Diversity Order of MMSE Single-Carrier Frequency Domain Linear Equalization

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***Abstract*— In this paper we investigate the diversity order of single-carrier frequency domain equalizers (SC-FDE). Specifically, we look at minimum mean square error (MMSE) linear equalizers utilizing block-transmission and cyclic prefix. It is shown that the diversity order in these systems depends on data transmission rate, channel memory length, as well as transmission block length. Analyses reveal that with memory length ν and transmission block length L, for the rates $R \leq \log \frac{L}{\nu}$ full diversity of $\nu+1$ is achievable. For higher rates the achievable diversity order is degraded and is equal to $\lfloor 2^{-R}L \rfloor + 1$. Therefore MMSE SC-FDE has a diversity that varies between 1 and $\nu+1$, and achieves full diversity only for a limited range of data rates.**

I. Introduction

In this paper we analyze the diversity of single-carrier frequency domain equalization. To begin with, we briefly review some of the previous results on the diversity of various classes of equalizers. For symbol-by-symbol equalization, it has been shown that full diversity can be achieved, therefore the only penalty with respect to maximum likelihood detection is a coding gain. This is true for both minimum mean square error (MMSE) decision feedback equalization [1] as well as MMSE linear equalizers [2].

On a different front, orthogonal frequency division multiplexing (OFDM) [3] and single-carrier frequency domain equalization (SC-FDE) [4] have emerged as popular schemes for broadband communication. These techniques process data in blocks that include a cyclic prefix. The advantage of SC-DFE over OFDM is that it does not suffer from peak-to-average ratio problem and high sensitivity to frequency synchronization.

While the diversity order of OFDM has been well studied (e.g. in [5], [6]), that of SC-FDE still needs more investigation. Hedayat *et. al.* in [7] have studied MMSE SC-FDE and have shown the achievability of full diversity for *very low rates* ($R \ll \log \frac{\nu+1}{\nu}$) and diversity one for *high rates* ($R \gg \log \frac{\nu+1}{\nu}$). In this study, the transmission block length is assumed to be fixed and equal to the number of channel taps.

In this paper we analytically show the dependence of diversity order on the data rate and transmission block length for *all rates* and *all transmission block lengths*. We find the tradeoff between data rate and diversity and demonstrate show that transmission block length shapes this tradeoff.

Our proofs consist of two steps. First we fix the transmission block length to be equal to the degrees of freedom, i.e. $L = \nu+1$ and find $\nu+1$ rate intervals for which diversity orders $\{1, \dots, \nu+1\}$ are achievable. Then we generalize the results to the case of arbitrary transmission block length and show how increasing this length affects the rate intervals we previously found.

II. System Model and Definitions

We consider a single carrier system with white Gaussian noise and frequency selective wireless quasi-static fading channel. The channel in the D domain is modeled as

$$H(D) = h_0 + h_1 D + \cdots + h_\nu D^\nu$$

where ν shows the channel memory length and h_i's are zero mean, unit variance complex Gaussian random variables. Assuming transmission block length L and exploiting the cyclic-prefix (CP) guard intervals, the inter-block interferences are removed and the system is governed by

$$\mathbf{y}_{L\times 1} = \mathbf{H}_{L\times L}\mathbf{x}_{L\times 1} + \mathbf{n}_{L\times 1} \tag{1}$$

where $\mathbf{H}$ is a circulant matrix and the channel coefficients are assumed to remain unchanged during each block transmission period.

$$\mathbf{H} = \begin{pmatrix} h_0 & h_1 & \dots & h_{\nu-1} & h_\nu & 0 & \dots & 0 \\ 0 & h_0 & h_1 & \dots & h_{\nu-1} & h_\nu & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ h_1 & h_2 & \dots & h_\nu & 0 & 0 & \dots & h_0 \end{pmatrix}$$

Definition 1: We denote the diversity order of a system with channel memory length ν, block transmission length L and data rate R by $d(R, \nu, L)$.

Definition 2: We say that $f(\rho)$ is *exponentially equal* to ρ^d, $f(\rho) \doteq \rho^d$ when

$$\lim_{\rho\to\infty} \frac{\log(f(\rho))}{\log \rho} = d$$

$\overset{\circ}{\leq}$ and $\overset{\circ}{\geq}$ are defined similarly.

Throughout the analysis, otherwise mentioned, by $\log$ we always mean a base 2 logarithm.

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III. MMSE Linear SC-FDE

Minimum mean-square error (MMSE) linear equalizers do not cancel the interference completely but rather balance a reduction in inter-symbol interference (ISI) and noise/interference power. For the transmission model of Equation (1), the MMSE SC-FDE has the matrix form of

$$\mathbf{W} = (\mathbf{H}^H\mathbf{H} + \rho^{-1}\mathbf{I})^{-1}\mathbf{H}^H$$

where ρ is the transmission signal to noise ratio. Consequently the transformed received signal is

$$\hat{\mathbf{y}} = (\mathbf{H}^H\mathbf{H} + \rho^{-1}\mathbf{I})^{-1}\mathbf{H}^H\mathbf{H}\mathbf{x} + \tilde{\mathbf{n}}$$

It's been shown that the unbiased decision-point SNR of MMSE linear block equalizers is (see e.g. [7], [8])

$$\gamma = \frac{1}{\frac{1}{L}\sum\limits_{k=1}^{L}\frac{1}{1+\rho|\lambda_k|^2}} - 1 \tag{2}$$

where λ_k's are the eigenvalues of matrix $\mathbf{H}$. It is noteworthy that for the circulant matrix $\mathbf{H}$, the eigenvalues are

$$\lambda_k = \sum_{i=0}^{\nu} h_i\omega_k^i \qquad k = 1, \ldots, \nu + 1 \tag{3}$$

where ω_k's are the L'th roots of 1, i.e. $\omega_k^L = 1$. The sequence of the eigenvalues of matrix $\mathbf{H}$, $\{\lambda_k\}$, is also the discrete Fourier transform of the sequence $\{h_0, \ldots, h_\nu, 0, \ldots, 0\}_{1\times L}$ which is the first row of matrix $\mathbf{H}$. The eigenvalues λ_k's are linear combinations of channel coefficients h_i's and therefore have zero mean complex Gaussian distribution.

Remark 1: For the case of $L = \nu + 1$, it can be shown that $E[\lambda_i\lambda_j] = 0$ for $\forall i \neq j$ and due to their Gaussian distribution it is concluded that all the eigenvalues are independent.

IV. Diversity Order Analysis

In this section we analytically find the diversity order achieved by MMSE linear frequency domain equalizers. As the analysis reveal, for a frequency selective channel, the diversity order depends on the data rate, channel memory length and block transmission length.

First we investigate the special case that transmission block length $L = \nu + 1$ and then we generalize the results to the general case of arbitrary transmission block length.

Lemma 1: For independent exponentially distributed random variables $x_1, \ldots, x_n$ with mean $\frac{1}{\lambda}$ we have:

$$\lim_{\lambda\to\infty} P(\sum_{i=1}^{n} x_i > n) = ne^{-n\lambda} \tag{4}$$

Proof: Since the x_i's are exponentially distributed, $\sum x_i$ has Gamma distribution.

$$\sum_{i=1}^{n} x_i \sim \text{Gamma}(n, 1/\lambda)$$

Therefore,

$$\begin{aligned} P(\sum_{i=1}^{n} x_i > n) &= \int_n^\infty \frac{x^{n-1}e^{-x\lambda}\lambda^n}{\Gamma(n)}\, dx \\ &= \frac{1}{(n-1)!}\int_{n\lambda}^\infty x^{n-1}e^{-x}\, dx \\ &= \frac{1}{(n-1)!}\,\Gamma(n, n\lambda) \\ &= \sum_{i=0}^{n-1}\frac{(n\lambda)^i}{i!\, e^{n\lambda}} \end{aligned} \tag{5}$$

Where, $\Gamma(n)$ and $\Gamma(n, n\lambda)$ are *Gamma* and *upper incomplete Gamma* functions respectively. Equation (5) is derived from the expansion of incomplete upper Gamma functions for integer values of n. Therefore we find the limit as follows.

$$\begin{aligned} \lim_{\lambda\to\infty} P(\sum_{i=1}^{n} x_i > n) &= \sum_{i=0}^{n-1}\lim_{\lambda\to\infty}\frac{(n\lambda)^i}{i!\, e^{n\lambda}} \\ &\overset{a}{=} \sum_{i=0}^{n-1}\lim_{\lambda\to\infty}\frac{1}{e^{n\lambda}} \\ &= ne^{-n\lambda} \end{aligned} \tag{6}$$

Equation (a) is derived by applying L'Hospital's rule i times on the term $\frac{(n\lambda)^i}{i!e^{n\lambda}}$ for $i = 0, \ldots, n-1$. ■

Remark 2: For $\lambda = \ln\rho$, $\lim\limits_{\rho\to\infty} P(\sum_{i=1}^{n} x_i > n) \doteq \rho^{-n}$.

Lemma 2: For i.i.d. complex Gaussian random variables $\lambda_1, \ldots, \lambda_n$ and real value $m \in (0, n)$, we have:

$$\lim_{\rho\to\infty} P\left(\sum_{k=1}^{n}\frac{1}{1+\rho|\lambda_k|^2} > m\right) \doteq \rho^{-(\lfloor m\rfloor+1)} \tag{7}$$

where $\lfloor m\rfloor$ denotes the greatest integer less than or equal to m.

Proof: For a complex Gaussian random variable λ and a certain value ρ we define $x \triangleq -\frac{\log|\lambda|^2}{\log\rho}$. It can be readily verified that

$$\lim_{\rho\to\infty} p_X(x) = (\ln\rho)\rho^{-x}$$

which shows x is exponentially distributed with mean $\frac{1}{\ln\rho}$. Now for all k's we define

$$x_k \triangleq -\frac{\log|\lambda_k|^2}{\log\rho} \quad \text{for } 1 \le k \le n \tag{8}$$

Since λ_k's are i.i.d, so are x_k's. We also define

$$u(m) \triangleq \delta(m - \lfloor m\rfloor)$$

where $\delta(\cdot)$ is Dirac's delta function and therefore $u(m)$ has only non-zero values for integer values of m. By expanding the LHS of Equation (7) and ordering the terms on the two sides of the inequality such that all the coefficients of ρ are strictly positive we have

$$\lim_{\rho\to\infty} P\left(\sum_{k=1}^{n}\frac{1}{1+\rho|\lambda_k|^2} > m\right) =$$

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$$\lim_{\rho\to\infty} P\Bigg(\sum_{k=n-\lfloor m\rfloor-u(m)}^{n} (m+k-n)\rho^k \sum_{\substack{A\subset\{1,\dots,n\}\\|A|=k}} \prod_{j\in A} |\lambda_j|^2 < \sum_{k=1}^{n-\lfloor m\rfloor-1} (n-k-m)\rho^k \sum_{\substack{A\subset\{1,\dots,n\}\\|A|=k}} \prod_{j\in A} |\lambda_j|^2 + (n-m) \Bigg)$$

Remark 3: For positive and fixed values of values of $\{a_i\}$ and $\{b_i\}$ and positive random variables $\{u_i\}$ and $\{v_i\}$:

$$\lim_{\rho\to\infty} P(\sum_i a_i\rho^{u_i} < \sum_i b_i\rho^{v_i}) = P(\max_i u_i < \max_i v_i)$$

The terms of the inequality inside $P(\cdot)$ have been ordered such that all ρ^k's have strictly positive coefficients. Now by using the definition in Equation (8), considering the remark above and without loss of generality assuming $x_1 \leq \cdots \leq x_n$ we have

$$\begin{aligned}
&\lim_{\rho\to\infty} P\Bigg(\sum_{k=1}^{n} \frac{1}{1+\rho|\lambda_k|^2} > m\Bigg) \\
&= P\Bigg(\max_{0\leq i\leq\lfloor m\rfloor-u(m)} \sum_{k=1}^{n-i}(1-x_k) < \max_{\lfloor m\rfloor+1\leq i\leq n} \sum_{k=1}^{n-i}(1-x_k)\Bigg) \\
&= \sum_{j=1}^{n-\lfloor m\rfloor-u(m)} P(x_j \geq 1, x_{j-1} < 1) \\
&\qquad \times P\Bigg(\sum_{k=j}^{n-\lfloor m\rfloor} x_k > n-\lfloor m\rfloor - j+1\Bigg) \\
&+ \sum_{j=n-\lfloor m\rfloor+1}^{n} P(x_j \geq 1, x_{j-1} < 1) \\
&\qquad \times P\Bigg(\sum_{k=n-\lfloor m\rfloor}^{j-1} x_k > j+\lfloor m\rfloor - n\Bigg) \\
&\overset{b}{\doteq} \sum_{j=1}^{n-\lfloor m\rfloor-u(m)} (e^{-\log\rho})^{(n-j+1)} \\
&\qquad \times (n-\lfloor m\rfloor - j+1)e^{-(n-\lfloor m\rfloor-j+1)\log\rho} \\
&+ \sum_{j=n-\lfloor m\rfloor+1}^{n} (e^{-\log\rho})^{(n-j+1)} \\
&\qquad \times (n-\lfloor m\rfloor - j+1)e^{-(j+\lfloor m\rfloor-n)\log\rho} \\
&\doteq \lim_{\rho\to\infty} \sum_{j=1}^{n-\lfloor m\rfloor-u(m)} \rho^{-(n-j+1)}\rho^{-(n-j-\lfloor m\rfloor+1)} \qquad (9) \\
&+ \lim_{\rho\to\infty} \sum_{j=n-\lfloor m\rfloor+1}^{n+1} \rho^{-(n-j+1)}\rho^{-(j+\lfloor m\rfloor-n)} \\
&\doteq \lim_{\rho\to\infty} \rho^{-(\lfloor m\rfloor+2+2u(m))} + \rho^{-(\lfloor m\rfloor+1)} \qquad (10) \\
&\doteq \lim_{\rho\to\infty} \rho^{-(\lfloor m\rfloor+1)} \qquad (11)
\end{aligned}$$

Equation (b) holds due to the exponential distribution of x_i's with mean $\frac{1}{\ln\rho}$ and the result of Lemma 1. For any arbitrary order of x_i's via the same approach the same results can be obtained and therefore the result is independent of the order of x_i's. ■

Now by using the results of lemmas 1 and 2, the main contribution of this paper is provided in the following theorem.

Theorem 1: For a frequency selective channel with $(\nu+1)$ taps, cyclic-prefix block transmission of length $L = \nu+1$, linear MMSE frequency domain equalization and data transmission rate R, the achievable diversity order is

$$d(R,\nu,\nu+1) = \lfloor 2^{-R}(\nu+1)\rfloor + 1 \qquad (12)$$

Proof: Given the unbiased decision point SNR in Equation (2) the outage probability is

$$\begin{aligned}
P(\mathcal{O}) &= P(\log(1+\gamma) < R) \\
&= P\Bigg(\sum_{k=1}^{\nu+1} \frac{1}{1+\rho|\lambda_k|^2} > 2^{-R}(\nu+1)\Bigg)
\end{aligned}$$

Since for $R > 0$ we have $2^{-R}(\nu+1) < \nu+1$ and as mentioned earlier in Remark 1, λ_k's are iid with complex Gaussian distribution, the necessary conditions of Lemma 2 are satisfied, therefore

$$\lim_{\rho\to\infty} P(\mathcal{O}) \doteq \rho^{-(\lfloor 2^{-R}(\nu+1)\rfloor+1)} \qquad (13)$$

which shows that the diversity order achieved for data transmission rate R and $L = \nu+1$ is

$$d(R,\nu,\nu+1) = \lfloor 2^{-R}(\nu+1)\rfloor + 1$$

and this concludes the theorem. ■

Figure 1 illustrates numerical results for the case of $\nu = 3$ and $L = 4$. It is shown that by changing the data transmission rate the diversity order varies between 1 and 4 and as stated in Equation (12).

Now we analyze the case that transmission block length exceeds $(\nu+1)$ and find its diversity order. For this purpose we first provide the following lemma which has a key role in linking the cases of $L = \nu+1$ and arbitrary L.

Lemma 3: For two sequences $\mathbf{h}_{1\times L}$ and $\mathbf{h}'_{1\times L'}$ of the form $\{h_0,\dots,h_\nu,0,\dots,0\}$ and discrete Fourier transforms $\{\lambda_k\}$ and $\{\lambda'_k\}$ respectively, we have

$$\lim_{\rho\to\infty} P(\sum_{k=1}^{L} \frac{1}{1+\rho|\lambda_k|^2} > m) \doteq \lim_{\rho\to\infty} P(\sum_{k=1}^{L'} \frac{1}{1+\rho|\lambda'_k|^2} > m) \qquad (14)$$

Proof: For the purpose of brevity, we provide a sketch of the approach to the proof of this lemma and omit the unnecessary justification details.

The two sequences $\mathbf{h}$ and $\mathbf{h}'$ only differ in the number of zeros padded, therefore according to Parseval's theorem we have:

$$\mathcal{E} \triangleq \frac{1}{L}\sum_{k=1}^{L}|\lambda_k|^2 = \frac{1}{L'}\sum_{k=1}^{L'}|\lambda'_k|^2 = \sum_{k=0}^{\nu}|h_k|^2 \qquad (15)$$

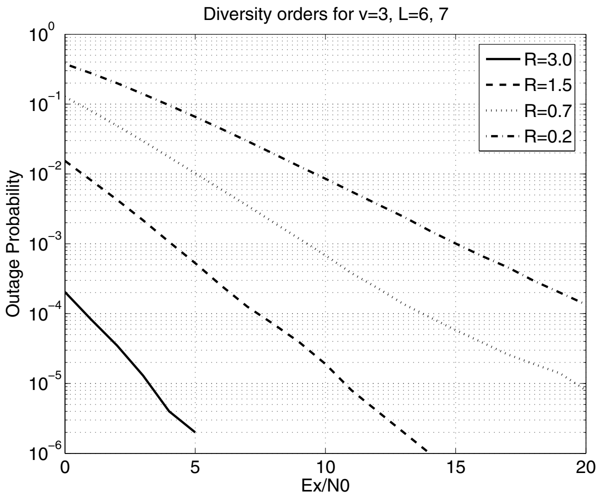

Fig. 1. Diversity order achieved by MMSE-SC-FDE in a channel with memory length $\nu = 3$ and block transmission $L = 4$.

Using the arithmetic-harmonic inequality provides

$$\frac{1}{L}\sum_{k=1}^{L}\frac{1}{1+\rho|\lambda_k|^2} \geq \frac{1}{1+\rho\mathcal{E}} \tag{16}$$

By defining $|\lambda_{\min}|^2 \triangleq \min_i |\lambda_i|^2$ and $|\lambda'_{\min}|^2 \triangleq \min_i |\lambda'_i|^2$ it can be readily shown that

$$\sum_{k=1}^{L'}\frac{1}{1+\rho|\lambda'_k|^2} \leq \frac{1/\rho+\mathcal{E}}{|\lambda'_{\min}|^2}\cdot\frac{L'}{L}\sum_{k=1}^{L}\frac{1}{1+\rho|\lambda_k|^2} \tag{17}$$

We also assume that

$$\lim_{\rho\to\infty} P\Big(\sum_{k=1}^{L}\frac{1}{1+\rho|\lambda_k|^2} > m\Big) \doteq \rho^{-d} \tag{18}$$

Now we define the random variable $U = \frac{L|\lambda'_{\min}|^2}{L'\mathcal{E}}$ which is a function of the channel coefficients h_i's. Also we define the event $\mathcal{A} \triangleq \{|\lambda_{\min}| \neq 0 \ \& \ |\lambda'_{\min}| \neq 0\}$. Considering that λ_k's and λ'_k's have complex Gaussian distribution we have $P(\mathcal{A}) = 1$.

By exploiting the inequality in Equation (17) we get

$$\begin{aligned}
&\lim_{\rho\to\infty} P\Big(\sum_{k=1}^{L'}\frac{1}{1+\rho|\lambda'_k|^2} > m\Big) \leq \\
&\lim_{\rho\to\infty} P\Big(\frac{L|\lambda'_{\min}|^2}{L'\mathcal{E}}\sum_{k=1}^{L}\frac{1}{1+\rho|\lambda_k|^2} > m\Big) = \\
&\lim_{\rho\to\infty} P\Big(\frac{1}{U}\sum_{k=1}^{L}\frac{1}{\rho|\lambda_k|^2} > m|\mathcal{A}\Big) = \\
&\lim_{\rho\to\infty}\int_u P\Big(\sum_{k=1}^{L}\frac{1}{U\rho|\lambda_k|^2} > m|\mathcal{A}, U=u\Big)P(u)du
\end{aligned} \tag{19}$$

Since the two probability terms inside the integral in Equation (19) are dominated by the function $g(u) = 1$, according to *dominated convergence theorem*, the limit and integral are interchangeable. Therefore,

$$\begin{aligned}
&\lim_{\rho\to\infty} P\Big(\sum_{k=1}^{L'}\frac{1}{1+\rho|\lambda'_k|^2} > m\Big) \\
&\leq \int_u \lim_{\rho\to\infty} P\Big(\sum_{k=1}^{L}\frac{1}{1+U\rho|\lambda_k|^2} > m|\mathcal{A}, U=u\Big)P(u)du \\
&\leq \int_u \lim_{\rho\to\infty} (\rho u)^{-d}p(u)du = \lim_{\rho\to\infty}\rho^{-d}\int_u u^{-d}p(u)du \\
&\doteq \rho^{-d}
\end{aligned} \tag{20}$$

Which shows that

$$\lim_{\rho\to\infty} P(\sum_{k=1}^{L'}\frac{1}{1+\rho|\lambda'_k|^2} > m) \mathrel{\dot{\leq}} \lim_{\rho\to\infty} P(\sum_{k=1}^{L}\frac{1}{1+\rho|\lambda_k|^2} > m) \tag{21}$$

Similarly it can be demonstrated that

$$\lim_{\rho\to\infty} P(\sum_{k=1}^{L}\frac{1}{1+\rho|\lambda_k|^2} > m) \mathrel{\dot{\leq}} \lim_{\rho\to\infty} P(\sum_{k=1}^{L'}\frac{1}{1+\rho|\lambda'_k|^2} > m) \tag{22}$$

Equations (21) and (22) prove that the LHS and RHS of the Equation (14) are exponentially equal and thus concludes the proof Lemma 3. ■

Figure 2 provides numerical simulation when $\nu = 3$ and compares two different cases of $L = 6$ and $L = 7$ for different values of $m = 0.5, 1.5, 2.5, 3.5$. Numerical results support the exponential equality in Equation (14).

Theorem 2: In a frequency selective channel with cyclic-prefix linear MMSE frequency domain equalization, increasing the block equalization length expands the rate interval for which a certain diversity order d is achievable:

$$d(R,\nu,L) = d(R + \log\frac{L'}{L}, \nu, L')$$

Proof:

$$\lim_{\rho\to\infty} P\Big(\frac{1}{L}\sum_{k=1}^{L}\frac{1}{1+\rho|\lambda_k|^2} > 2^{-R}\Big) \tag{23}$$

$$= \lim_{\rho\to\infty} P\Big(\sum_{k=1}^{L}\frac{1}{1+\rho|\lambda_k|^2} > L2^{-R}\Big)$$

$$\doteq \lim_{\rho\to\infty} P\Big(\sum_{k=1}^{L'}\frac{1}{1+\rho|\lambda'_k|^2} > L2^{-R}\Big) \tag{24}$$

$$= \lim_{\rho\to\infty} P\Big(\frac{1}{L'}\sum_{k=1}^{L'}\frac{1}{1+\rho|\lambda'_k|^2} > \frac{L}{L'}2^{-R}\Big)$$

$$= \lim_{\rho\to\infty} P\Big(\frac{1}{L'}\sum_{k=1}^{L'}\frac{1}{1+\rho|\lambda'_k|^2} > 2^{-(\log\frac{L'}{L}+R)}\Big) \tag{25}$$

Equation (24) holds according to Lemma (3) for $m = L2^{-R}$. Exponential equality of equations (23) and (25) proves the theorem. ■

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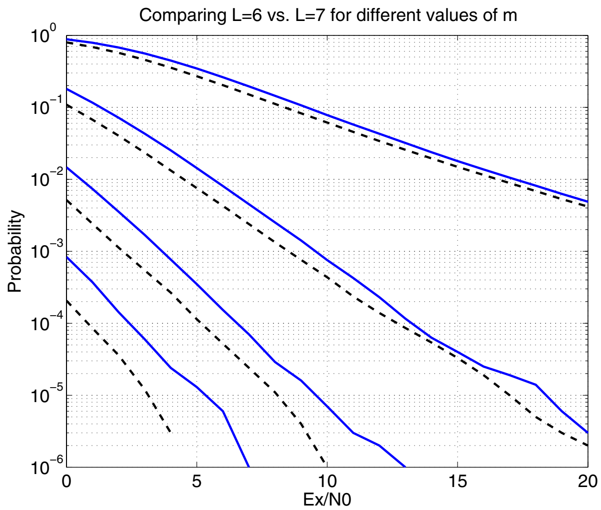

Fig. 2. Numerical simulation for the case $\nu = 3$ and $L = 6, 7$. Solid lines and dashed lines correspond to the case $L = 6$ and $L = 7$ respectively. The plots from top to bottom correspond to $m = 0.5, 1.5, 2.5, 3.5$.

So far in Theorem 1 we have demonstrated that when $L = \nu + 1$, how increasing the data rate can force the diversity order change between 1 and $\nu + 1$ and then in Theorem 2 we have analyzed the effect of transmission block length. Putting these results together leads to the following statement.

Corollary 1: In a frequency selective fading channel with memory length $\nu + 1$ and MMSE frequency domain linear equalization and transmission block length L, for data transmission rate R the achievable diversity order is:

$$d(R, \nu, L) = \begin{cases} \nu + 1 & R \leq \log \frac{L}{\nu} \\ \lfloor 2^{-R} L \rfloor + 1 & \text{o.w.} \end{cases} \quad (26)$$

As stated above, for data transmission rate below $\log \frac{L}{\nu}$ achieving full diversity is guaranteed and as the rate increases the diversity order is degraded. Specifically the diversity order for the rates lying in the interval $[\log \frac{L}{i}, \log \frac{L}{i-1})$ is i for $i = 1, \ldots, \nu$.

V. Conclusion

In this paper we show that unlike symbol-by-symbol equalization, single-carrier frequency domain equalizers with cyclic-prefix block transmission may not fully capture the diversity inherent in the frequency-selective channels. We prove that the diversity order is affected by data rate as well as transmission block length. The results show that at high rates and low block-lengths, only diversity 1 is achieved, but by increasing the transmission block length and/or decreasing data rate, diversity order can be increased up to a maximum level of $\nu + 1$. We characterize the dependence on these two parameters in our results.

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