Part I

The LVCSR Acoustic Model
What is LVCSR?

- Large vocabulary.
  - Phone-based modeling vs. word-based modeling.
- Continuous.
  - No pauses between words.

The Fundamental Equation of ASR

\[
\text{class}(x) = \arg\max_{\omega} P(\omega|x)
\]
\[
= \arg\max_{\omega} \frac{P(\omega)P(x|\omega)}{P(x)}
\]
\[
= \arg\max_{\omega} P(\omega)P(x|\omega)
\]

- \(P(x|\omega)\) — acoustic model.
- \(P(\omega)\) — language model.

The Acoustic Model: Small Vocabulary

\[
P_{\omega}(x) = \sum_{A} P_{\omega}(x, A) = \sum_{A} P_{\omega}(A) \times P_{\omega}(x|A)
\]
\[
\approx \max_{A} P_{\omega}(A) \times P_{\omega}(x|A)
\]
\[
= \max_{A} \prod_{t=1}^{T} P(a_t) \prod_{t=1}^{T} P(\tilde{x}_t|a_t)
\]
\[
\log P_{\omega}(x) = \max_{A} \left[ \sum_{t=1}^{T} \log P(a_t) + \sum_{t=1}^{T} \log P(\tilde{x}_t|a_t) \right]
\]
\[
P(\tilde{x}_t|a_t) = \sum_{m=1}^{M} \lambda_{a_t,m} \prod_{\text{dim } d} \mathcal{N}(x_{t,d}; \mu_{a_t,m,d}, \sigma_{a_t,m,d})
\]

The Acoustic Model: Large Vocabulary

\[
P_{\omega}(x) = \sum_{A} P_{\omega}(x, A) = \sum_{A} P_{\omega}(A) \times P_{\omega}(x|A)
\]
\[
\approx \max_{A} P_{\omega}(A) \times P_{\omega}(x|A)
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\[
P(\tilde{x}_t|a_t) = \sum_{m=1}^{M} \lambda_{a_t,m} \prod_{\text{dim } d} \mathcal{N}(x_{t,d}; \mu_{a_t,m,d}, \sigma_{a_t,m,d})
\]
What Has Changed?

- The HMM.
  - Each alignment $A$ describes a path through an HMM.
  - Its parameterization.
    - In $P(x|a)$, how many GMM's to use? (Share between HMM's?)

Describing the Underlying HMM

- Fundamental concept: how to map a word (or baseform) sequence to its HMM.
  - In training, map reference transcript to its HMM.
  - In decoding, glue together HMM's for all allowable word sequences.

The HMM: Small Vocabulary

- One HMM per word.
- Glue together HMM for each word in word sequence.

The HMM: Large Vocabulary

- One HMM per phone.
- Glue together HMM for each phone in phone sequence.
  - Map word sequence to phone sequence using baseform dictionary.
I Still Don’t See What's Changed

- HMM topology typically doesn’t change.
- HMM parameterization changes.

Parameterization

- Small vocabulary.
  - One GMM per state (three states per phone).
  - No sharing between phones in different words.
- Large vocabulary, context-independent (CI).
  - One GMM per state.
  - Tying between phones in different words.
- Large vocabulary, context-dependent (CD).
  - Many GMM’s per state; GMM to use depends on phonetic context.
  - Tying between phones in different words.

Context-Dependent Parameterization

- Each phone HMM state has its own decision tree.
  - Decision tree asks questions about phonetic context. (Why?)
  - One GMM per leaf in the tree. (Up to 200+ leaves/tree.)
- How will tree for first state of a phone tend to differ . . .
  - From tree for last state of a phone?
- Terminology.
  - tripphone model — ±1 phones of context.
  - quinphone model — ±2 phones of context.

A Real-Life Tree

Tree for fememe AA_1:

node 0: quest-P 23[-1] --> true: node 1, false: node 2
  quest: AR AR R B BO CH D DD DR DS ER F G GD GG JH K KD M N NG P PD R S
  SH S TD TH TS UW V W X Z ZH
node 1: quest-P 66[-1] --> true: node 3, false: node 4
  quest: AO AR ER IY L M N NG OY R OSH UW Y
node 2: quest-P 36[-2] --> true: node 5, false: node 6
  quest: DS X
node 3: quest-P 13[-1] --> true: node 7, false: node 8
  quest: AR ER R
node 5: leaf 0
node 6: quest-P 15[-1] --> true: node 11, false: node 12
  quest: AR ER L ON R UW W
node 7: quest-P 49[-2] --> true: node 13, false: node 14
  quest: D K P T
node 11: leaf 1
node 12: quest-P 15[-2] --> true: node 21, false: node 22
  quest: AR ER L ON R UW W
node 13: leaf 2
node 14: leaf 3
...
Pop Quiz

- Pretend you are Keanu Reeves.
- System description:
  - 1000 words in lexicon; average word length = 5 phones.
  - There are 50 phones; each phone HMM has three states.
  - Each decision tree contains 100 leaves on average.
- How many GMM’s are there in:
  - A small vocabulary system (word models)?
  - A CI large vocabulary system?
  - A CD large vocabulary system?

Context-Dependent Phone Models

- Typical model sizes:
  - | type       | HMM      | GMM’s/  | GMM’s  |
    |           | state    |        | Gaussians |
    | word      | per word | 1      | 10–500   |
    | CI phone  | per phone| 1      | ∼150     |
    | CD phone  | per phone| 1–200  | 1k–3k    |
  - 39-dimensional feature vectors $\Rightarrow \sim 80$ parameters/Gaussian.
  - Big models can have tens of millions of parameters.

What About Transition Probabilities?

- This slide only included for completeness.
- Small vocabulary.
  - One set of transition probabilities per state.
  - No sharing between phones in different words.
- Large vocabulary.
  - One set of transition probabilities per state.
  - Sharing between phones in different words.
- What about context-dependent transition modeling?
Recap

- Main difference between small vocabulary and large vocabulary:
  - Allocation of GMM’s.
  - Sharing GMM’s between words: needs less GMM’s.
  - Modeling context-dependence: needs more GMM’s.
  - Hybrid allocation is possible.
- Training and decoding for LVCSR.
  - In theory, any reason why small vocabulary techniques won’t work?
  - In practice, yikes!

Points to Ponder

- Why deterministic mapping?
  - DID YOU ⇒ D I H D J H UW
  - The area of pronunciation modeling.
- Why decision trees?
  - Unsupervised clustering.

Part II

Acoustic Model Training for LVCSR

- Phase 1: Collect underpants.
  - Initialize all Gaussian means to 0, variances to 1.
- Phase 2: Iterate over training data.
  - For each word, train associated word HMM . . .
  - On all samples of that word in the training data . . .
  - Using the Forward-Backward algorithm.
- Phase 3: Profit!
Large Vocabulary Training

- What’s changed going to LVCSR?
  - Same HMM topology; just more Gaussians and GMM's.
  - Can we just use the same training algorithm as before?

Flat or Random Start

- Why does this work for small models?
  - We believe there’s a huge global minimum . . .
  - In the “middle” of the parameter search space.
  - With a neutral starting point, we’re apt to fall into it.
  - (Who knows if this is actually true.)
- Why doesn’t this work for large models?

Where Are We?

- The Local Minima Problem
- Training GMM’s
- Building Phonetic Decision Trees
- Details
- The Final Recipe

Case Study: Training a Simple GMM

- Front end from Lab 1; first two dimensions; 546 frames.
Training a Mixture of Two 2-D Gaussians

- Flat start?
  - Initialize mean of each Gaussian to 0, variance to 1.

- Random seeding?
  - Picked 8 random starting points ⇒ 3 different optima.

---

Training Hidden Models

- (MLE) training of models with hidden variables has local minima.
- What are the hidden variables in ASR?
  - *i.e.*, what variables are in our model . . .
  - That are not observed.
How To Spot Hidden Variables

\[ P_\omega(x) = \sum_A P_\omega(x, A) = \sum_A P_\omega(A) \times P_\omega(x|A) \]
\[ \approx \max_A P_\omega(A) \times P_\omega(x|A) \]
\[ = \max_A \prod_{t=1}^T P(a_t) \prod_{t=1}^T P(\tilde{x}_t|a_t) \]

\[ \log P_\omega(x) = \max_A \left[ \sum_{t=1}^T \log P(a_t) + \sum_{t=1}^T \log P(\tilde{x}_t|a_t) \right] \]

\[ P(\tilde{x}_t|a_t) = \sum_{m=1}^M \lambda_{a_t,m} \prod_{d} N(x_{t,d}; \mu_{a_t,m,d}, \sigma_{a_t,m,d}) \]

What To Do?

- Insight: If we know the “correct” hidden values for a model:
  - e.g., which arc and which Gaussian for each frame . . .
  - Training is easy! (No local minima.)
  - Remember Viterbi training given fixed alignment in Lab 2.
- Is there a way to guess the correct hidden values for a large model?

Gradient Descent and Local Minima

- EM training does hill-climbing/gradient descent.
- Finds “nearest” optimum to where you started.

Bootstrapping Alignments

- Recall that all of our acoustic models, from simple to complex:
  - Generally use the same HMM topology!
  - (All that differs is how we assign GMM’s to each arc.)
- Given an alignment (from arc/phone states to frames) for simple model . . .
  - It is straightforward to compute analogous alignment for complex model!
Recipe:
- Start with model simple enough that flat start works.
- Iteratively build more and more complex models . . .
- By using last model to seed hidden values for next.
- Need to come up with sequence of successively more complex models . . .
- With related hidden structure.

How To Seed Next Model From Last
- Directly via hidden values, e.g., alignment.
  - e.g., single-pass retraining.
  - Can be used between very different models.
- Via parameters.
  - Seed parameters in complex model so that . . .
  - Initially, will yield same/similar alignment as in simple model.
  - e.g., moving from CI to CD GMM's.

Recurring motif in acoustic model training.
- The reason why state-of-the-art systems . . .
  - Require many, many training passes, as you will see.
- Recipes handed down through the generations.
  - Discovered via sweat and tears.
  - Art, not science.
  - But no one believes these find global optima . . .
  - Even for small problems.
Where Are We?

1. The Local Minima Problem
2. Training GMM’s
3. Building Phonetic Decision Trees
4. Details
5. The Final Recipe

Case Study: Training a GMM

- Recursive mixture splitting.
  - A sequence of successively more complex models.
- k-means clustering.
  - Seed means in one shot.

Gaussian Mixture Splitting

- Start with single Gaussian per mixture (trained).
- Split each Gaussian into two.
  - Perturb means in opposite directions; same variance.
  - Train.
- Repeat until reach desired number of mixture components (1, 2, 4, 8, ...).
  - (Discard Gaussians with insufficient counts.)
- Assumption: c-component GMM gives good guidance ...
  - On how to seed 2c-component GMM.

Mixture Splitting Example

- Train single Gaussian.
Mixture Splitting Example

- Split each Gaussian in two ($\pm 0.2 \times \sigma$)

Mixture Splitting Example

- Train, yep.
Applying Mixture Splitting in ASR

- Recipe:
  - Start with model with 1-component GMM’s (à la Lab 2).
  - Split Gaussians in each output distribution simultaneously.
  - Do many iterations of FB.
  - Repeat.
- Real-life numbers:
  - Five splits spread within 30 iterations of FB.

Another Way: Automatic Clustering

- Use unsupervised clustering algorithm to find clusters.
- Given clusters . . .
  - Use cluster centers to seed Gaussian means.
  - FB training.
  - (Discard Gaussians with insufficient counts.)

k-Means Clustering

- Select desired number of clusters $k$.
- Choose $k$ data points randomly.
  - Use these as initial cluster centers.
- “Assign” each data point to nearest cluster center.
- Recompute each cluster center as . . .
  - Mean of data points “assigned” to it.
- Repeat until convergence.

k-Means Example

- Pick random cluster centers; assign points to nearest center.
**k-Means Example**

- Recompute cluster centers.

- Assign each point to nearest center.

- Repeat until convergence.

- Use centers as means of Gaussians; train, yep.
The Final Mixtures, Splitting vs. $k$-Means

Technical Aside: $k$-Means Clustering

- When using Euclidean distance . . .
- $k$-means clustering is equivalent to . . .
  - Seeding Gaussian means with the $k$ initial centers.
  - Doing Viterbi EM update, keeping variances constant.

Applying $k$-Means Clustering in ASR

- To train each GMM, use $k$-means clustering . . .
  - On what data? Which frames?
- Huh?
  - How to decide which frames align to each GMM?
- This issue is evaded for mixture splitting.
  - Can we avoid it here?

Forced Alignment

- Viterbi algorithm.
  - Finds most likely alignment of HMM to data.

<table>
<thead>
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<th>frame</th>
<th>0</th>
<th>1</th>
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<th>12</th>
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<tbody>
<tr>
<td>arc</td>
<td>$P_1$</td>
<td>$P_1$</td>
<td>$P_2$</td>
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<td>$P_3$</td>
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</tbody>
</table>

- Need existing model to create alignment. (Which?)
Recap

- You can use single Gaussian models to seed GMM models.
- Mixture splitting: use c-component GMM to seed 2c-component GMM.
- k-means: use single Gaussian model to find alignment.
- Both of these techniques work about the same.
- Nowadays, we primarily use mixture splitting.

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What Do We Need?

- For each tree/phone state...
  - List of frames/feature vectors associated with that tree.
  - (This is the data we are optimizing the likelihood of.)
  - For each frame, the phonetic context.
- A list of candidate questions about the phonetic context.
  - Ask about phonetic concepts; e.g., vowel or consonant?
  - Expressed as list of phones in set.
  - Allow same questions to be asked about each phone position.
  - Handed down through the generations.
Training Data for Decision Trees

- Forced alignment/Viterbi decoding!
- Where do we get the model to align with?
  - Use CI phone model or other pre-existing model.

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<th>8</th>
<th>9</th>
<th>...</th>
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<td>DH2</td>
<td>AH1</td>
<td>AH2</td>
<td>D1</td>
<td>D1</td>
<td>D2</td>
<td>D2</td>
<td>D2</td>
<td>AO1</td>
<td>...</td>
</tr>
</tbody>
</table>

Building the Tree

- A set of events \( \{(\vec{x}_i, p_L, p_R)\} \) (possibly subsampled).
- Given current tree:
  - Choose question of the form …
  - “Does the phone in position \( j \) belong to the set \( q \)?” …
  - That optimizes \( \prod_i P(\vec{x}_i|\text{leaf}(p_L, p_R)) \) …
  - Where we model each leaf using a single Gaussian.
- Can efficiently build whole level of tree in single pass.
- See Lecture 6 slides and readings for the gory details.

Seeding the Context-Dependent GMM’s

- Context-independent GMM’s: one GMM per phone state.
- Context-dependent GMM’s: \( l \) GMM’s per phone state.
- How to seed context-dependent GMM’s?
  - \( e.g. \), so that initial alignment matches CI alignment?

Where Are We?

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Where Are We?

Details
- Maximum Likelihood Training?
- Viterbi vs. Non-Viterbi Training
- Graph Building

Maximum Likelihood Training?
- Regular training iterations (FB, Viterbi EM).
  - Increase (Viterbi) likelihood of data.
- Seeding last model from next model.
  - Mixture splitting.
  - CI $\Rightarrow$ CD models.
- (Decision-tree building.)

The Original Story, Small Vocabulary
- One HMM for each word; flat start.
- Collect all examples of each word.
  - Run FB on those examples to do maximum likelihood training of that HMM.

The New Story
- One HMM for each word sequence!?
  - But tie parameters across HMM’s!
- Do complex multi-phase training.
- Are we still doing anything resembling maximum likelihood training?
Maximum Likelihood Training?

- Just as LM’s need to be smoothed or regularized.
  - So do acoustic models.
  - Prevent extreme likelihood values (e.g., 0 or $\infty$).
- ML training maximizes training data likelihood.
  - We actually want to optimize test data likelihood.
  - Let’s call the difference the overfitting penalty.
- The overfitting penalty tends to increase as . . .
  - The number of parameters increase and/or . . .
  - Parameter magnitudes increase.

Regularization/Capacity Control

- Limit size of model.
  - Will training likelihood continue to increase as model grows?
  - Limit components per GMM.
  - Limit number of leaves in decision tree, i.e., number of GMM’s.
- Variance flooring.
  - Don’t let variances go to 0 $\Rightarrow$ infinite likelihood.

Where Are We?

Details
  - Maximum Likelihood Training?
  - Viterbi vs. Non-Viterbi Training
  - Graph Building

Two Types of Updates

- “Full” EM.
  - Compute true posterior of each hidden configuration.
- Viterbi EM.
  - Use Viterbi algorithm to find most likely hidden configuration.
  - Assign posterior of 1 to this configuration.
- Both are valid updates; instances of generalized EM.
Examples

- Training GMM's.
  - Mixture splitting vs. k-means clustering.
- Training HMM's.
  - Forward-backward vs. Viterbi EM (Lab 2).
- Everywhere you do a forced alignment.
  - Refining the reference transcript.
  - What is non-Viterbi version of decision-tree building?

When To Use One or the Other?

- Which version is more expensive computationally?
  - Optimization: need not realign every iteration.
- Which version finds better minima?
- If posteriors are very sharp, they do almost the same thing.
  - Remember example posteriors in Lab 2?
- Rule of thumb:
  - When you're first training a “new” model, use full EM.
  - Once you’re “locked in” to an optimum, Viterbi is fine.

Where Are We?

- Maximum Likelihood Training?
- Viterbi vs. Non-Viterbi Training
- Graph Building

Building HMM's For Training

- When doing Forward-Backward on an utterance . . .
  - We need the HMM corresponding to the reference transcript.
- Can we use the same techniques as for small vocabularies?
**Word Models**

- Reference transcript
- Replace each word with its HMM

**Context-Independent Phone Models**

- Reference transcript
- Pronunciation dictionary.
  - Maps each word to a sequence of phonemes.
- Replace each phone with its HMM

**Context-Dependent Phone Models**

- The Pronunciation Dictionary
  - Need pronunciation of every word in training data.
  - Including pronunciation variants
    - THE(01) DH AH
    - THE(02) DH IY
  - Listen to data?
  - Use automatic spelling-to-sound models?
  - Why not consider multiple baseforms/word for word models?
But Wait, It’s More Complicated Than That!

- Reference transcripts are created by humans . . .
  - Who, by their nature, are human (i.e., fallible)
- Typical transcripts don’t contain everything an ASR system wants.
  - Where silence occurred; noises like coughs, door slams, etc.
  - Pronunciation information, e.g., was THE pronounced as DH UH or DH IY?

Pronunciation Variants, Silence, and Stuff

- How can we produce a more “complete” reference transcript?
  - Viterbi decoding!
    - Build HMM accepting all word (HMM) sequences consistent with reference transcript.
    - Compute best path/word HMM sequence.
  - Where does this initial acoustic model come from?

Another Way

- Just use the whole expanded graph during training.

The problem: how to do context-dependent phone expansion?
  - Use same techniques as in building graphs for decoding.

Where Are We?

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Prerequisites

- Audio data with reference transcripts.
- What two other things?

The Training Recipe

- Find/make baseforms for all words in reference transcripts.
- Train single Gaussian models (flat start; many iters of FB).
- Do mixture splitting, say.
  - Split each Gaussian in two; do many iterations of FB.
  - Repeat until desired number of Gaussians per mixture.
- (Use initial system to refine reference transcripts.)
  - Select pronunciation variants, where silence occurs.
  - Do more FB training given refined transcripts.
- Build phonetic decision tree.
  - Use CI model to align training data.
- Seed CD model from CI; train using FB or Viterbi EM.
  - Possibly doing more mixture splitting.

How Long Does Training Take?

- It’s a secret.
- We think in terms of *real-time factor*.
  - How many hours does it take to process one hour of speech?

Whew, That Was Pretty Complicated!

- Adaptation (VTLN, fMLLR, mMLLR)
- Discriminative training (LDA, MMI, MPE, fMPE)
- Model combination (cross adaptation, ROVER)
- Iteration.
  - Repeat steps using better model for seeding.
  - Alignment is only as good as model that created it.
**Recap: Acoustic Model Training for LVCSR**

- **Take-home messages.**
  - Hidden model training is fraught with local minima.
  - Seeding more complex models with simpler models helps avoid terrible local minima.
  - People have developed many recipes/heuristics to try to improve the minimum you end up in.
  - Training is insanely complicated for state-of-the-art research models.
- **The good news . . .**
  - I just saved a bunch on money on my car insurance by switching to GEICO.

**Decoding for LVCSR (Inefficient)**

\[
\text{class}(x) = \arg \max_{\omega} P(\omega|x) = \arg \max_{\omega} \frac{P(\omega)P(x|\omega)}{P(x)} = \arg \max_{\omega} P(\omega)P(x|\omega)
\]

- Now that we know how to build models for LVCSR . . .
  - CD acoustic models via complex recipes.
  - n-gram models via counting and smoothing.
- How can we use them for decoding?
  - Let’s ignore memory and speed constraints for now.
Decoding: Small Vocabulary

- Take graph/WFSA representing language model
  - *i.e.*, all allowable word sequences.
- Expand to underlying HMM
- Run the Viterbi algorithm!

Issue 1: Are N-Gram Models WFSA’s?

- Yup.
- Invariants.
  - One state for each \((n - 1)\)-gram history.
  - All paths ending in state for \((n - 1)\)-gram \(\omega\) . . .
  - Are labeled with word sequence ending in \(\omega\).
  - State for \((n - 1)\)-gram \(\omega\) has outgoing arc for each word \(w\) . . .
  - With arc probability \(P(w|\omega)\).

Bigram, Trigram LM’s Over Two Word Vocab

Pop Quiz

- How many states in FSA representing \(n\)-gram model . . .
  - With vocabulary size \(|V|\)?
- How many arcs?
**Issue 2: Graph Expansion**

- **Word models.**
  - Replace each word with its HMM.
- **CI phone models.**
  - Replace each word with its phone sequence(s)
  - Replace each phone with its HMM.

**Context-Dependent Graph Expansion**

- How can we do context-dependent expansion?
  - Handling branch points is tricky.
- Other tricky cases.
  - Words consisting of a single phone.
  - Quinphone models.

**Triphone Graph Expansion Example**

**Word-Internal Acoustic Models**

- Simplify acoustic model to simplify graph expansion.
- **Word-internal** models.
  - Don’t let decision trees ask questions across word boundaries.
  - Pad contexts with the unknown phone.
  - Hurts performance (e.g., coarticulation across words).
- As with word models, just replace each word with its HMM.
Context-Dependent Graph Expansion

- Is there some elegant theoretical framework . . .
- That makes it easy to do this type of expansion . . .
- And also makes it easy to do lots of other graph operations useful in ASR?
- ⇒ Finite-state transducers (FST’s)! (Part IV)

Recap: Decoding for LVCSR (Inefficient)

- In theory, do same thing as we did for small vocabularies.
  - Start with LM represented as word graph.
  - Expand to underlying HMM.
  - Viterbi.
- In practice, computation and memory issues abound.
- How to do the graph expansion? FST’s (Part IV)
- How to make decoding efficient? search (Part V)

Part IV

Introduction to Finite-State Transducers

Overview

- FST’s are closely related to finite-state automata (FSA).
  - An FSA is a graph.
  - An FST . . .
  - Takes an FSA as input . . .
  - And produces a new FSA.
- Natural technology for graph expansion . . .
  - And much, much more.
- FST’s for ASR pioneered by AT&T in late 1990’s
**Review: What is a Finite-State Acceptor?**

- It has states.
  - Exactly one initial state; one or more final states.
- It has arcs.
  - Each arc has a label, which may be empty ($\epsilon$).
- Ignore probabilities for now.
- Meaning: a (possibly infinite) list of strings.

![Diagram of a finite-state acceptor]

**Review: Pop Quiz**

- What are the differences between the following:
  - HMM's with discrete output distributions.
  - FSA's with arc probabilities.

**What is a Finite-State Transducer?**

- It's like a finite-state acceptor, except...
- Each arc has two labels instead of one.
  - An *input* label (possibly empty)
  - An *output* label (possibly empty)
- Meaning: a (possibly infinite) list of pairs of strings...
  - An input string and an output string.

![Diagram of a finite-state transducer]

**Terminology**

- *finite-state acceptor* (FSA): one label on each arc.
- *finite-state transducer* (FST): input and output label on each arc.
- *finite-state machine* (FSM): FSA or FST.
  - Also, *finite-state automaton*
- Incidentally, an FSA can act like an FST.
  - Pretend input label is both input and output label.
Transforming a Single String

- Let’s say you have a string, *e.g.*, THE DOG
- Let’s say we want to apply a transformation.
  - *e.g.*, map words to their baseforms.
  
  DH AH D AO G
- This is easy, *e.g.*, use `sed` or `perl` or ...

Transforming Lots of Strings At Once

- Let’s say you have a (possibly infinite) list of strings . . .
  
  Expressed as an FSA, as this is compact.
- Let’s say we want to apply a transformation.
  - *e.g.*, map words to their baseforms.
- On all of these strings.
- And have the (possibly infinite) list of output strings . . .
  
  Expressed as an FSA, as this is compact.
  
  Efficiently.

The Composition Operation

- FSA: represents a list of strings \( \{i_1 \cdots i_N\} \).
- FST: represents a list of strings pairs \( \{(i_1 \cdots i_N, o_1 \cdots o_M)\} \).
  
  A compact way of representing string transformations.
- Composing FSA \( A \) with FST \( T \) to get FSA \( A \circ T \).
  
  If string \( i_1 \cdots i_N \in A \) and . . .
  
  Input/output string pair \( (i_1 \cdots i_N, o_1 \cdots o_M) \in T \), . . .
  
  Then, string \( o_1 \cdots o_M \in A \circ T \).

Rewriting a Single String

\[
A \circ T
\]

\[
A \quad 1 \quad a \quad 2 \quad b \quad 3 \quad d \quad 4
\]

\[
T \quad 1 \quad a:A \quad 2 \quad b:B \quad 3 \quad d:D \quad 4
\]

\[
A \circ T \quad 1 \quad A \quad 2 \quad B \quad 3 \quad D \quad 4
\]
Rewriting a Single String

\[ A \]

\[ T \]

\[ A \circ T \]

Rewriting Many Strings At Once

\[ A \]

\[ T \]

\[ A \circ T \]

Rewriting A Single String Many Ways

\[ A \]

\[ T \]

\[ A \circ T \]

Rewriting Some Strings Zero Ways

\[ A \]

\[ T \]

\[ A \circ T \]
And a Dessert Topping!

- Composition seems pretty versatile.
- Can it help us build decoding graphs?

Example: Inserting Optional Silences

Example: Mapping Words To Phones

Example: Rewriting CI Phones as HMM's
Computing Composition

- For now, pretend no $\epsilon$-labels
- For every state $s \in A$, $t \in T$, create state $(s, t) \in A \circ T$
- Create arc from $(s_1, t_1)$ to $(s_2, t_2)$ with label $o$ iff . . .
  - There is an arc from $s_1$ to $s_2$ in $A$ with label $i$
  - There is an arc from $t_1$ to $t_2$ in $T$ with input label $i$ and output label $o$
- $(s, t)$ is initial iff $s$ and $t$ are initial; similarly for final states.
- (Remove arcs and states that cannot reach both an initial and final state.)
- What is time complexity?

Example: Computing Composition

\[
A = \begin{array}{ccc}
1 & a & 2 \\
& & b \\
& a & 3
\end{array}
\]
\[
T = \begin{array}{ccc}
1 & a:A & 2 \\
& b:B \\
& & 3
\end{array}
\]
\[
A \circ T = \begin{array}{ccc}
1,1 & a & 2,2 \\
& & b \\
& a & 3,3
\end{array}
\]

Optimization: start from initial state, build outward.

Another Example

\[
A = \begin{array}{ccc}
1 & a & 2 \\
& a:A & 3 \\
& b & 1
\end{array}
\]
\[
T = \begin{array}{ccc}
1 & a:A & 2 \\
& b:B \\
& & 3
\end{array}
\]
\[
A \circ T = \begin{array}{ccc}
1,1 & a & 2,2 \\
& & b \\
& a & 3,3
\end{array}
\]

Composition and $\epsilon$-Transitions

- Basic idea: can take $\epsilon$-transition in one FSM without moving in other FSM.
  - A little tricky to do exactly right.
  - Do the readings if you care: (Pereira, Riley, 1997)

\[
A, T = \begin{array}{ccc}
1 & \text{<epsilon>} & 2,2 \\
& B & 3,3 \\
& A & 3,1
\end{array}
\]
\[
A \circ T = \begin{array}{ccc}
1,1 & \text{<epsilon>} & 2,2 \\
& B & 3,3 \\
& A & 3,1
\end{array}
\]
Step 1: Rewrite each phone as a triphone.
   - Rewrite AX as DH_AX_R if DH to left, R to right.
Step 2: Rewrite each triphone with correct context-dependent HMM for center phone.
   - Just like rewriting a CI phone as its HMM.
   - Need to precompute HMM for each possible triphone (~ 50^3).

Point: composition automatically expands FSA to correctly handle context!
   - Makes multiple copies of states in original FSA . . .
   - That can exist in different triphone contexts.
   - (And makes multiple copies of only these states.)

Graph expansion can be expressed as series of composition operations.
   - Need to build FST to represent each expansion step, e.g.,
     1  2  THE
     2  3  DOG
     3
   - With composition operation, we’re done!
Composition is efficient.
Context-dependent expansion can be handled effortlessly.
What About Those Probability Thingies?

- *e.g.*, to hold language model probs, transition probs, etc.
- FSM’s ⇒ weighted FSM’s
  - WFSA’s, WFST’s
- Each arc has a score or cost.
  - So do final states.

\[ 1 \xrightarrow{a/0.3} 2/1 \xrightarrow{b/1.3} 3/0.4 \]

Arc Costs vs. Probabilities

- Typically, we take costs to be negative log probabilities.
  - Costs can move back and forth along a path.
  - The cost of a path is sum of arc costs plus final cost.

\[ 1 \xrightarrow{a/0.5} 2 \xrightarrow{b/0} 3/0.5 \]

- If two paths have same labels, can be combined into one.
  - Typically, use min operator to compute new cost.

\[ 1 \xrightarrow{a/0} 2 \xrightarrow{b} 3/0 \]

- Operations (+, min) form a semiring (the tropical semiring).
  - Other semirings are possible.

The Meaning of Life

- WFSA: a list of (unique) string and cost pairs \( \{(i_1 \cdots i_N, c)\} \).
- WFST: a list of triples \( \{(i_1 \cdots i_N, o_1 \cdots o_M, c')\} \).

Which Is Different From the Others?

\[ 1 \xrightarrow{<\text{epsilon}>/1} 2 \xrightarrow{a/0} 3/0 \]

\[ 1 \xrightarrow{a/3} 2/2 \xrightarrow{b/1} 3 \]
Weighted Composition

- Composing WFSA $A$ with WFST $T$ to get WFSA $A \circ T$.
- If $(i_1 \cdots i_N, c) \in A$ and \ldots
- $(i_1 \cdots i_N, o_1 \cdots o_M, c') \in T$, \ldots
- Then, $(o_1 \cdots o_M, c + c') \in A \circ T$.
- Combine costs for all different ways to produce same $o_1 \cdots o_M$.

Weighted Graph Expansion

- Start with weighted FSA representing language model.
- Use composition to apply weighted FST for each level of expansion.
  - Scores/logprobs will be accumulated.
  - Log probs may move around along paths.
  - All that matters for Viterbi is total score of paths.

Recap: Composition

- Like sed, but can operate on all paths in a lattice simultaneously.
- Rewrite symbols as other symbols.
  - e.g., rewrite words as phone sequences (or vice versa).
- Context-dependent rewriting of symbols.
  - e.g., rewrite CI phones as their CD variants.
- Add in new scores.
  - e.g., language model lattice rescoring.
- Restrict the set of allowed paths/intersection.
  - e.g., find all paths in lattice containing word NOODGE.
- Or all of the above at once.
Road Map

- Part I: The LVCSR acoustic model.
- Part II: Acoustic model training for LVCSR.
- Part III: Decoding for LVCSR (inefficient).
  - Part IV: Introduction to finite-state transducers.
- Part V: Search (Lecture 8).
  - Making decoding for LVCSR efficient.

Course Feedback

- Was this lecture mostly clear or unclear? What was the muddiest topic?
- Other feedback (pace, content, atmosphere)?