A Broadcasting Relay for Orthogonal Multiuser Channels

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***Abstract*— This paper introduces broadcasting relay nodes for orthogonal multiuser channels. The underlying idea is that a single relay node is shared by multiple source-destination pairs. In this scheme, the relay node receives the messages of multiple independent sources, and broadcasts a single superimposed signal to multiple destinations. Compared to dedicated relay scenarios, large gains in capacity region and outage capacity is possible with the shared relay scenario. We consider the special case of two pairs, and examine discrete memoryless channels and Gaussian channels assuming degradedness for the relay channels and physically degradedness for the broadcast channel. Upper bounds on capacity are obtained and shown to be achievable. The analysis is also extended to Rayleigh fading channels, where outage regions are investigated.**

I. Introduction

Relay channels [1], [2], have provided new techniques for improving the bandwidth efficiency and reliability of wireless networks. Cooperative diversity protocols and relaying schemes in uplink and downlink channels have been studied in [3], [4], [5].

We consider a multiuser network with multiple pairs of source-destination nodes, each communicating through their dedicated orthogonal channel. For taking the advantage of relaying in these systems, a straightforward approach is to assign one relay node to each source-destination pair. In this paper we investigate an alternative approach for half-duplex relays: to share one common relay among multiple source-destination pairs. In this scheme each source transmits to its corresponding destination in an orthogonal channel; the relay listens to all these transmissions and broadcasts to all destination nodes in a separate orthogonal channel.

The motivation for our study, in part, arises from the well-known loss of multiplexing gain in half-duplex relaying. It is possible to re-capture some of the rate, which was lost due to channel slots occupied by dedicated half-duplex relays, by the efficiencies of a joint relay.

The basic idea is that, if we combine all these relays into one, it is possible to utilize relay resources more efficiently. First, broadcast communication is in general more efficient than orthogonal communication, so the broadcast relay will achieve some gains in that respect. Second, a common relay will allow a more efficient allocation of relay resources, compared with dedicated relays. For example, since all relaying activity is concentrated in one node, its component powers and rates can be flexibly divided so that the user that needs more assistance will receive more help from the relay. Therefore the capacity region will be enhanced significantly.

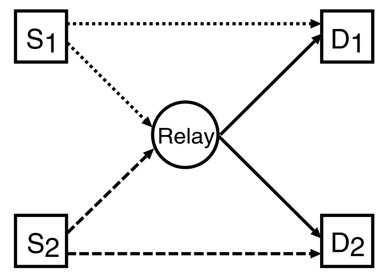

Fig. 1. Broadcasting relay for two orthogonal channels

Aside from the bandwidth efficiency and outage performance, from a practical point of view, this method can be helpful when there are not a large number of dedicated relays in a network, so that relays must be shared.

Clearly broadcast relaying will require more capable receiver nodes, compared to orthogonal relaying. Therefore, one of the goals of this study has been to quantify the gains of broadcast relays, to see if they are potentially worth the added complexity. Numerical results, to be shown in the sequel, indicate that the gains are indeed significant and may be worthwhile.

The outline of this paper is as following: section II describes the system model under consideration. Section III concentrates on finding the capacity in discrete memroyless channels. The results will be examined for Gaussian channels in section IV. In section V we introduce Rayleigh fading to gaussian channels and look into the capacity region and outage behavior. Section VI concludes the paper.

II. System Model

We consider a network, as depicted in Fig. 1, with two pairs of source-destination nodes, each communicating through its dedicated orthogonal channel and a time division multiplexing scheme is used to orthogonalize the channels. Also it is assumed that the relay node can not receive and transmit at the same time in the same frequency band. The transmission is accomplished in three steps:

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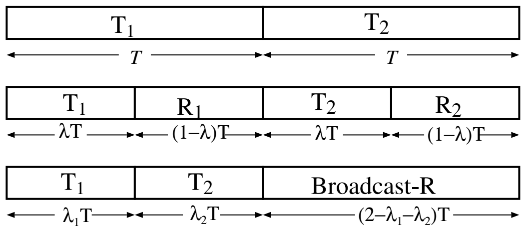

Fig. 2. Three transmission schemes:Non-Relay Dedicated Relay,Joint Relay

- *Source 1* in transmit mode. *Relay* and *Destination 1* in receive mode.
- *Source 2* in transmit mode. *Relay* and *Destination 2* in receive mode.
- *Relay* in broadcast mode. *Destination 1* and *Destination 2* in receive mode.

As Fig. 2 illustrates, two time slots of durations $\lambda_1 T$ and $\lambda_2 T$ are allocated for each of the transmissions in the first and second steps, and the broadcast is performed in the third time slot with duration $(2 - \lambda_1 - \lambda_2)T$. Later we will see that for obtaining the capacity region, the optimum duration for each of the above three transmission phases needs to be calculated.

Now based on the notation of the random variables $X_1, X_2, X_3, Y_1, Y_2, Y_3$ illustrated in Fig. 4 we provide the following definitions.

Definition 1: A *broadcasting relay* channel consists of two channel input alphabets $\mathcal{X}_1$ and $\mathcal{X}_2$, relay channel input $\mathcal{X}_3$ and channel outputs $\mathcal{Y}_1$, $\mathcal{Y}_2$. The conditional joint probability distribution $p(y_1, y_2|x_1, x_2, x_3)$ describes the input output transition function.

Definition 2: A *broadcasting relay* channel is said to be degraded if $X_1 \rightarrow X_3 \rightarrow Y_1$ and $X_2 \rightarrow X_3 \rightarrow Y_1 \rightarrow Y_2$ are Markov chains. Hence

$$p(y_1, y_2|x_1, x_2, x_3) = p(y_1|x_2, x_3)p(y_2|x_1, y_1) \quad (1)$$

In other words, given x_3, y_1 is independent of x_1 and given y_1, y_2 is independent of x_2 and x_3.

III. Discrete Memoryless Channel

We assume discrete memoryless channels. The main contribution of this paper is the following theorem:

Theorem 1: The capacity region of a degraded broadcasting relay channel for transmitting independent source messages X_1 and X_2 to the receivers Y_1 and Y_2 correspondingly is the convex hull of all rates (R_1, R_2) which satisfy:

$$R_1 \leq \min\{I(X_1; Y_{31}|X_3), I(X_1; Y_1|X_2, X_3) + I(X_3; Y_1|U)\} \quad (2)$$

$$R_2 \leq \min\{I(X_2; Y_{32}|X_3), I(X_2; Y_2|X_1, X_3) + I(U; Y_2)\} \quad (3)$$

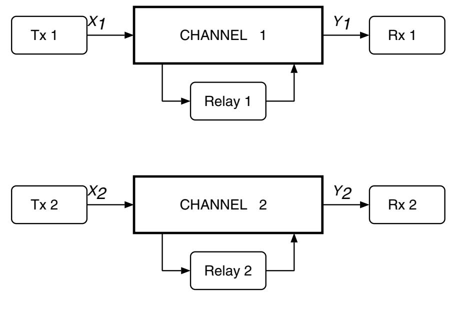

Fig. 3. Dedicated relays for orthogonal multiuser channels

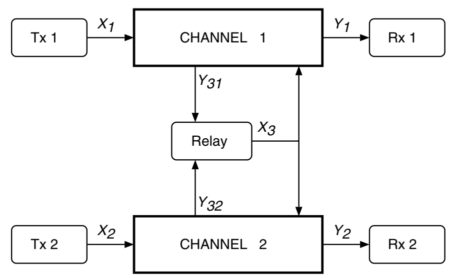

Fig. 4. Broadcast relay for orthogonal multiuser system

over all joint distribution $p(u)p(x_3|u)p(y_1, y_2|x_3)$ where Y_{31} and Y_{32} are the components of Y_3 received via channels 1 and 2 respectively. U is an auxiliary random variable such that its cardinality $|U|$ is upper bounded by $\min\{|X_3|, |Y_1|, |Y_2|\}$.

Proof: An outline of the proof is provided in two steps.

A. Upper Bounds

The procedure of finding upper bounds on rates involves three steps. First the results of max flow min cut theorem [6] are applied. Eight upper bounds are found for each R_i, $i = 1, 2$. After some simplifications it will be observed that two of them impose stronger conditions. Considering the degradedness of relay channels we have this upper bound for R_1:

$$R_1 \leq \min\{I(X_1; Y_3|X_2, X_3), I(X_1, X_3; Y_1|X_2)\} \quad (4)$$

Then by incorporating the impacts of orthogonal transmissions and physically degradedness of the broadcast channel [7] the upper bound in equation (2) is found for R_1. Taking the same approach for R_2 will lead to the expression shown in equation (3).

B. Achievability

In this section an outline for proving the achievability of the upper bounds obtained in the previous section is provided.

1) Regular Encoding: We use the regular encoding approach of [8]. B blocks, each of n symbol pairs are considered. A sequence of $B-1$ message pairs (w_i^1, w_i^2) where $w_i^1 \in [1, 2^{nR_1}]$ and $w_i^2 \in [1, 2^{nR_2}]$ is sent over the channels. The transmission is performed by using codewords $x_1(i, j)$,

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TABLE I
CODE CONSTRUCTION SCHEME

Block 1	Block 2	Block 3	Block 4
$U(1)$	$U(w_1^2)$	$U(w_2^2)$	$U(w_3^2)$
$X_2(1, w_1^2)$	$X_2(w_1^2, w_2^2)$	$X_2(w_2^2, w_3^2)$	$X_2(w_3^2, 1)$
$X_3(1, 1)$	$X_3(w_1^1, w_1^2)$	$X_3(w_2^1, w_2^2)$	$X_3(w_3^1, w_3^2)$
$X_1(1, w_1^1)$	$X_1(w_1^1, w_2^1)$	$X_1(w_2^1, w_3^1)$	$X_1(w_3^1, 1)$

$x_2(m, n)$ and $x_3(i, m)$ where $i, j \in [1, 2^{nR_1}]$ and $m, n \in [1, 2^{nR_2}]$. As $B \to \infty$ rate $R_i.\frac{B}{B-1}$ is arbitrarily close to R_i.

In the first block, *Source 1* transmits codeword $x_1(1, w_1^1)$, *Source 2* transmits codeword $x_2(1, w_1^2)$ and *Relay* transmits codeword $x_3(1, 1)$. As far as $R_i \leq (X_i; Y_{3i}|X_3)$, for sufficiently large n, the relay node is capable of decoding codewords x_1 and x_2 perfectly. Assuming the relay node has decoded x_1 and x_2 correctly, the next codewords to be transmitted are $x_1(w_1^1, w_2^1)$, $x_2(w_1^2, w_2^2)$ and $x_3(w_1^1, w_1^2)$. The same code construction scheme continues to the last block where the codewords $x_1(w_{B-1}^1, 1)$, $x_2(w_{B-1}^2, 1)$ and $x_3(w_{B-1}^1, w_{B-1}^2)$ are transmitted. Table 1. illustrates the code construction scheme.

2) Backward Decoding: Destination nodes start decoding after receiving all $B - 1$ blocks. By performing Willems's backward decoding [9], the receivers first decode w_{B-1}^1 and w_{B-1}^2 from y_{1_B}, y_{2_B} which only depend on $x_1(w_{B-1}^1, 1)$, $x_2(w_{B-1}^2, 1)$ and $x_3(w_{B-1}^1, w_{B-1}^2)$. Willems has shown that for sufficiently large n receiver nodes can decode reliably as long as

$$R_1 \leq I(X_1, X_3; Y_1|X_2) \tag{5}$$

$$R_2 \leq I(X_2, X_3; Y_2|X_1) \tag{6}$$

Assuming w_{B-1}^1 and w_{B-1}^2 are known to receivers 1 and 2 respectively, next w_{B-2}^1 and w_{B-2}^2 will be decoded from $y_{1_{B-1}}$, $y_{2_{B-1}}$ which only depend on $x_1(w_{B-2}^1, w_{B-1}^1)$, $x_2(w_{B-2}^2, w_{B-1}^2)$ and $x_3(w_{B-2}^1, w_{B-2}^2)$. Again as long as conditions in equations (5) and (6) are satisfied w_{B-2}^1 and w_{B-2}^2 are reliably decodable. and the backward procedure continues until all message words are decoded. ■

IV. THE GAUSSIAN CHANNEL

In this section we find the capacity region for Gaussian broadcasting relay channel. The main result in this section is expressed in *Theorem 2*. But first we look at the channel model utilized throughout the analysis in this section.

As mentioned earlier we consider a time division scheme to orthogonalize the channels. The source i, $i \in \{1, 2\}$, will be in transmission mode during the time slot i with duration $\lambda_i T$. During the third time slot, which its duration is $(2-\lambda_1-\lambda_2)T$, the relay node will be in broadcast mode.

The source nodes, destination nodes and the relay node are referred by s_i, d_i and r respectively. We use the baseband equivalent discrete-time model for the channel. First we consider the channel for direct transmissions during time slots 1 and 2. For the user i we have:

$$Y_{i1}(k) = X_i(k) + Z_{s_i,d_i}(k) \tag{7}$$

where $X_i(k)$ is the transmitted signal by source i and Y_{i1} is the received signal by destination i through the direct channel. The AWGN is modeled by Z_{ij} with variance N_{ij}.

During the time slots 1 and 2, the relay node is in receive mode. The channel linking the source i and the relay is:

$$Y_{3i}(k) = X_i(k) + Z_{s_i,r}(k) \tag{8}$$

And finally the broadcast channel between the relay and destination node i is modeled as:

$$Y_{i2}(k) = X_3(k) + Z_{r,d_i}(k) \tag{9}$$

We assume that the transmission power at the source nodes as well as the relay node is equal to the orthogonal transmission case, P. Based on the channel model defined, we have the following theorem:

Theorem 2: The capacity region of a degraded broadcasting relay channel is the convex hull of all rates $\{R_1, R_2\}$ given :

$$R_1+R_2 \leq \max_{0\leq\lambda_1,\lambda_2\leq 1} \sum_{i=1}^{2} \min\{\mathcal{C}_{1_i}(\lambda_i), \mathcal{C}_{2_i}(\lambda_i)+\mathcal{C}_{3_i}(\lambda_1, \lambda_2, \theta)\} \tag{10}$$

over all values of $\theta \in [0, 1]$ where

$$\mathcal{C}_{1_i}(\lambda_i) = \lambda_i \mathcal{C}\left(\frac{P}{N_{s_i,r}}\right) \tag{11}$$

$$\mathcal{C}_{2_i}(\lambda_i) = \lambda_i \mathcal{C}\left(\frac{P}{N_{s_i,d_i}}\right) \tag{12}$$

$$\mathcal{C}_{3_1}(\lambda_1, \lambda_2, \theta) = (1 - \lambda_1 - \lambda_2)\mathcal{C}\left(\frac{\theta P}{N_{r,d_1}}\right) \tag{13}$$

$$\mathcal{C}_{3_2}(\lambda_1, \lambda_2, \theta) = (1 - \lambda_1 - \lambda_2)\mathcal{C}\left(\frac{\overline{\theta} P}{N_{r,d_2} + \theta P}\right) \tag{14}$$

where $\mathcal{C}(x) = \frac{1}{2}\log_2(1 + x)$.

The parameter θ reflects the power allocation ratio for the broadcast channel. θP is power of the part of the broadcasted signal carrying the information of *Source 1* and $\overline{\theta}P = (1-\theta)P$ is the power of the part carrying information of *Source 2*. Values of $\theta = 0$ and $\theta = 1$ show the special cases where there is no information at the relay node intended to destination 1 and 2 respectively. As $\theta \to 1$, more power is dedicated to transmit the first user's information. Corresponding to each value of $0 \leq \theta \leq 1$, an achievable rate for each user is defined. The convex hull of all these rates identifies the capacity region.

Equations (11) and (12) represent the maximum rate of error free transmission from the source nodes to the relay node and destination nodes scaled by the factor λ_i which captures the effect of changing the duration that the channel is used by each of the source nodes as depicted in Figure 2.

Equations (13) and (14) demonstrate the contribution of the relay to the achievable rates. During the third time slot

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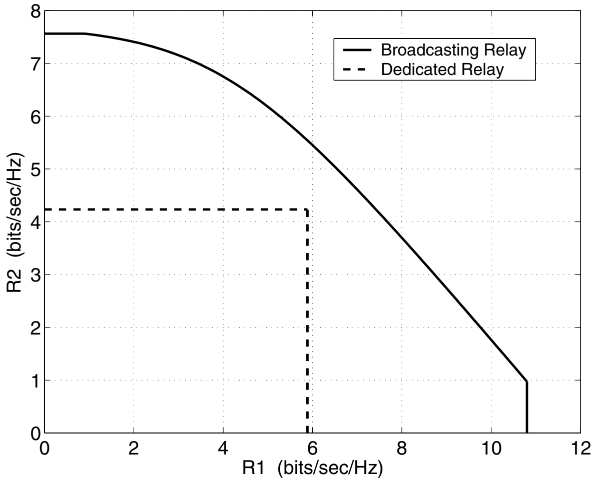

Fig. 5. Capacity Region

when the relay is in transmit mode and destination nodes are in receive mode, *Destination 1* is capable of decoding the part of the information intended to *Destination 2* as well as that part intended to itself. On the other side, *Destination 2* only decodes its own data so that the signal containing the information of user 1 seems to it as interference and is treated as noise, so that the term θP shows up in equation (14).

V. FADING CHANNEL

A. Analytical Results

In the previous section, the channel fading effects were ignored, assuming AWGN channels. In this section, we consider wireless fading channels with Rayleigh distribution. Channel coefficients are denoted by $\alpha_{i,j}$. Throughout the analysis of fading channels, $\alpha_{i,j}$ represents the effects of fading, path loss and shadowing of the wireless channel between two nodes i and j, where $i, j \in \{s_1, s_2, r, d_1, d_2\}$. Channel gains $|\alpha_{i,j}|^2$ are assumed to be independent and exponentially distributed with parameter $\gamma_{i,j}$:

$$p_{\alpha_{i,j}}(u) = \frac{1}{\gamma_{i,j}} \quad \exp\,(-\frac{u}{\gamma_{i,j}}) \tag{15}$$

We study the performance of the relaying scheme introduced in terms of outage events. Specifically we analyze the outage probability and outage capacity region and compare the results to those of the scheme with dedicated relays in Fig. 3.

We use the baseband equivalent discrete-time model for the fading channel. For the first and second time slots for direct transmission from user i we have:

$$Y_{i1}(k) = \sqrt{P}\alpha_{s_i,d_i} X_i(k) + Z_{s_i,d_i}(k) \tag{16}$$

The channel linking the source nodes with the relay node is modeled as below:

$$Y_{3i}(k) = \sqrt{P}\alpha_{s_i,r} X_i(k) + Z_{s_i,r}(k) \tag{17}$$

And finally, the model of the broadcast channel between the relay node and destination nodes is:

$$Y_{i2}(k) = \sqrt{P}\alpha_{r,d_i} X_3(k) + Z_{r,d_i}(k) \tag{18}$$

where as mentioned earlier, a fraction θ of the power is allocated to *Destination 1* and the remainder is allocated to the other user.

The mutual information I_i as a function of fading coefficients which are random variables, is itself a random variable. Compared to specific rates R_i, the outage event is defined as:

$$I_i(\theta) = \min\{\mathcal{C}_{1_i}(\lambda_i^*), \mathcal{C}_{2_i}(\lambda_i^*) + \mathcal{C}_{3_i}(\lambda_1^*, \lambda_2^*, \theta)\} < R_i \tag{19}$$

$\mathcal{C}_{i_j}$s are modified from Equations (11) to (14) such that the effect of channel gains are included , i.e. $P \to |\alpha|^2 P$. λ_1^* and λ_2^* reflect the optimum durations for the three phases of transmissions discussed in the system model and according to *Theorem* 2, λ_1^* and λ_2^* maximize the sum rate. Therefore the outage probability is given by:

$$\begin{aligned} P_{out_i}(\theta) &= P(I_i(\theta) < R_i) \\ &= 1 - P(\mathcal{C}_{1_i} \geq R_i)\; P(\mathcal{C}_{2_i} + \mathcal{C}_{3_i}(\theta) \geq R_i) \\ &= 1 - P_{i_1}\; P_{i_2} \end{aligned} \tag{20}$$

The expansions of P_{i_1} and P_{i_2} are included in Appendix.

The term $\mathcal{C}_{3_1}(\theta)$ is an increasing function of θ and so is the mutual information function $I_1(\theta)$. Therefore, Equation (20) shows that $P_{out_1}(\theta)$ is decreasing in θ. On the other side, $P_{out_2}(\theta)$ is an increasing function of θ. A good metric for finding the best power allocation scheme is the outage probability of a transmission consisting of two frames, one from each users. We allocate the power such that this outage probability is minimized.

$$P_{out} = \min_{0 \leq \theta \leq 1} 1 - (1 - P_{out_1}(\theta))(1 - P_{out_2}(\theta)) \tag{21}$$

The value of θ leading to P_{out} corresponds to the optimum power allocation scheme. The analytical approach for solving this problem is discussed in [10].

B. Numerical Results

Figure 6 shows the outage probability versus SNR and figure 7 shows the the tradeoff between P_{out_1} and P_{out_2}. P denotes the transmit power at each node and the average noise density is denoted as $N_{i,j}$.

The numerical results shown in Figures 6 and 7 are based on the following setup:

- *Transmission rates* : $R_1 = 0.6$, $R_2 = 0.4$ bits/sec/Hz.
- *Noise power* : $N_{i,j} = 1$ for $(i, j) \neq (r, d_2)$ and $N_{r,d_2} = 4$; the broadcast channel is degraded
- *Distribution of fading coefficients* : $\gamma_{s_i,r} = 0.8$ and for other channels, $\gamma_{i,j} = 0.6$; the relay channels are degraded.

Figure 6 compares the outage probability behavior in the two cases of dedicated relaying and broadcasting relay. As

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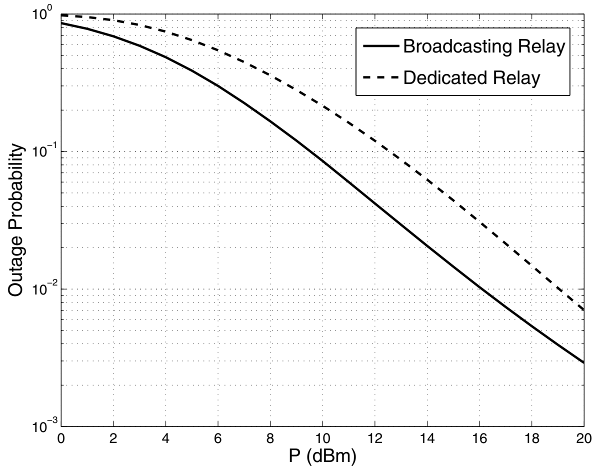

Fig. 6. Outage Probability

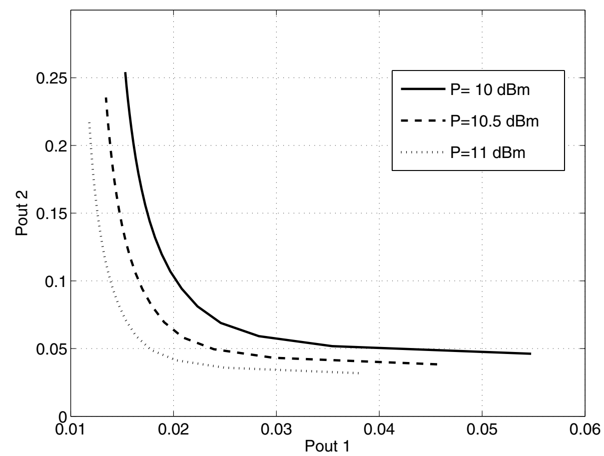

Fig. 7. Outage probabilities, unequal transmit powers

depicted in the figure, the broadcasting relay channel shows an improved outage behavior and the improvement is increasing with SNR.

Figure 7 illustrates the outage probability region for the rates $(R_1, R_2) = (0.6, 0.4)$ and compares it for three different transmit power values. The achievable regions shown in the figureare the regions above the curves. For a fixed transmit power, when the outage probability of the second user decreases, there will be a rapid increase in the outage probability of the first user. The reason is that we have assumed that the second user is using a worse channel compared to the first users, so any improvement for the second channel costs more power compared to the power needed for the same improvement for the first user.

VI. Conclusion

In this paper we develop a relaying scheme for distributed networks. Achievable rates are significantly improved, as shown by analytical results and simulations. Outage probability is also improved significantly. This work may be extended to multiple source-destination pairs in a straightforward manner.

Appendix

$$P_{i_1} = \exp\left(-\frac{2^{2R}-1}{\gamma_{s_i,r}\cdot\frac{P}{\sigma^2_{s_i,r}}}\right) \tag{22}$$

$$P_{1_2} = \int_1^{+\infty}\int_{\frac{2^{2R}}{u_1}}^{+\infty} \frac{u_1^{\frac{1-\lambda_1^*}{\lambda_1^*}}}{\frac{\lambda_1^* P\gamma_{s_1,d_1}}{N_{s_1,d_1}}} \cdot \frac{u_2^{\frac{\lambda_1^*+\lambda_2^*}{1-\lambda_1^*-\lambda_2^*}}}{\frac{(1-\lambda_1^*-\lambda_2^*)\theta P\gamma_{r,d_1}}{N_{r,d_1}}} \times \exp\left(-\frac{u_1^{\frac{1}{\lambda_1^*}}-1}{\frac{P\gamma_{s_1,d_1}}{N_{s_1,d_1}}} - \frac{u_2^{\frac{1}{1-\lambda_1^*-\lambda_2^*}}-1}{\frac{\theta P\gamma_{r,d_1}}{N_{r,d_1}}}\right) du_2 du_1 \tag{23}$$

$$P_{2_2} = \int_1^{+\infty}\int_{\frac{2^{2R}}{u_1}}^{+\infty} \frac{u_1^{\frac{1-\lambda_2^*}{\lambda_2^*}}}{\frac{\lambda_2^* P\gamma_{s_2,d_2}}{N_{s_2,d_2}}} \times \frac{(1-\theta)u_2^{\frac{\lambda_1^*+\lambda_2^*}{1-\lambda_1^*-\lambda_2^*}}}{\frac{(1-\lambda_1^*-\lambda_2^*)P\gamma_{r,d_2}}{N_{r,d_2}}(\theta u_2^{\frac{1}{1-\lambda_1^*-\lambda_2^*}}-1)^2} \times \exp\left(-\frac{u_1^{\frac{1}{\lambda_2^*}}-1}{\frac{P\gamma_{s_2,d_2}}{N_{s_2,d_2}}} - \frac{u_2^{\frac{1}{1-\lambda_1^*-\lambda_2^*}}-1}{\frac{P\gamma_{r,d_2}}{N_{r,d_2}}(\theta u_2^{\frac{1}{1-\lambda_1^*-\lambda_1^*}}-1)}\right) du_2 du_1 \tag{24}$$

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