SPECTRUM ACCESS (CSA)

We consider a network where the primary user as well as the secondary transmitter and receiver are randomly distributed. We develop a half-duplex regenerative (decode-and-forward) cooperation protocols such that

1. cooperation is carried out only if it is guaranteed to be beneficial to the secondary users in detecting the beacon message,

2. one of the secondary nodes is opportunistically selected as a relay for the other such that both the one acting as the relay and the one assisted by the relay yield a second order diversity gain. Note that if we fix one of the as the relay only the one assisted by the relay will achieve second order diversity gain, which motivates dynamic selection of the relay.

As mentioned earlier, in a non-cooperative case, the primary user broadcasts a beacon $x_b$ of length $N$ upon releasing the spectrum band. In contrast, the underlying idea behind the cooperation is that the secondary users, instead of being silent and listening to the beacon message during the entire $N$ channel uses, listen to the primary for only $N_1 < N$ channel uses and then exploit the remaining $N_2 \triangleq N - N_1$ channel uses to cooperate among themselves. Therefore, the primary user transmits a weaker beacon (codeword) $x'_b$ of length $N_1 < N$ and then the remaining $N_2$ channel uses is dedicated to cooperation. We can show that by the cooperation protocol as detailed in the sequel, we can uniformly improve upon the non-cooperative case, where each secondary node senses the vacancy of the channel individually. Note that for fair comparison we constrain the total channel uses ($N$) and power consumption be identical in both cooperative and non-cooperative schemes. We define the ration $\alpha \triangleq \frac{N_1}{N}$ as the level of cooperation.

The cooperation protocol, like most existing ones, consists of two phases as follows.

**Phase 1:** The primary user broadcasts a reserved codeword in its codebook ($x'_b$) by $N_1$ channel uses when it is willing to release the channel. Meanwhile, the secondary transmitter-receiver pair are listening the primary user and at the end of $N_1$ channel uses try to decode $x'_b$.

**Phase 2:** All the three nodes (primary transmitter and the pair of secondary transmitter-receiver) step in a competition for broadcasting additional party bits for $x'_b$ during the next $N_2$ channel uses. The competition is carried out as follows. We first assign the metrics $t_p \triangleq |\gamma_{p,t}|^2 + |\gamma_{p,r}|^2$, $t_t \triangleq |\gamma_{t,t}|^2 + |\gamma_{t,r}|^2$ and $t_r \triangleq |\gamma_{r,t}|^2 + |\gamma_{r,r}|^2$ to the primary user, secondary transmitter and secondary receiver, respectively. Then the $N_2$ channel uses is allocated to

1. the secondary transmitter if it successfully decodes $x'_b$ and $t_t > \max\{t_p, t_r\}$;

2. the secondary receiver if it successfully decodes $x'_b$ and $t_r > \max\{t_p, t_t\}$;

3. to the primary transmitter if either $t_p > \max\{t_t, t_r\}$ or both secondary nodes fail to decode $x'_b$.

We provide the proofs for the gains attained by this cooperation model in the next section. However, the intuitive argument for such gains is as follows. The choices of the metrics $t$ have key roles in justifying the gains. For instance, one possible situation that primary transmitter wins the competition in the second phase is when $t_p > \max\{t_t, t_r\}$ or equivalently $\min\{\gamma_{p,t}, \gamma_{p,r}\} > \gamma_{r,p}$, which means that both channels $\gamma_{p,t}$ and $\gamma_{r,t}$ are more reliable than the direct channel from the primary transmitter to the secondary receiver. Therefore, the secondary receiver can recover the additional parity bits from the secondary transmitter more reliably, which justifies having the secondary transmitter act as relay for the secondary receiver.

Note that including the primary transmitter in the competition has the advantage that first, it guarantees to deploy cooperation only when it is helpful and secondly, even when cooperation can be helpful ($t_p < \max\{t_t, t_r\}$) in some instances it might so happen that neither of the secondary users can decode the beacon successfully and thereof neither can act as relay. Note that cooperative protocols are devised to provide higher diversity gains by providing multipath diversity. However, diversity as a high SNR measure, does not guarantee improvement for low or medium SNR regimes, whereas we show that our proposed scheme exhibits improvement over all SNR regimes.

Finally, a simple technique for identifying the winner in a distributed way without having a central controller is to equip each secondary user with a backoff timer, with its initial value set inversely proportional to the gain of its incoming channel from the primary user. Therefore the timer of the node with a better incoming channel from the primary user goes off sooner and will start relaying. This techniques has also been used in other contexts [8, 9].

4. DIVERSITY ANALYSIS

In this section we characterize the performance of the proposed protocol in terms of the probability of erroneous detection of the beacon message denoted by $P_e(x)$, and the achievable diversity order. Diversity order measures how rapidly the error probability decays with increasing SNR and is defined as $\lim_{\rho \to \infty} -\frac{\log P_e(x)}{\log \rho}$.

4.1. Non-Cooperative Scheme

As a baseline for performance comparisons, we first consider non-cooperative transmission by the primary user over the channels given by (2) and (3). For a coded transmission with coherent detection, the pairwise error probability (PEP) of erroneously detecting the beacon

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codeword $x^b$ in favor of the codeword $\hat{x}^b$ for a channel realization $\gamma_{p,t}$ is given by \([10, (12.13)]\)
\[
P_{\rho}(e_t \mid \gamma_{p,t}) \triangleq P(x^b \rightarrow \hat{x}^b \mid \gamma_{p,t}) = Q\left(\sqrt{2d}\rho_{\gamma_{p,t}}\right),
\]
where $d$ is the Hamming distance between $x^b$ and $\hat{x}^b$. Throughout the diversity order analyses, we will frequently use the following result.

**Lemma 1** For integer $M > 0$ and real $k_t > 0$,
\[
\int_0^\infty \prod_{i=1}^M \left(1 - e^{-k_t^2 v^2 / \rho}\right) \frac{1}{\sqrt{2\pi}} e^{-v^2 / 2} dv \geq \rho^{-M}.
\]

Therefore, the detection error probability at node $T_1$ is $(u_t = \mid \gamma_{p,t}\mid^2)$
\[
P_{\rho}^{NC}(e_t) = \mathbb{E}_{\rho\gamma_{p,t}} \left[ Q\left(\sqrt{2d]\rho_{u_t}\right) \right]
\]
\[
= \int_0^\infty \left(1 - e^{-\frac{1}{2\rho_{u_t}} v^2}\right) \frac{1}{\sqrt{2\pi}} e^{-v^2 / 2} dv \geq \rho^{-1}.
\]
which shows that the diversity order is $1$. The same can be concluded for $T_i$, i.e. $P_{\rho}^{NC}(e_t) \geq \rho^{-1}$.

**4.2. CSA Protocol**

We consider a base-band discrete-time channel model. We denote the winner node in the second phase and its transmitted signal by $T_w$ and $x_w$, respectively. During the first phase we have
\[
y_t[n] = \gamma_{p,t} x^b[n] + z_t[n],
\]
and
\[
y_t[n] = \gamma_{p,t} x^b[n] + z_t[n], \quad n = 1, \ldots, N_1.
\]

For the second phase, $T_w$ will be in transmission mode and we have
\[
y_t[n] = \gamma_{w,t} x_w[n] + z_t[n] \quad \text{if} \quad T_w \neq T_i \quad \text{and} \quad 0 \quad \text{if} \quad T_w = T_i.
\]
and
\[
y_t[n] = \gamma_{w,t} x_w[n] + z_t[n] \quad \text{if} \quad T_w \neq T_i \quad \text{and} \quad 0 \quad \text{if} \quad T_w = T_i,
\]
for $n = N_1 + 1, \ldots, N$, where $\gamma_{p,t}$ and $\gamma_{w,t}$ denote the channels from $T_w$ to $T_i$ and $T_t$, respectively.

**Theorem 1** For all values of SNR, level of cooperation and channel realizations, we have $P_{\rho}^C(e_t) < P_{\rho}^{NC}(e_t)$ and $P_{\rho}^C(e_r) < P_{\rho}^{NC}(e_r)$.

**Proof:** We define $u \triangleq \mid \gamma_{p,t}\mid$. The probability of missing the beacon message by the secondary transmitter is
\[
P_{\rho}^C(e_t) = \sum_{i=0}^{M} P_{\rho}^C(e_t \mid T_w = T_i) P(T_w = T_i).
\]

Note that $P_{\rho}^C(e_t \mid T_w = T_i) = 0$. Also when $T_w = T_r$ we have $t_r > t_p$, or $u > u_t$. Hence,
\[
P_{\rho}^C(e_t) = \mathbb{E}_{u\gamma_{p,t}} \left[ Q\left(\sqrt{2d]\rho_{u_t}\right) \right] P(T_w = T_p)
\]
\[
+ \mathbb{E}_{u\gamma_{p,t}} \left[ Q\left(\sqrt{2d]\rho_{u_t} + 2d\rho_{u_{t}}} \right) \right] P(T_w = T_r)
\]
\[
< \mathbb{E}_{u\gamma_{p,t}} \left[ Q\left(\sqrt{2d]\rho_{u_t}\right) \right] [P(T_w = T_p) + P(T_w = T_r)]
\]
\[
< \mathbb{E}_{u\gamma_{p,t}} \left[ Q\left(\sqrt{2d]\rho_{u_t}\right) \right] P_{\rho}^{NC}(e_t).
\]

$d_1$ and $d_2$ are the hamming distances of the codewords sent in the first and second phase and we have $d_1 + d_2 = d$. By following the same lines we can show the same for $P_{\rho}^C(e_r)$.

**Theorem 2** Both secondary transmitter and receiver achieve a second order diversity gain for detecting the beacon message, i.e.,
\[
P_{\rho}^C(e_t) \equiv P_{\rho}^{NC}(e_t) \equiv \rho^{-2}.
\]

**Proof:** We define $A$ as the event that $t_p > \max\{t_t, t_r\}$. From (10) and (11) we get
\[
P_{\rho}^C(e_t \mid T_w = T_p, A)
\]
\[
= \mathbb{E}_{u\gamma_{p,t}} \left[ Q\left(\sqrt{2d]\rho_{u_t} + 2d\rho_{u_{t}}} \right) \right] P(T_w = T_r)
\]
\[
\leq P_{\rho}^C(e_t \mid T_w = T_p, A) + P(A^C)
\]
\[
+ \mathbb{E}_{u\gamma_{p,t}} \left[ Q\left(\sqrt{2d]\rho_{u_t} + 2d\rho_{u_{t}}} \right) \right] P(T_w = T_r)
\]
\[
= \rho^{-2},
\]
where the last step follows from Lemma 1. We only need to show that $P(A^C) \equiv \rho^{-2}$. Note that $A^C$ corresponds to the case that both secondary transmitter and receiver fail to decode $x^b$ during the first phase. Hence, by defining $u \triangleq \mid \gamma_{p,t}\mid$ we get
\[
P(A^C) = \mathbb{E}_{u\gamma_{p,t}} \left[ Q\left(\sqrt{2d]\rho_{u_t}} \right) \right] E_{u\gamma_{p,t}} \left[ Q\left(\sqrt{2d]\rho_{u_{t}}} \right) \right] \geq \rho^{-2},
\]
which completes the proof. Similar argument holds for $P_{\rho}^C(e_r)$.

**5. CAPACITY ANALYSIS**

As a meaningful measure to assess how efficiently the proposed cooperation model improves the quality of exploiting communication opportunities, we analyze the capacity of the cognitive link. Capacity of cognitive radios is influenced by the spectral activity of the primary users and the efficiency of the cognitive radios in detecting the unused channels. In an earlier study in [11], it is also demonstrated how the cognitive radio capacity is affected by dissimilar perception of cognitive radios of primary user’s spectral activity in their vicinities, where it has been shown that more correlated perceptions lead to higher channel capacity for cognitive users. We will show that the cooperative protocols are also effective in increasing such correlation. Lower and upper bounds on the capacity of the cognitive channel when the received power at the cognitive receiver is $P$ are
\[
C^U(P) = \mathbb{Pr}(S_t = S_r = 1) \log \left(1 + \frac{P}{\Pr(S_r = 1)}\right),
\]
\[
C^L(P) = \mathbb{Pr}(S_t = S_r = 1) \log \left(1 + \frac{P}{\Pr(S_r = 1)}\right) - \frac{1}{T_c},
\]

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where the states $S_t, S_r \in \{0, 1\}$, assigned to the secondary transmitter and receiver respectively, indicate the perception of the primary user about the activity of the primary user. $S_t = 0$ and $S_t = 1$ mean that the cognitive transmitter has sensed the channel to be busy and idle, respectively, and $S_r$ is defined accordingly. These state variables retain their states for a period of $T_c$ channel uses and vary to an i.i.d. state afterwards and as seen above, $T_c$ only affects the lower bound.

For further analysis we assume that all channel channels (i.e., $h_{t,r}, h_{t,r}, h_{t,r}$) follow the same fading model and therefore the states $S_t$ and $S_r$ have the same time variations as the cognitive channel $h_{t,r}$ which means that all remain unchanged for $T_c$ channel uses and change to independent states afterwards. Now corresponding to different values of $T_c$ we will have slow and fast fading processes and need to look into meaningful notions of capacity for each case.

1) Fast fading: Small values of $T_c$ correspond to fast fading for which a meaningful notion of capacity is given by ergodic capacity and is the obtained by averaging over all channel fluctuations.

$$\bar{C}_e^U(\rho) = \mathbb{E}_{h_{t,r}}[C^U(\rho_{h_{t,r}})] = \mathbb{E}_{h_{t,r}}[C^L(\rho_{h_{t,r}})]$$

2) Slow fading: Corresponding to large values of $T_c \gg 1$ we consider the $\epsilon$-outage capacity $C_\epsilon$ as the performance measure for which the bounds are given by

$$P\left(C(\rho, h_{t,r}) < C^U(\rho) \right) \leq \epsilon \quad \text{and} \quad P\left(C(\rho, h_{t,r}) < C^L(\rho) \right) \leq \epsilon$$

We introduce the random variables $\theta_t, \theta_r \in \{0, 1\}$ to account for modeling the spectral activities. $\theta_t = 1$ ($\theta_r = 1$) states that in the vicinity of the cognitive transmitter (receiver) no primary user is using the channel and the channel may be used by the cognitive radios. $\theta_t = 0$ and $\theta_r = 0$ are defined accordingly for busy channels. Now based on the definitions above and those of $S_t$ and $S_r$ which indicate the perception of secondary nodes of the activity of the primary user we have

$$\Pr(S_t = 1) = \Pr(\theta_t = 1, \hat{\epsilon}_t) = \Pr(\theta_t = 1)(1 - P(\epsilon_t)),$$

and

$$\Pr(S_r = 1) = \Pr(\theta_r = 1, \hat{\epsilon}_r) = \Pr(\theta_r = 1)(1 - P(\epsilon_r)).$$

The following theorem provides the main result of this section. The extensive proofs are omitted for brevity purposes.

**Theorem 3** For all SNR regimes we have

$$\bar{C}_e^U(\rho) > C_e^{NC}(\rho), \quad \text{and} \quad \bar{C}_e^L(\rho) > C_e^{NC}(\rho),$$

$$C_e^U(\rho) > C_e^{NC}(\rho), \quad \text{and} \quad C_e^L(\rho) > C_e^{NC}(\rho).$$

Figure 1 demonstrates numerical evaluations for the outage capacity, denoted by $C_\epsilon(\rho)$, where it demonstrates a considerable improvement attained by the proposed cooperation protocol.

$$P\left(C(\rho, h_{t,r}) < C_\epsilon(\rho) \right) \leq \epsilon.$$

**6. CONCLUSIONS**

In this paper we have proposed an opportunistic protocol for spectrum access in cognitive networks where the secondary users are only allowed to use the licensed band when the primary users are silent. The proposed protocol, exploits cooperation between secondary transmitter and its corresponding secondary receiver and exhibits gains in terms of more reliable detection of spectrum holes as well as achieving higher secondary channel capacity. The improvements attained are mainly due to the opportunistically deploying cooperation only when it is deemed to be beneficial and a combination of multipath diversity and dynamic relay assignment.

**7. REFERENCES**


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**Fig. 1.** Lower and upper bounds on outage capacity for different schemes.