Multiuser Diversity Gain in Cognitive Networks with Distributed Spectrum Access

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Abstract—Opportunistic allocation of resources to the best link in large multiuser networks offers considerable improvement in spectral efficiency, which is often referred to as multiuser diversity gain and can be cast as double logarithmic growth of the network throughput with the number of users. In this paper we consider large decentralized cognitive networks granted concurrent spectrum access with license-holding users. We assume that the primary network affords to accommodate one secondary user per any under-utilized spectrum band and seek allocating such spectrum bands to a subset of the existing secondary users. We first consider the optimal spectrum-secondary user pairing, which is supervised by a central entity fully aware of the instantaneous channel conditions, and show that the throughput of the cognitive network scales double logarithmically with the number of secondary users (N) and linearly with the number of available spectrum bands (M), i.e. M log log N. Next, we propose a distributed spectrum allocation scheme, which does not necessitate a central controller or any information exchange between different secondary users and obeys the optimal throughput scaling law. This scheme requires that some secondary transmitter-receiver pairs exchange log M information bits among themselves. We also show that the aggregate amount of information exchange between secondary transmitter-receiver secondary pairs is also asymptotically equal to M log M. Finally, we show that our distributed scheme also guarantees fairness among the secondary users, meaning that they are equally likely to get access to an available spectrum band.

Index Terms—Cognitive radio, distributed, fairness, multiuser diversity, spectrum allocation.

I. INTRODUCTION

Dense multiuser networks offer significant spectral efficiency improvement by dynamically identifying and allocating the resources to the best link. The improvements attained are often referred to as multiuser diversity gain and rest on the basis of opportunistically allocating all the resources to the most reliable link, which indeed, supports the highest data rate among all the links. The performance in such schemes relies on the peak, rather than average, channel conditions and improves by adding more number of users, as it gets more likely to have a user with an instantaneously strong link.

The recent advances in secondary spectrum leasing [1] and cognitive networks [2] consider accommodating unlicensed users (secondary users or cognitive radios), within the license-holding networks and providing them with the privilege of accessing under-utilized spectrum bands. Among different spectrum sharing schemes, underlaid spectrum access [3] has received a great deal of attention. This scheme allows for simultaneous spectrum access by the primary and secondary users, provided that the power of secondary users is limited such does not harm the primary transmission.

In this paper we investigate opportunistic underlaid spectrum access by secondary users and find the multiuser diversity gain by analyzing the sum-rate throughput scaling of cognitive networks. Such analysis for cognitive networks differs from those of primary networks studied in [4]–[8] in two directions. First, the transmissions in the cognitive network are contaminated by the interferences induced by the primary users. The existence of such interference does not make opportunistic communication possible by merely finding the strongest secondary link and necessitates also accounting for the effect of interference. Secondly, and more importantly, the uplink/downlink transmissions in the networks referenced above, entail feedback from the users to the base station and it is the base station which decides to whom the resources should be allocated. Cognitive networks, in contrary, often are assumed to lack any infrastructure or central entity which requires that spectrum allocation be carried out in a distributed way.

To address these two issues, we first focus on only accounting for the effects of interference and assume that the cognitive network benefits from having a central decision-making entity, who is fully aware of all cognitive users' instantaneous channel realizations, and find the sum-rate throughput scaling factor. This result, providing the optimal scaling factor, presents an upper bound on the throughput yielded by any distributed spectrum allocation scheme. In the next step, we offer our distributed algorithm where the cognitive users decide about accessing a channel merely based on their own observation of instantaneous network conditions.

The analyses reveal that, interestingly, in both centralized and distributed setups, the sum-rate throughput scales double logarithmically with the number of users, which is the optimal growth and is the same as that of centralized primary networks. Therefore, the interference from the primary network incurs no loss on the multiuser diversity gain of the cognitive network.

We also examine how fairness is maintained in our distributed scheme. In general, in opportunistic communication schemes, there exist a conflict between fairness and multiuser diversity gain, as the network tends to reserve the resources for the most reliable links, which leads the network to be dominated by the users with strong links. We show that, however, in
our distributed scheme, we can ensure fairness among the cognitive users by providing them with the same chance for accessing an available spectrum band.

Another line of research, other than diversity gain analysis, on throughput scaling is the notion of scaling laws of ad-hoc networks first introduced by Gupta and Kumar [9]. While in diversity gain analysis all the system resources are dynamically allocated to the best link for establishing a point-to-point communication, in analyzing the scaling laws of ad-hoc networks it is assumed that all the users are active simultaneously and may exploit different physical-layer capabilities, including relaying and multi-hop transmission. The idea of scaling law of ad-hoc networks has been extended for cognitive networks in [10], [11].

II. DESCRIPTIONS

A. System Model

We consider a decentralized cognitive network comprising of N secondary transmitter-receiver pairs coexisting with the primary transmitters via underlaid [3] spectrum access, which involves simultaneous transmissions by primary and secondary users on the same spectrum band. The primary network affords to accommodate 1 ≤ M ≪ N secondary users and allows them to access the non-overlapping spectrum bands B1, . . . , BM such that each band is allocated to exactly one secondary transmitter-receiver pair. We also assume that each secondary transceiver is potentially capable of operating on each of the M spectrum bands, which can be facilitated by having appropriate reconfigurable hardware.

We consider single-antenna cognitive and primary users. We assume quasi-static flat fading channels and denote the channel from the i-th primary transmitter to the j-th secondary receiver in the m-th spectrum band (Bm) by h^m_{i,j} ∈ C and denote the channel between the i-th secondary transmitter-receiver pair in the m-th spectrum band (Bm) by g^m_{i} ∈ C. Let x^p_i(t) and x^s_i(t) represent the transmitted signals by the i-th primary transmitter and the i-th secondary transmitter, respectively. We assume that there might be a group of users active on each spectrum band Bm and define the set Sm such that it contains the indices of such users, where clearly |Sm| ≥ 0. If the n-th secondary pair transmits on Bm, then the received signal at the n-th secondary receiver is given by

\[ y_n = \sqrt{\gamma_m} g^m_n x_n^s + \sum_{j \in S_m} \sqrt{\gamma_{m,j}} h^m_{n,j} x^p_j + z_n^m, \]

where \( z_n^m \sim \mathcal{CN}(0, N_0) \) is the additive white Gaussian noise at the n-th receiver. In a non-homogeneous network, the users experience different path-loss and shadowing effects, which we account for by incorporating the terms \( \{\gamma_{i,j}\} \) and \( \{\eta_i\} \). Also, we assume that the primary and secondary transmitters satisfy average power constraints \( P_p \) and \( P_s \), respectively, i.e., \( E[|x^p_i|^2] \leq P_p \) and \( E[|x^s_i|^2] \leq P_s \) and the channel coefficients \( \{h^m_{i,j}\}_{i,j,m} \) and \( \{g^m_{i}\}_{i,m} \) are distributed as i.i.d. complex Gaussian \( \mathcal{CN}(0, 1) \). Each secondary receiver treats all undesired signals (interference from the primary users) as Gaussian interferers. Therefore, the signal-to-interference plus noise-ratio (SINR) of the communication that the n-th secondary pair can carry out on the spectrum band Bm can be computed as

\[ \text{SINR}_{m,n} = \frac{P_s \eta_n |g^m_n|^2}{N_0 + P_p \sum_{j \in S_m} \gamma_{m,j} |h^m_{n,j}|^2} \]

We define the transmission signal-to-noise ratio (SNR) by \( \rho \triangleq \frac{P_s}{N_0} \). Throughout the paper we say that \( a_K \) and \( b_K \) are asymptotically equal, denoted by \( a_K \approx b_K \) if \( \lim_{K \to \infty} \frac{a_K}{b_K} = 1 \), and define ≤ and ≥, accordingly. We also define the set of secondary users indices by \( \mathcal{N} = \{1, \ldots, N\} \).

B. Problem Statement

Our goal is to identify M secondary transmitter-receiver pairs out of N available ones and assign each of them one spectrum band B_i, such that the cognitive network throughput is maximized. We assume that all the spectrum bands B_i are of the same bandwidth. Therefore, the maximum throughput is given by

\[ R_{\text{max}} = \mathbb{E} \left[ \max_{A \subset \mathcal{N}, |A|=M} \sum_{m=1}^{M} \log \left( 1 + \text{SINR}_{m,A_m} \right) \right], \]

where \( A_m \) denotes the m-th element of set A, for \( m = 1, \ldots, M \) and maximization is taken over all ordered subsets of \( \mathcal{N} \). We are interested in analyzing how the maximum throughput scales as the number of secondary transmitter-receiver pairs \( N \), increases. For solving this problem we propose two approaches as follows.

1) Centralized Spectrum Allocation: In order to find the optimal diversity gain in the cognitive network we first consider a centralized setup. We assume that there exist a central decision-making entity in the cognitive network, which is fully and instantaneously aware of all the channel conditions of all the cognitive users. The central node solves the problem cast in (2) by an exhaustive search for selecting and pairing up M cognitive users with the M available channels. For such cognitive user-channel pairs we analyze how the sum-rate of the cognitive network scales as the number of cognitive users (N) increases. Such centralized setup imposes extensive information exchange which can be prohibitive for large network sizes.

2) Distributed Spectrum Allocation: Motivated by alleviating the amount of information exchange imposed by the centralized setup and noting that our cognitive network is ad-hoc in nature and lacks a central-decision making entity we propose a decentralized spectrum allocation scheme, which is provided in Section IV-A.

In the distributed scheme each cognitive user takes action for taking over a channel solely based on its own perception of the network realization. We prove that the proposed distributed scheme retains the same throughput scaling law as in the centralized setup, i.e., is asymptotically optimal.

1M real numbers per user.
II. CENTRALIZED SPECTRUM ALLOCATION

The central decision-making unit has access to all \{\text{SINR}_{m,n}\} and by an exhaustive search over all \(\sum_{m=1}^{N} \text{SINR}_{m,n}\) possible user-channel (spectrum band) matches finds the one which maximizes the throughput given in (2). The assumptions made in this section, while not being practical, are useful in shedding light on the sum-throughput limit of such cognitive networks. The results provided in this section can be exploited as the benchmark to quantify the efficiency of our distributed algorithm proposed in the following section.

In order to find the throughput scaling for the centralized setup, we establish lower and upper bounds on \(R_{max}\), denoted by \(R_{max}^l\) and \(R_{max}^u\), respectively, and show that these bounds are asymptotically equal, i.e., \(R_{max}^u \approx R_{max}^l\), which in turn provides the optimal throughput scaling law of the cognitive network.

We define the most favorable user of the \(m^{th}\) spectrum band as the user with the largest SINR on this band, i.e., \(n_m = \arg\max_{1 \leq n \leq N} \text{SINR}_{m,n}\). Also let us define

\[
R_{max}^u \triangleq \mathbb{E} \left[ \sum_{m=1}^{M} \log \left( 1 + \text{SINR}_{m,n_m} \right) \right].
\] (3)

**Lemma 1:** \(R_{max}^l\) and \(R_{max}^u\) are asymptotically equal, i.e.,

\[
R_{max}^u \approx R_{max}^l.
\]

Now, we find how \(R_{max}^u\) scales as \(N\) increases. Note that the SINRs are statistically independent for all users and spectrum bands; but since the users and spectrum bands experience different path-losses and shadowing effects, they are not identically distributed. Hence, for more mathematical tractability we build two other sets whose elements provide lower and upper bounds on \(\text{SINR}_{m,n}\) and are i.i.d. For this purpose we define \(\gamma_{max} = \max_{i,j} \gamma_{i,j} \) and \(\gamma_{min} = \min_{i,j} \gamma_{i,j} \) and

\[
\gamma_{max} \triangleq \max_{i,j} \left\{ \frac{\gamma_{i,j}}{\eta_i} \right\}, \quad \gamma_{min} \triangleq \min_{i,j} \left\{ \frac{\gamma_{i,j}}{\eta_i} \right\},
\]

and for \(m = 1, \ldots, M\) also define \(S_l(m) \triangleq \{S_l(m,n)\}\) and \(S_u(m) \triangleq \{S_u(m,n)\}\) where

\[
S_l(m,n) = \frac{1}{\rho_{min}} + \frac{|g_{n_m}^m|^2}{\rho_{max}} + \frac{P_n}{P_x} \gamma_{max} \sum_j \left| h_{n_j}^m \right|^2,
\]

and

\[
S_u(m,n) = \frac{1}{\rho_{max}} + \frac{|g_{n_m}^m|^2}{\rho_{min}} + \frac{P_n}{P_x} \gamma_{min} \sum_j \left| h_{n_j}^m \right|^2.
\]

It can be readily verified that \(S_l(m,n) \leq \text{SINR}_{m,n} \leq S_u(m,n)\). We use the notations \(S_l(i)(m)\) and \(S_u(i)(m)\) to refer to the \(i^{th}\) biggest elements of \(S_l(m)\) and \(S_u(m)\), respectively and use \(\text{SINR}^{(i)}_m\) to denote the \(i^{th}\) biggest element of \(\text{SINR}_{m,n}\). In the following lemma we show how these ordered elements are related.

**Lemma 2:** For any spectrum band \(B_m\) and any \(i = 1, \ldots, N\) we have \(S_l(i)(m) \leq \text{SINR}^{(i)}_m \leq S_u(i)(m)\). Now, by recalling the definition of \(R_{max}^u\) given in (3) and noting that \(\text{SINR}_{m,n}^{(i)} = \text{SINR}^{(i)}_m\) and by invoking the result of Lemma 2 we get

\[
\mathbb{E} \left[ \sum_{m=1}^{M} \log \left( 1 + S_l(i)(m) \right) \right] \leq R_{max}^u
\]

\[
\leq \mathbb{E} \left[ \sum_{m=1}^{M} \log \left( 1 + S_u(i)(m) \right) \right].
\]

In order to further simplify the bounds on \(R_{max}^u\) given in (7) we provide the cumulative density functions (CDF) of \(S_l(m,n)\) and \(S_u(m,n)\) in the following lemma.

**Lemma 3:** The elements of \(S_l(m)\) and \(S_u(m)\) are i.i.d. with CDF \((K_m \triangleq |B_m|)\)

\[
S_l(m,n) \sim F_l(x;m) \triangleq 1 - \frac{e^{-x/P_{\min}}}{P_x P_{\max} x + 1} K_m, \quad \text{for} \quad m \geq 1,
\]

and

\[
S_u(m,n) \sim F_u(x;m) \triangleq 1 - \frac{e^{-x/P_{\max}}}{P_x P_{\min} x + 1} K_m, \quad \text{for} \quad m \geq 1.
\]

By using the result of the lemma above, the CDFs of \(S_l(i)(m)\) and \(S_u(i)(m)\) as the \(i^{th}\) order statistics of the statistical samples \(S_l(m)\) and \(S_u(m)\) with parent distributions given in (8)-(9) can be found as [12]

\[
F_l(i)(x;m) \triangleq \sum_{j=0}^{i-1} \binom{N}{j} \left( F_l(x;m) \right)^{N-j} \left( 1 - F_l(x;m) \right)^{j},
\]

and

\[
F_u(i)(x;m) \triangleq \sum_{j=0}^{i-1} \binom{N}{j} \left( F_u(x;m) \right)^{N-j} \left( 1 - F_u(x;m) \right)^{j}.
\]

By invoking the above definitions, (7) can be re-written as

\[
\sum_{m=1}^{M} \int_{0}^{\infty} \log(1 + x) \, dF_l(i)(x;m) \leq R_{max}^u
\]

\[
\leq \sum_{m=1}^{M} \int_{0}^{\infty} \log(1 + x) \, dF_u(i)(x;m).
\]

We also define

\[
G(x) \triangleq 1 - e^{-x},
\]

and let \(G(i)(x)\) denote the CDF of the \(i^{th}\) order statistic of an statistical sample with \(N\) members and with parent distribution \(G(x)\). By using this definition we offer the following lemma which is a key step in finding how \(R_{max}^u\) scales with increasing \(N\).

**Lemma 4:** For the distributions \(F_l(i)(x;m), F_u(i)(x;m)\) and \(G(i)(x)\) we have

\[
\int \log(1 + x) \, dF_u(i)(x;m) \leq \int \log(1 + \rho_{max} x) \, dG(i)(x),
\]

\[
\int \log(1 + x) \, dF_l(i)(x;m) \geq \int \log(1 + \rho_{min} x) \, dG(i)(x)
\]

\[
- \log \left[ 1 + \frac{K_m P_n}{P_x} \gamma_{max} \rho_{min} \right].
\]

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Next, by using the result of the following lemma, we find the scaling behavior of the right-hand sides of the expressions in Lemma 4.

Lemma 5: For a family of exponentially distributed random variables of size \( N \) and parent distribution \( G(x) \) (CDF) and for any positive real number \( a \in \mathbb{R}_+ \) we have

\[
\int_0^\infty \log(1 + ax)dG^{(1)}(x) = \log \log N + \log a.
\] (12)

Now, by recalling the bounds provided in (10) and taking into account the results of Lemmas 1, 4 and 5 we provide the optimal throughput scaling law of the cognitive network with optimal user-channel assignments in the following theorem.

Theorem 1: In a centralized cognitive network with \( N \) secondary transmitter-receiver pairs and \( M \) available spectrum bands, by optimal user-channel assignments, the sum-rate throughput of the network scales like \( R_{\text{max}} = M \log \log N \).

To this end, we have assumed that there exist a decision-making center who fully knows all the the instantaneous channel realizations, i.e., \( \{h_{i,j}^m\} \), \( \{\gamma_{i,j}\} \) and \( \{\eta_i\} \) and has no complexity constraints which makes it possible to exhaust all the possible user-channel assignments and choose the one which maximizes the sum-throughput of the network.

IV. DISTRIBUTED SPECTRUM ALLOCATION

In this section we offer our distributed algorithm, where each user independently of others, makes decision regarding taking over transmission on any specific spectrum band. We analyze the achievable sum-throughput of the cognitive network when this distributed algorithm is utilized and show that it is asymptotically optimal.

A. Distributed Algorithm

In this algorithm, at the first step, to prevent from exhaustively searching for the best user-channel matches, we only consider assigning any specific channel to the users which can carry out communication on this specific channel with a pre-determined minimum level of quality. Specifically, to each user \( n = 1, \ldots, N \), we assign a minimum acceptable level of SINR, denoted by \( \lambda(m, n) \), which is defined as follows. Let \( T(x; m, n) \) denote the CDF of SINR\( m,n \), as given in (2). \( \lambda(m, n) \) is set such that

\[
T\left(\lambda(m, n); m, n\right) = 1 - \frac{1}{N}.
\] (13)

Note that for any given \( m \) and \( n \), \( T(x; m, n) \) is a non-decreasing function on \( [0, +\infty) \times [0, 1] \) which ensures that there always exist a unique solution for \( \lambda(m, n) \). Also it is noteworthy that as SINR\( m,n \) only depends on the incoming channels to the \( n^{th} \) secondary receiver on the \( m^{th} \) spectrum band, \( \lambda(m, n) \) can be computed locally at the \( n^{th} \) secondary receiver and does not in any way exchange information between the secondary users.

Now, each user \( n \) computes SINR\( n\), \( \cdots \), SINR\( M,n \), and identifies the channel corresponding to the largest SINR, and denotes it index by \( m_n^* \), i.e.,

\[
m_n^* = \arg \max_{m=1, \ldots, M} \{\text{SINR}_{m,n}\}.
\] (14)

In the next step, the \( n^{th} \) user compares \( \text{SINR}_{m_n^*,n} = \max_m \{\text{SINR}_{m,n}\} \) against \( \lambda(m, n) \) and if \( \text{SINR}_{m_n^*,n} \geq \lambda(m, n) \), then deems itself as a candidate for accessing the channel indexed by \( m_n^* \). Next, we define the mutually disjoint sets \( \mathcal{H}_m \) for \( m = 1, \ldots, M \), such that \( \mathcal{H}_m \) contains the indices of the users deemed as candidates for taking over the \( m^{th} \) channel, i.e.,

\[
\mathcal{H}_m \triangleq \{ n \mid m = \arg \max_{m} \text{SINR}_{m,n}\}.
\] (15)

Finally, a user with its index in \( \mathcal{H}_m \) is randomly selected for utilizing the \( m^{th} \) channel. This can be facilitated in a distributed way via any contention based random media access method, e.g., Aloha, carrier sense multiple access, etc. As soon as one user takes a channel, the other users will no longer try to access that channel. In the following section, we analyze the sum-rate throughput scaling factor of the proposed algorithm.

B. Sum-Rate Throughput Scaling

We denote the sum-rate throughput by \( R_{\text{sum}} \) and refer to the throughput of the \( m^{th} \) channel by \( R_m \). Note that the construction of \( \mathcal{H}_m \) guarantees that no single user will be regarded as a candidate for more than one channel and also we have \( R_{\text{sum}} = \sum_{m=1}^M R_m \). By defining \( R_m | \mathcal{H}_m \) as the throughput achieved for the \( m^{th} \) beam conditioned on having users with indices in \( \mathcal{H}_m \) be candidates for taking over \( B_m \) we have

\[
R_m = \sum_{\mathcal{H}_m \subseteq N} \frac{R_m | \mathcal{H}_m}{P(\mathcal{H}_m)}.
\] (16)

On the other hand, by noting that one member of \( \mathcal{H}_m \) will be randomly picked for accessing \( B_m \) we get

\[
R_m | \mathcal{H}_m = \frac{1}{|\mathcal{H}_m|} \sum_{i \in \mathcal{H}_m} \log \left(1 + \text{SINR}_m(i)\right) \mid \mathcal{H}_m \right].
\] (17)

By further defining \( Q^m_i \triangleq \sum_{n=1}^N \frac{1}{n} P(|\mathcal{H}_m| = n) \) and

\[
R^m_i \triangleq \sum_{n=1}^N Q^m_i \log \left(1 + \text{SINR}_m\right),
\] (18)

we can show that \( R_m \geq R^m_i \). If we also define \( Q^m_0 = \sum_{n=1}^N Q^m_n \) we get

\[
R^m_i = \sum_{n=0}^N \log \left(1 + \text{SINR}(i)\right) = 1,
\]

which suggests that \( \{Q^m_n\}_{n=0}^N \) is a valid probability mass function (pmf). In the sequel, we concentrate on finding the scaling behavior of \( R_m \). By using the definitions of \( S(m) \) and \( S_n(m) \) provided in (5) and (6) and exploiting Lemma 2 we have

\[
\sum_{n=1}^N Q^m_i \log \left(1 + S^m_n(i)\right) \leq R^m_i
\]
\[
\sum_{i=1}^{N} Q_i^m \left[ \log \left( 1 + S_u^{(i)}(m) \right) \right].
\]

By recalling that the CDFs of \( S_l^{(i)}(m) \) and \( S_u^{(i)}(m) \) are \( F_l^{(i)}(x; m) \) and \( F_u^{(i)}(x; m) \) provided in Lemma 3, for the given set of \( \{ Q_i^m \} \) we define

\[
F_l^N(x; m) \triangleq \sum_{i=1}^{N} Q_i^m F_l^{(i)}(x; m),
\]

and \( F_u^N(x; m) \triangleq \sum_{i=1}^{N} Q_i^m F_u^{(i)}(x; m) \),

and \( G^N(x) \triangleq \sum_{i=1}^{N} Q_i^m G^{(i)}(x) \).

Since \( \{ Q_i^m \} \) is a valid pmf, \( F_l^N(x; m) \), \( F_u^N(x; m) \), and \( G^N(x) \) can be cast as valid CDFs. Therefore, (19) can be stated as

\[
\sum_{i=1}^{N} Q_i^m \int_0^1 \log(1+x) \, dF_l^{(i)}(x; m) \leq R_m^l
\]

or equivalently,

\[
\int_0^1 \log(1+x) \, dF_l^N(x; m) \leq R_m^l
\]

The two subsequent lemmas are key in finding how \( R_m^l \) scales with increasing \( N \).

**Lemma 6:** For the distributions \( F_l^N(x) \), \( F_u^N(x) \) and \( G^N(x) \) we have

\[
\int \log(1+x) \, dF_u^N(x; m) \leq \int \log(1+\rho_{\max} x) \, dG^N(x),
\]

\[
\int \log(1+x) \, dF_l^N(x; m) \geq \int \log(1+\rho_{\min} x) \, dG^N(x)
\]

\[- \log \left[ 1 + \frac{K_m P_s}{P_o} \gamma_{\max} \rho_{\min} \right].
\]

**Lemma 7:** For a family of exponentially distributed random variables of size \( N \) and parent distribution \( G(x) \) (CDF) and for any set of \( \{ Q_i^m \}_{i=1}^{N} \) such that \( \sum_{i=1}^{N} Q_i^m = 1 \), if

\[
\lim_{N \to \infty} \sum_{i=1}^{N} i Q_i^m / N = 0
\]

then for any positive real number \( a \in \mathbb{R}_+ \) we have

\[
\int_0^\infty \log(1+ax) \, dG^N(x) \doteq \log \log N + \log a.
\]

By using the results of lemmas 6 and 7 we offer the the main result of the distributed algorithm in the following theorem

**Theorem 2:** The sum-rate throughput of the cognitive network by exploiting the proposed distributed algorithm scales as

\[
R_{\text{sum}} \doteq M \log \log N
\]

The simulation results in Fig. 1 demonstrates the sum-rate throughput achieved under the centralized and the distributed setups. We consider a primary network consisting of 4 users and look at the throughput scaling for the cases that there exist \( M = 1, \ldots, 4 \) available spectrum bands to be utilized by the secondary users. We set all path-loss terms \( \{ \eta_i \} \) and \( \{ \gamma_{i,j} \} \) equal to 1 and find the sum-rate throughput as the number of secondary users increases. As shown in Fig. 1, as the number of users increases, the sum-rate throughput achieved by the centralized and distributed schemes exhibit the same scaling factor.

The throughput achieved under the distributed setup is uniformly less than that of the centralized setup. This is justified by recalling that the centralized scheme finds the best secondary user for each available spectrum band, whereas the distributed network finds all the secondary users whose quality of communication on a specific channel satisfies a constraint \( (\lambda(m, n)) \) and among all such secondary user one is randomly selected to access the spectrum band. This, not necessarily guarantees finding the best user for each available spectrum band, which as a result leads to some degradation in the sum-rate throughput.

Figure 2 demonstrates the dependence of \( \lambda(m, n) \) on the transmission SNR denoted by \( \rho \). It is seen that \( \lambda(m, n) \) monotonically increases with \( \rho \). Intuitively, this is due to the fact that as the as \( \rho \) increases, the users are expected to have more reliable communication and as a result the algorithm will impose more stringent conditions on the secondary users for considering themselves as a candidate for accessing any specific spectrum band. More stringent conditions will translate to having higher values of \( \lambda(m, n) \) such that the condition in (13) is satisfied.

V. INFORMATION EXCHANGE

In our distributed algorithm we had assumed that the \( n^{th} \) secondary receivers measure \( \{ \text{SNR}_{m,n} \}_{m=1}^{M} \) corresponding
Fig. 2. $\lambda(m, n)$ versus the number of secondary users for $M = 4$ available spectrum bands and $K_m = 4$ primary users.

to different spectrum bands, selects the largest one, whose index is denoted by $m^*_j$, and compares it against a pre-determined quality metric $\lambda(m, n)$. If $\text{SINR}_{m^*_j, n} \geq \lambda(m, n)$, this secondary receiver should notify its designated secondary transmitter to participate in a contention-based competition for taking over channel $m^*_j$. Such notification requires exchanging $\log M$ information bits from the secondary receiver to its respective secondary transmitter.

Although not all of the secondary pairs will be involved in such information exchange, it is imperative to analyze the aggregate amount of such information for large networks ($N \to \infty$). In the following theorem we demonstrate that for the choice of $\lambda(m, n)$ provided in (13), the asymptotic amount of information exchange is a constant independent of $N$ and therefore does not harm the sum-rate throughput of the cognitive network.

**Theorem 3:** In the cognitive network with distributed spectrum access, when $\lambda(m, n)$ satisfies

$$T(\lambda(m, n); m, n) = 1 - \frac{1}{N},$$

the aggregate amount of information exchange between secondary transmitter-receiver pairs is asymptotically equal to $M \log M$.

**VI. FAIRNESS**

In our distributed algorithm we have considered the users with a minimum level of reception quality to be candidates for taking over the channels. In general, such opportunistic user selections might lead to the situation that the network be dominated by the secondary pairs with their receiver far from the primary users so that see less amount of interference from them, or by those pairs where the transmitter and the receiver are closely located and enjoy a good communication channel.

In despite of these facts, we show that in our network, by appropriate choice of $\lambda(m, n)$ we can control all the users such that all are equiprobable for being assigned a channel. This can be made possible by enforcing more stringent conditions (higher $\lambda(m, n)$) for the users benefitting from smaller path-loss and shadowing effects. In the following theorem we show that by the choice of $\lambda(m, n)$ provided in (13) all the users have the same opportunities for accessing a channel.

**Theorem 4:** In the cognitive network with distributed spectrum allocation, when $\lambda(m, n)$ satisfies

$$T(\lambda(m, n); m, n) = 1 - \frac{1}{N},$$

then all the users have the same probability of being allocated a channel.

**VII. CONCLUSIONS**

In this paper we investigated the multiuser diversity gain in cognitive networks. We first obtained the optimal gain achieved in a network with a central authority and show that the gain achieved in such cognitive networks is similar to that of the interference-free networks, i.e. the network throughput scales double logarithmically with the number of users. Then we proposed a distributed spectrum access scheme which is proven to achieve the optimal throughput scaling factor. This scheme imposes the exchange of $\log M$ information bits per transmitter-receiver cognitive pair for some pairs, and no information exchange for the others. The other specification of the distributed algorithm are that the network-wide average aggregate amount of information bits it requires is asymptotically equal to $M \log M$, and it ensures fairness among the secondary users.

**REFERENCES**