Lecture 7

LVCSR Training and Decoding (Part A)

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The Big Picture

- Weeks 1–4: Small vocabulary ASR.
- Weeks 5–8: Large vocabulary ASR.
  - Week 5: Language modeling (for large vocabularies).
  - Week 6: Pronunciation modeling — acoustic modeling for large vocabularies.
  - Week 7, 8: Training, decoding for large vocabularies.
- Weeks 9–13: Advanced topics.

Outline

- Part I: The LVCSR acoustic model.
- Part II: Acoustic model training for LVCSR.
- Part III: Decoding for LVCSR (inefficient).
  - Part IV: Introduction to finite-state transducers.
- Part V: Search (Lecture 8).
  - Making decoding for LVCSR efficient.

Part I

The LVCSR Acoustic Model

What is LVCSR?

- Large vocabulary.
  - Phone-based modeling *vs.* word-based modeling.
- Continuous.
  - No pauses between words.

The Fundamental Equation of ASR

$$\begin{aligned}
\text{class}(\mathbf{x}) &= \arg\max_{\omega} P(\omega|\mathbf{x}) \\
&= \arg\max_{\omega} \frac{P(\omega)P(\mathbf{x}|\omega)}{P(\mathbf{x})} \\
&= \arg\max_{\omega} P(\omega)P(\mathbf{x}|\omega)
\end{aligned}$$

- $P(\mathbf{x}|\omega)$ — acoustic model.
- $P(\omega)$ — language model.

The Acoustic Model: Small Vocabulary

$$\begin{aligned}
P_{\omega}(\mathbf{x}) &= \sum_{A} P_{\omega}(\mathbf{x}, A) = \sum_{A} P_{\omega}(A) \times P_{\omega}(\mathbf{x}|A) \\
&\approx \max_{A} P_{\omega}(A) \times P_{\omega}(\mathbf{x}|A) \\
&= \max_{A} \prod_{t=1}^{T} P(a_t) \prod_{t=1}^{T} P(\vec{x}_t|a_t) \\
\log P_{\omega}(\mathbf{x}) &= \max_{A} \left[\sum_{t=1}^{T} \log P(a_t) + \sum_{t=1}^{T} \log P(\vec{x}_t|a_t) \right] \\
P(\vec{x}_t|a_t) &= \sum_{m=1}^{M} \lambda_{a_t,m} \prod_{\text{dim } d}^{D} \mathcal{N}(x_{t,d}; \mu_{a_t,m,d}, \sigma_{a_t,m,d})
\end{aligned}$$

The Acoustic Model: Large Vocabulary

$$\begin{aligned}
P_{\omega}(\mathbf{x}) &= \sum_{A} P_{\omega}(\mathbf{x}, A) = \sum_{A} P_{\omega}(A) \times P_{\omega}(\mathbf{x}|A) \\
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\end{aligned}$$

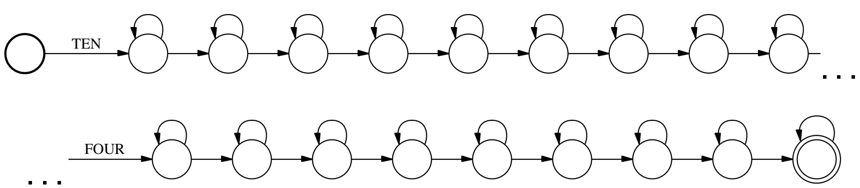
What Has Changed?

- The HMM.
  - Each alignment A describes a path through an HMM.
- Its parameterization.
  - In $P(\vec{x}_t|a_t)$, how many GMM's to use? (Share between HMM's?)

Describing the Underlying HMM

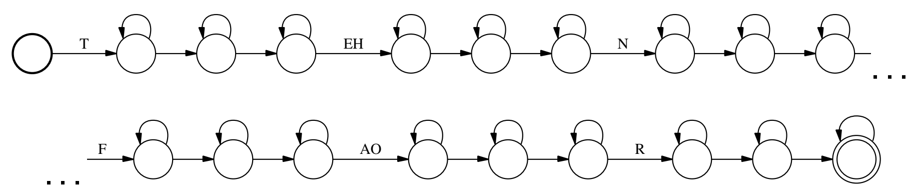
- Fundamental concept: how to map a word (or baseform) sequence to its HMM.
  - In training, map reference transcript to its HMM.
  - In decoding, glue together HMM's for all allowable word sequences.

The HMM: Small Vocabulary



- One HMM per word.
- Glue together HMM for each word in word sequence.

The HMM: Large Vocabulary



- One HMM per phone.
- Glue together HMM for each phone in phone sequence.
  - Map word sequence to phone sequence using baseform dictionary.

I Still Don't See What's Changed

- HMM topology typically doesn't change.
- HMM parameterization changes.

Parameterization

- Small vocabulary.
  - One GMM per state (three states per phone).
  - No sharing between phones in different words.
- Large vocabulary, context-independent (CI).
  - One GMM per state.
  - *Tying* between phones in different words.
- Large vocabulary, context-dependent (CD).
  - Many GMM's per state; GMM to use depends on phonetic context.
  - Tying between phones in different words.

Context-Dependent Parameterization

- Each phone HMM state has its own decision tree.
  - Decision tree asks questions about phonetic context. (Why?)
  - One GMM per leaf in the tree. (Up to 200+ leaves/tree.)
- How will tree for first state of a phone tend to differ ...
  - From tree for last state of a phone?
- Terminology.
  - *triphone* model — ± 1 phones of context.
  - *quinphone* model — ± 2 phones of context.

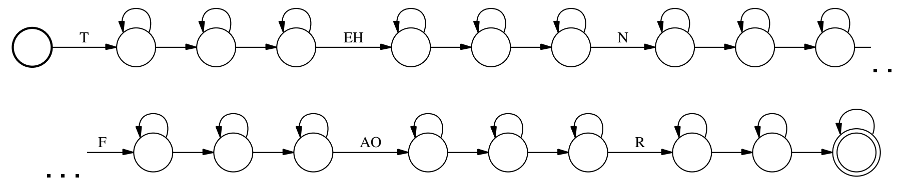
A Real-Life Tree

```
Tree for feneme AA_1:
node   0: quest-P  23[-1] --> true: node   1, false: node   2
  quest: AX AXR B BD CH D DD DH DX D$ ER F G GD HH JH K KD M N NG P PD R S
    SH T TD TH TS UW V W X Z ZH
node   1: quest-P  66[-1] --> true: node   3, false: node   4
  quest: AO AXR ER IY L M N NG OW OY R UH UW W Y
node   2: quest-P  36[-2] --> true: node   5, false: node   6
  quest: D$ X
node   3: quest-P  13[-1] --> true: node   7, false: node   8
  quest: AXR ER R
node   4: quest-P  13[+1] --> true: node   9, false: node  10
  quest: AXR ER R
node   5: leaf    0
node   6: quest-P  15[-1] --> true: node  11, false: node  12
  quest: AXR ER L OW R UW W
node   7: quest-P  49[-2] --> true: node  13, false: node  14
  quest: DX K P T
node   8: quest-P  20[-1] --> true: node  15, false: node  16
  quest: B BD CH D DD DH F G GD IY JH K KD M N NG P PD S SH T TD TH TS V X Y
    Z ZH
node   9: quest-P  43[-2] --> true: node  17, false: node  18
  quest: CH DH F HH JH S SH TH TS V Z ZH
node  10: quest-P  49[-1] --> true: node  19, false: node  20
  quest: DX K P T
node  11: leaf    1
node  12: quest-P  15[-2] --> true: node  21, false: node  22
  quest: AXR ER L OW R UW W
node  13: leaf    2
node  14: leaf    3
...
```

Pop Quiz

- Pretend you are Keanu Reeves.
- System description:
  - 1000 words in lexicon; average word length = 5 phones.
  - There are 50 phones; each phone HMM has three states.
  - Each decision tree contains 100 leaves on average.
- How many GMM's are there in:
  - A small vocabulary system (word models)?
  - A CI large vocabulary system?
  - A CD large vocabulary system?

Any Questions?



- Given a word sequence, you should understand how to . . .
  - Layout the corresponding HMM topology.
  - Determine which GMM to use at each state, for CI and CD models.

Context-Dependent Phone Models

- Typical model sizes:

type	HMM	GMM's/ state	GMM's	Gaussians
word	per word	1	10–500	100–10k
CI phone	per phone	1	∼150	1k–3k
CD phone	per phone	1–200	1k–10k	10k–300k

- 39-dimensional feature vectors $\Rightarrow$ $\sim$80 parameters/Gaussian.
- Big models can have tens of millions of parameters.

What About Transition Probabilities?

- This slide only included for completeness.
- Small vocabulary.
  - One set of transition probabilities per state.
  - No sharing between phones in different words.
- Large vocabulary.
  - One set of transition probabilities per state.
  - Sharing between phones in different words.
- What about context-dependent transition modeling?

Recap

- Main difference between small vocabulary and large vocabulary:
  - Allocation of GMM's.
  - Sharing GMM's between words: needs less GMM's.
  - Modeling context-dependence: needs more GMM's.
  - Hybrid allocation is possible.
- Training and decoding for LVCSR.
  - In theory, any reason why small vocabulary techniques won't work?
  - In practice, yikes!

Points to Ponder

- Why deterministic mapping?
  - DID YOU ⇒ D IH D JH UW
  - The area of *pronunciation modeling*.
- Why decision trees?
  - Unsupervised clustering.

Part II

Acoustic Model Training for LVCSR

Small Vocabulary Training — Lab 2

- Phase 1: Collect underpants.
  - Initialize all Gaussian means to 0, variances to 1.
- Phase 2: Iterate over training data.
  - For each word, train associated word HMM . . .
  - On all samples of that word in the training data . . .
  - Using the Forward-Backward algorithm.
- Phase 3: Profit!

Large Vocabulary Training

- What's changed going to LVCSR?
  - Same HMM topology; just more Gaussians and GMM's.
- Can we just use the same training algorithm as before?

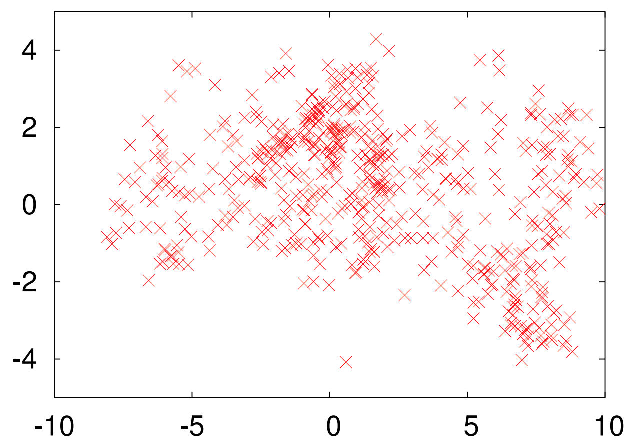
Where Are We?

Flat or Random Start

- Why does this work for small models?
  - We believe there's a huge global minimum . . .
  - In the "middle" of the parameter search space.
  - With a neutral starting point, we're apt to fall into it.
  - (Who knows if this is actually true.)
- Why doesn't this work for large models?

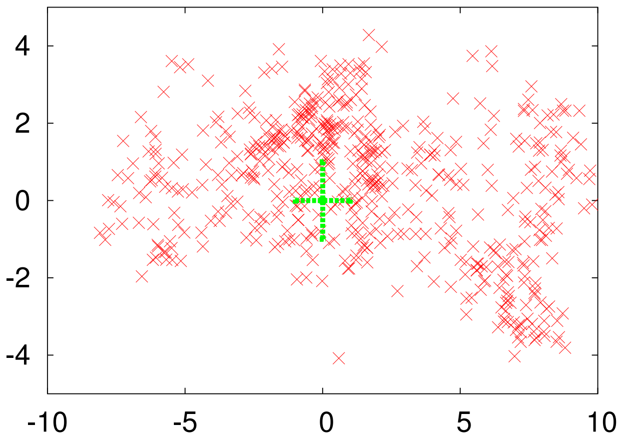
Case Study: Training a Simple GMM

- Front end from Lab 1; first two dimensions; 546 frames.



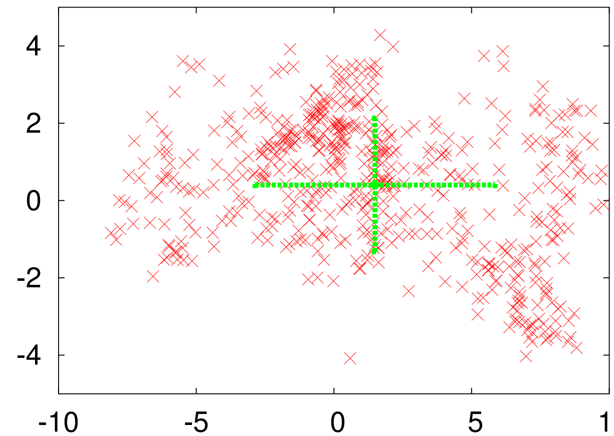
Training a Mixture of Two 2-D Gaussians

- Flat start?
  - Initialize mean of each Gaussian to 0, variance to 1.



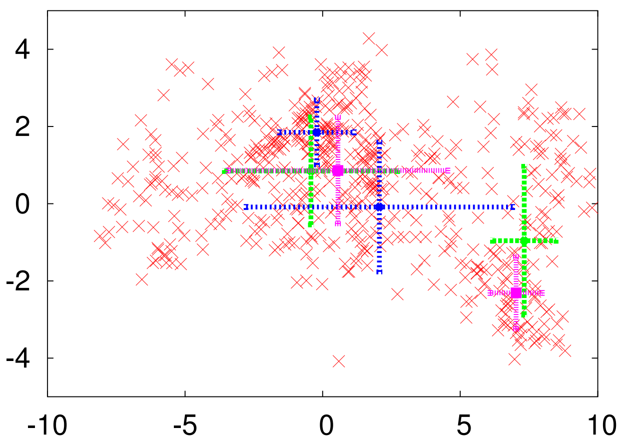
Training a Mixture of Two 2-D Gaussians

- "*At the Mr. O level, symmetry is everything.*"
  - At the GMM level, symmetry is a bad idea.



Training a Mixture of Two 2-D Gaussians

- Random seeding?
  - Picked 8 random starting points $\Rightarrow$ 3 different optima.



Training Hidden Models

- (MLE) training of models with hidden variables has local minima.
- What are the hidden variables in ASR?
  - *i.e.*, what variables are in our model ...
  - That are not observed.

How To Spot Hidden Variables

$$
\begin{aligned}
P_\omega(\mathbf{x}) &= \sum_A P_\omega(\mathbf{x}, A) = \sum\nolimits_A P_\omega(A) \times P_\omega(\mathbf{x}|A) \\
&\approx \max_A P_\omega(A) \times P_\omega(\mathbf{x}|A) \\
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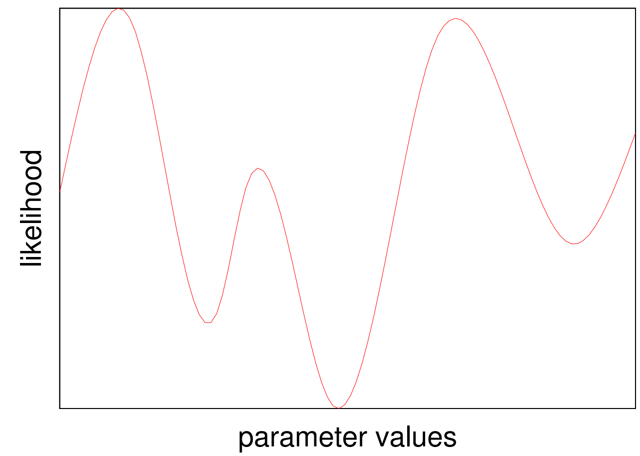
P(\vec{x}_t|a_t) &= \sum\nolimits_{m=1}^{M} \lambda_{a_t,m} \prod_{\text{dim } d}^{D} \mathcal{N}(x_{t,d}; \mu_{a_t,m,d}, \sigma_{a_t,m,d})
\end{aligned}
$$

Gradient Descent and Local Minima

- EM training does hill-climbing/gradient descent.
  - Finds "nearest" optimum to where you started.



What To Do?

- Insight: If we know the "correct" hidden values for a model:
  - *e.g.*, which arc and which Gaussian for each frame . . .
  - Training is easy! (No local minima.)
  - Remember Viterbi training given fixed alignment in Lab 2.
- Is there a way to guess the correct hidden values for a large model?

Bootstrapping Alignments

- Recall that all of our acoustic models, from simple to complex:
  - Generally use the same HMM topology!
  - (All that differs is how we assign GMM's to each arc.)
- Given an alignment (from arc/phone states to frames) for simple model . . .
  - It is straightforward to compute analogous alignment for complex model!

Bootstrapping Big Models From Small

- Recipe:
  - Start with model simple enough that flat start works.
  - Iteratively build more and more complex models . . .
  - By using last model to seed hidden values for next.
- Need to come up with sequence of successively more complex models . . .
  - With related hidden structure.

How To Seed Next Model From Last

- Directly via hidden values, *e.g.*, alignment.
  - *e.g.*, *single-pass retraining*.
  - Can be used between very different models.
- Via parameters.
  - Seed parameters in complex model so that . . .
  - Initially, will yield same/similar alignment as in simple model.
  - *e.g.*, moving from CI to CD GMM's.

Bootstrapping Big Models From Small

- Recurring motif in acoustic model training.
- The reason why state-of-the-art systems . . .
  - Require many, many training passes, as you will see.
- Recipes handed down through the generations.
  - Discovered via sweat and tears.
  - Art, not science.
  - But no one believes these find global optima . . .
  - Even for small problems.

Overview of Training Process

- Build CI single Gaussian model from flat start.
- Use CI single Gaussian model to seed CI GMM model.
- Build phonetic decision tree (using CI GMM model to help).
- Use CI GMM model to seed CD GMM model.

Where Are We?

Case Study: Training a GMM

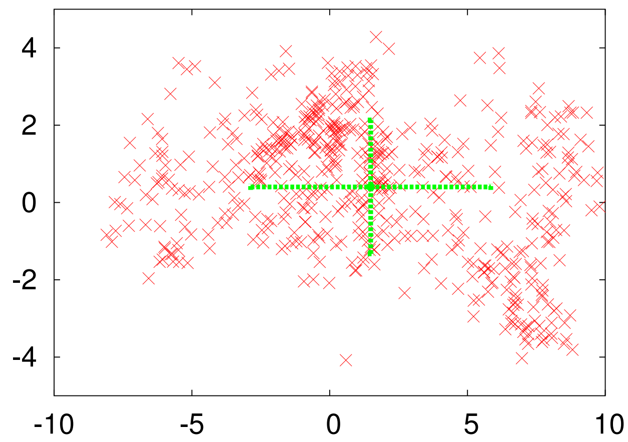
- Recursive mixture splitting.
  - A sequence of successively more complex models.
- *k*-means clustering.
  - Seed means in one shot.

Gaussian Mixture Splitting

- Start with single Gaussian per mixture (trained).
- Split each Gaussian into two.
  - Perturb means in opposite directions; same variance.
  - Train.
- Repeat until reach desired number of mixture components (1, 2, 4, 8, . . .).
  - (Discard Gaussians with insufficient counts.)
- Assumption: c-component GMM gives good guidance . . .
  - On how to seed $2c$-component GMM.

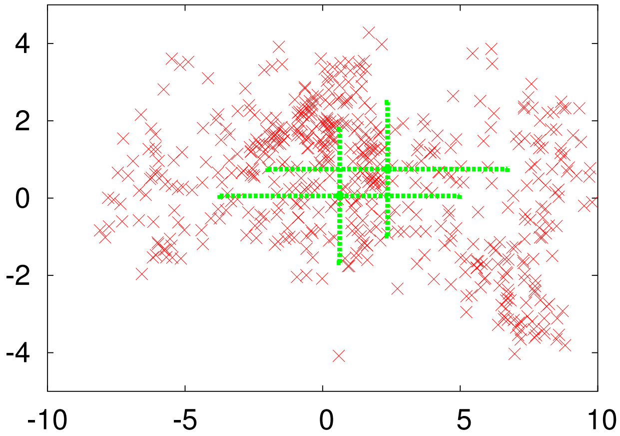
Mixture Splitting Example

- Train single Gaussian.



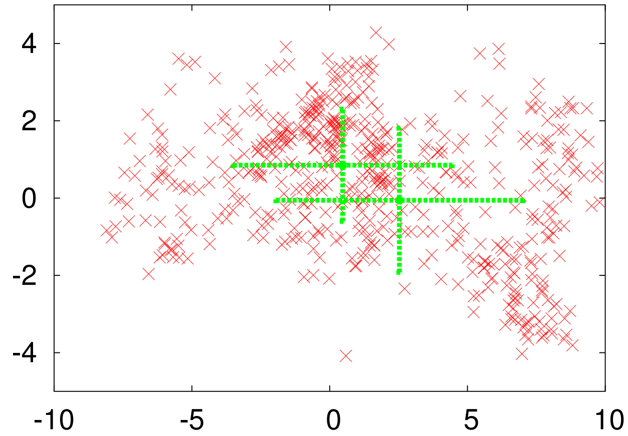
Mixture Splitting Example

- Split each Gaussian in two ($\pm 0.2 \times \vec{\sigma}$)



Mixture Splitting Example

- Train, yep.



Mixture Splitting Example

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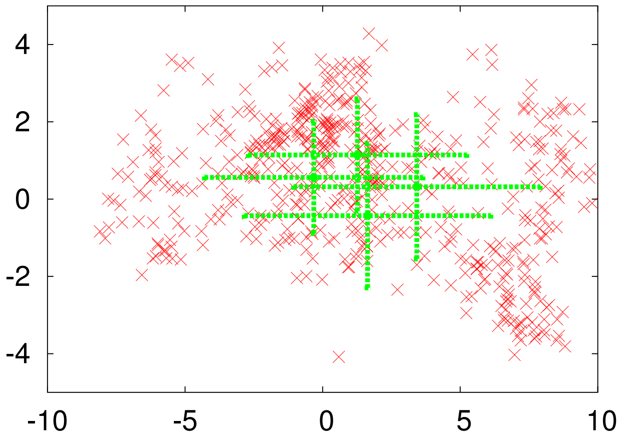
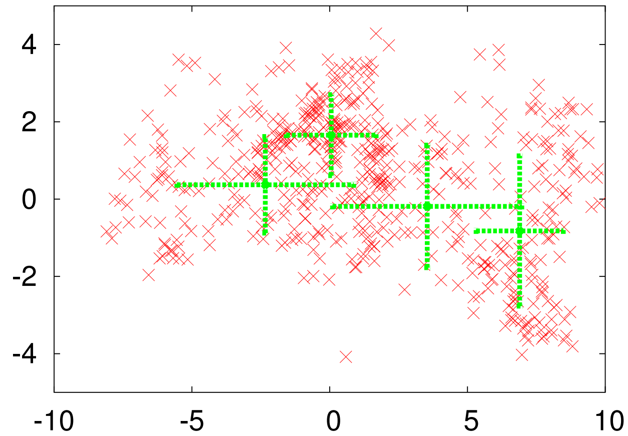


Mixture Splitting Example

- Train, yep.



Applying Mixture Splitting in ASR

- Recipe:
  - Start with model with 1-component GMM's (à la Lab 2).
  - Split Gaussians in each output distribution simultaneously.
  - Do many iterations of FB.
  - Repeat.
- Real-life numbers:
  - Five splits spread within 30 iterations of FB.

Another Way: Automatic Clustering

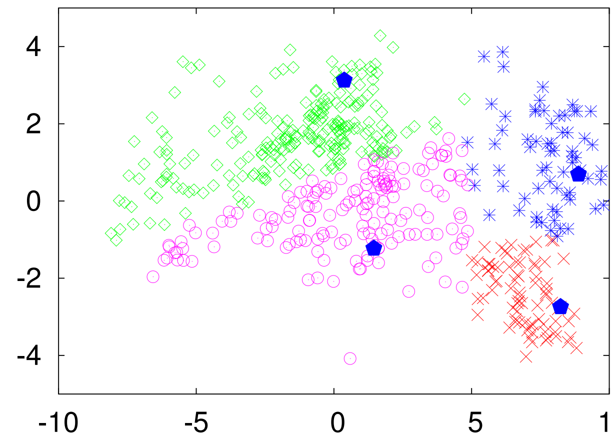
- Use unsupervised clustering algorithm to find clusters.
- Given clusters . . .
  - Use cluster centers to seed Gaussian means.
  - FB training.
  - (Discard Gaussians with insufficient counts.)

k-Means Clustering

- Select desired number of clusters k.
- Choose k data points randomly.
  - Use these as initial cluster centers.
- "Assign" each data point to nearest cluster center.
- Recompute each cluster center as . . .
  - Mean of data points "assigned" to it.
- Repeat until convergence.

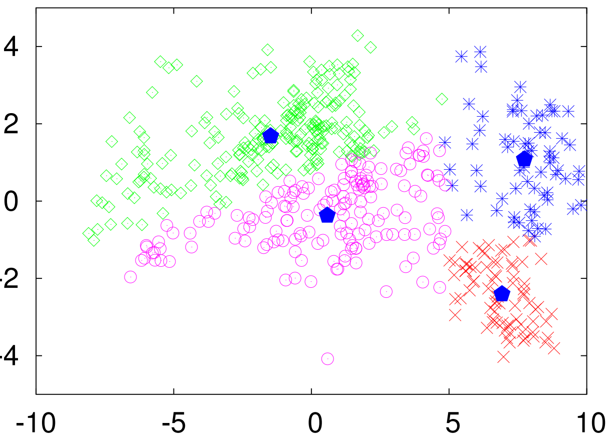
k-Means Example

- Pick random cluster centers; assign points to nearest center.



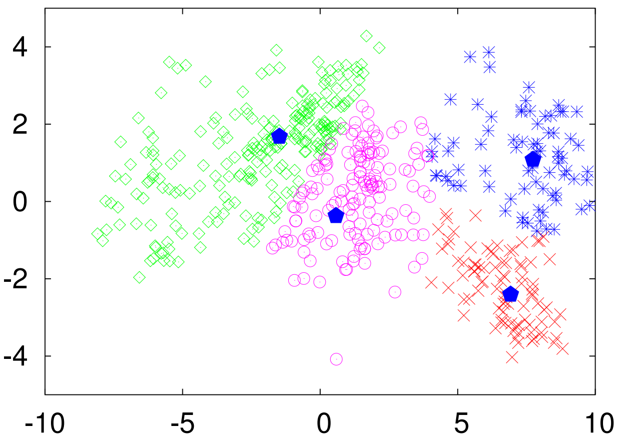
k-Means Example

- Recompute cluster centers.



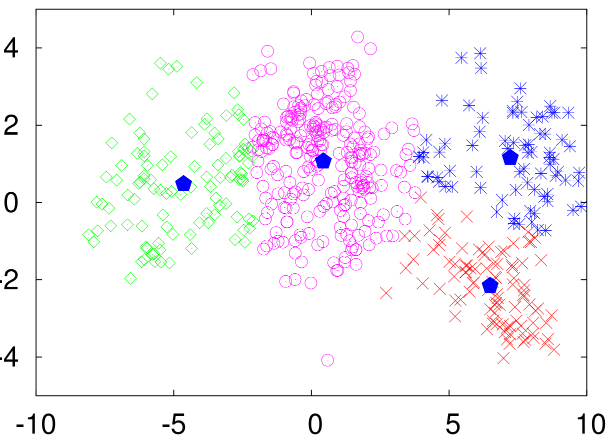
k-Means Example

- Assign each point to nearest center.



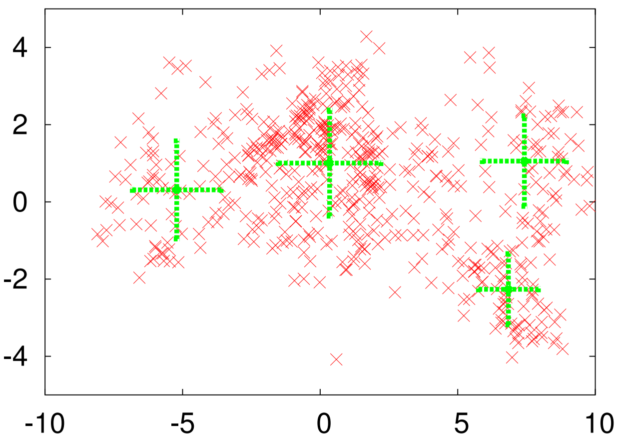
k-Means Example

- Repeat until convergence.

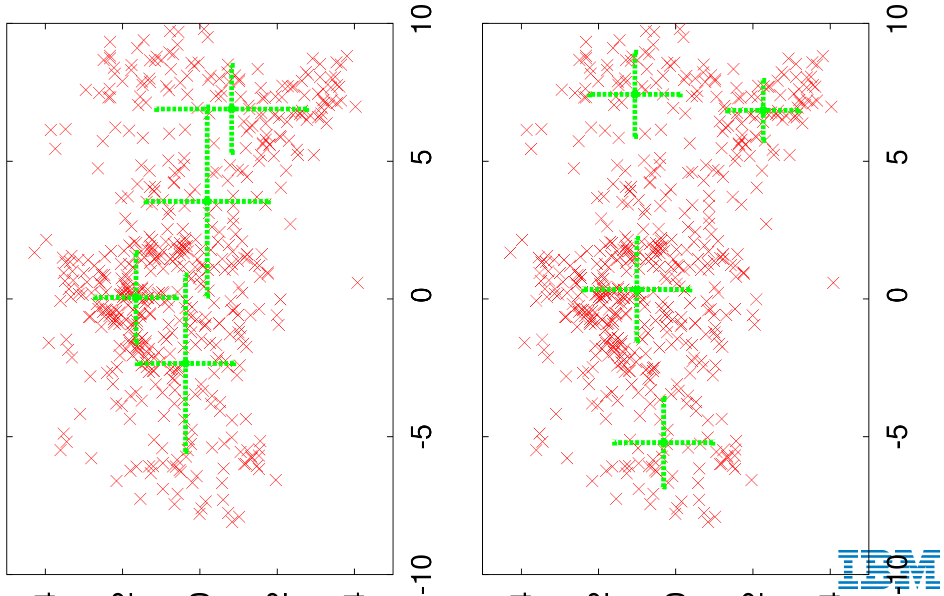


k-Means Example

- Use centers as means of Gaussians; train, yep.



The Final Mixtures, Splitting *vs.* k-Means



Technical Aside: k-Means Clustering

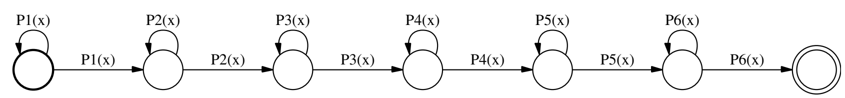
- When using Euclidean distance ...
- k-means clustering is equivalent to ...
  - Seeding Gaussian means with the k initial centers.
  - Doing Viterbi EM update, keeping variances constant.

Applying k-Means Clustering in ASR

- To train each GMM, use k-means clustering ...
  - On what data? Which frames?
- Huh?
  - How to decide which frames *align* to each GMM?
- This issue is evaded for mixture splitting.
  - Can we avoid it here?

Forced Alignment

- Viterbi algorithm.
  - Finds most likely alignment of HMM to data.



frame	0	1	2	3	4	5	6	7	8	9	10	11	12
arc	P_1	P_1	P_1	P_2	P_3	P_4	P_4	P_5	P_5	P_5	P_5	P_6	P_6

- Need existing model to create alignment. (Which?)

Recap

- You can use single Gaussian models to seed GMM models.
  - Mixture splitting: use c-component GMM to seed $2c$-component GMM.
  - k-means: use single Gaussian model to find alignment.
- Both of these techniques work about the same.
  - Nowadays, we primarily use mixture splitting.

Where Are We?

What Do We Need?

- For each tree/phone state ...
  - List of frames/feature vectors associated with that tree.
  - (This is the data we are optimizing the likelihood of.)
  - For each frame, the phonetic context.
- A list of candidate questions about the phonetic context.
  - Ask about phonetic concepts; *e.g.*, vowel or consonant?
  - Expressed as list of phones in set.
  - Allow same questions to be asked about each phone position.
  - Handed down through the generations.

A Real-Life Tree

```
Tree for feneme AA_1:
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node  13: leaf    2
node  14: leaf    3
...
```
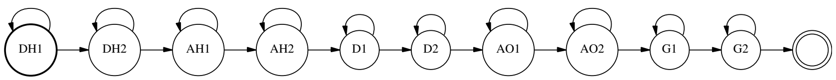

Training Data for Decision Trees

- Forced alignment/Viterbi decoding!
- Where do we get the model to align with?
  - Use CI phone model or other pre-existing model.



frame	0	1	2	3	4	5	6	7	8	9	$\cdots$
arc	DH_1	DH_2	AH_1	AH_2	D_1	D_1	D_2	D_2	D_2	AO_1	$\cdots$

Building the Tree

- A set of events $\{(\vec{x}_i, p_L, p_R)\}$ (possibly subsampled).
- Given current tree:
  - Choose question of the form ...
  - "*Does the phone in position j belong to the set q?*" ...
  - That optimizes $\prod_i P(\vec{x}_i | \text{leaf}(p_L, p_R))$ ...
  - Where we model each leaf using a single Gaussian.
- Can efficiently build whole level of tree in single pass.
- See Lecture 6 slides and readings for the gory details.

Seeding the Context-Dependent GMM's

- Context-independent GMM's: one GMM per phone state.
- Context-dependent GMM's: l GMM's per phone state.
- How to seed context-dependent GMM's?
  - *e.g.*, so that initial alignment matches CI alignment?

Where Are We?

Where Are We?

4 Details
- Maximum Likelihood Training?
- Viterbi *vs.* Non-Viterbi Training
- Graph Building

The Original Story, Small Vocabulary

- One HMM for each word; flat start.
- Collect all examples of each word.
  - Run FB on those examples to do maximum likelihood training of that HMM.

The New Story

- One HMM for each word sequence!?
  - But tie parameters across HMM's!
- Do complex multi-phase training.
- Are we still doing anything resembling maximum likelihood training?

Maximum Likelihood Training?

- Regular training iterations (FB, Viterbi EM).
  - Increase (Viterbi) likelihood of data.
- Seeding last model from next model.
  - Mixture splitting.
  - CI $\Rightarrow$ CD models.
- (Decision-tree building.)

Maximum Likelihood Training?

- Just as LM's need to be smoothed or *regularized*.
  - So do acoustic models.
  - Prevent extreme likelihood values (*e.g.*, 0 or ∞).
- ML training maximizes training data likelihood.
  - We actually want to optimize test data likelihood.
  - Let's call the difference the *overfitting penalty*.
- The overfitting penalty tends to increase as . . .
  - The number of parameters increase and/or . . .
  - Parameter magnitudes increase.

Regularization/Capacity Control

- Limit size of model.
  - Will training likelihood continue to increase as model grows?
  - Limit components per GMM.
  - Limit number of leaves in decision tree, *i.e.*, number of GMM's.
- Variance flooring.
  - Don't let variances go to 0 $\Rightarrow$ infinite likelihood.

Where Are We?

Two Types of Updates

- "Full" EM.
  - Compute true posterior of each hidden configuration.
- Viterbi EM.
  - Use Viterbi algorithm to find most likely hidden configuration.
  - Assign posterior of 1 to this configuration.
- Both are valid updates; instances of generalized EM.

Examples

- Training GMM's.
  - Mixture splitting *vs.* k-means clustering.
- Training HMM's.
  - Forward-backward *vs.* Viterbi EM (Lab 2).
- Everywhere you do a forced alignment.
  - Refining the reference transcript.
  - What is non-Viterbi version of decision-tree building?

When To Use One or the Other?

- Which version is more expensive computationally?
  - Optimization: need not realign every iteration.
- Which version finds better minima?
- If posteriors are very sharp, they do almost the same thing.
  - Remember example posteriors in Lab 2?
- Rule of thumb:
  - When you're first training a "new" model, use full EM.
  - Once you're "locked in" to an optimum, Viterbi is fine.

Where Are We?

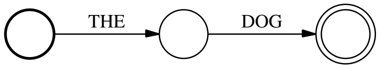
4 Details
- Maximum Likelihood Training?
- Viterbi *vs.* Non-Viterbi Training
- Graph Building

Building HMM's For Training

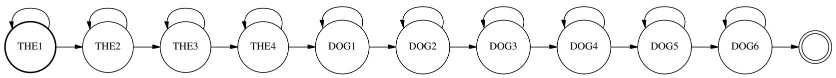
- When doing Forward-Backward on an utterance ...
  - We need the HMM corresponding to the reference transcript.
- Can we use the same techniques as for small vocabularies?

Word Models

- Reference transcript

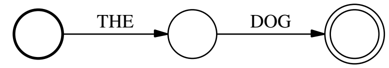


- Replace each word with its HMM

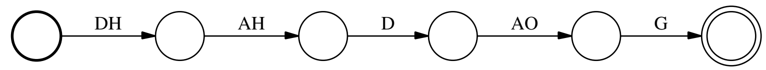


Context-Independent Phone Models

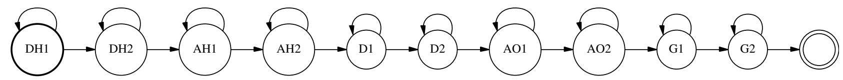
- Reference transcript



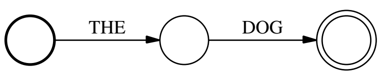
- Pronunciation dictionary.
  - Maps each word to a sequence of phonemes.

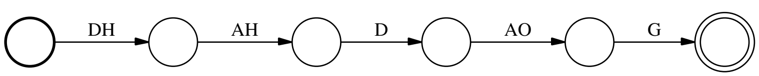


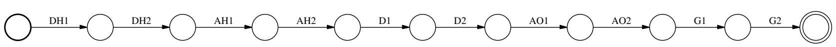
- Replace each phone with its HMM

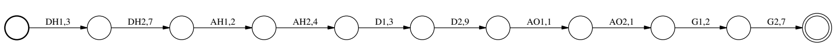


Context-Dependent Phone Models









The Pronunciation Dictionary

- Need pronunciation of *every* word in training data.
  - Including pronunciation variants

```
THE(01)    DH  AH
THE(02)    DH  IY
```

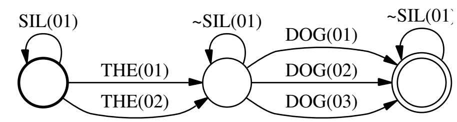
  - Listen to data?
  - Use automatic spelling-to-sound models?
- Why not consider multiple baseforms/word for word models?

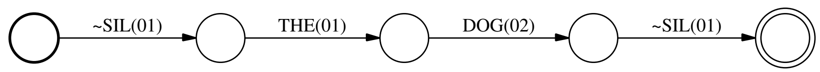
But Wait, It's More Complicated Than That!

- Reference transcripts are created by humans . . .
  - Who, by their nature, are *human* (*i.e.*, fallible)
- Typical transcripts don't contain everything an ASR system wants.
  - Where silence occurred; noises like coughs, door slams, etc.
  - Pronunciation information, *e.g.*, was `THE` pronounced as `DH UH` or `DH IY`?

Pronunciation Variants, Silence, and Stuff

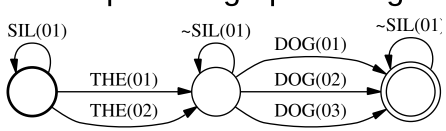
- How can we produce a more "complete" reference transcript?
- Viterbi decoding!
  - Build HMM accepting all word (HMM) sequences consistent with reference transcript.
  - Compute best path/word HMM sequence.
  - Where does this initial acoustic model come from?





Another Way

- Just use the whole expanded graph during training.



- The problem: how to do context-dependent phone expansion?
  - Use same techniques as in building graphs for decoding.

Where Are We?

1. The Local Minima Problem
2. Training GMM's
3. Building Phonetic Decision Trees
4. Details
5. The Final Recipe

Prerequisites

- Audio data with reference transcripts.
- What two other things?

The Training Recipe

- Find/make baseforms for all words in reference transcripts.
- Train single Gaussian models (flat start; many iters of FB).
- Do mixture splitting, say.
  - Split each Gaussian in two; do many iterations of FB.
  - Repeat until desired number of Gaussians per mixture.
- (Use initial system to refine reference transcripts.)
  - Select pronunciation variants, where silence occurs.
  - Do more FB training given refined transcripts.
- Build phonetic decision tree.
  - Use CI model to align training data.
- Seed CD model from CI; train using FB or Viterbi EM.
  - Possibly doing more mixture splitting.

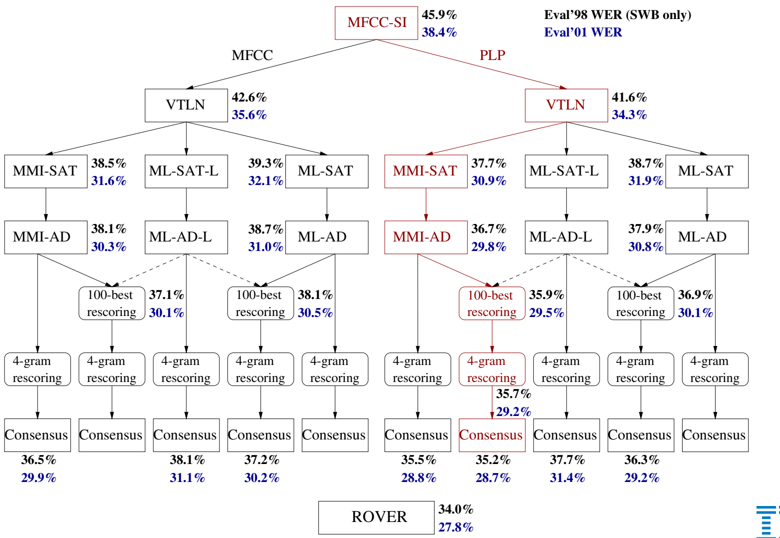
How Long Does Training Take?

- It's a secret.
- We think in terms of *real-time factor*.
  - How many hours does it take to process one hour of speech?

Whew, That Was Pretty Complicated!

- Adaptation (VTLN, fMLLR, mMLLR)
- Discriminative training (LDA, MMI, MPE, fMPE)
- Model combination (cross adaptation, ROVER)
- Iteration.
  - Repeat steps using better model for seeding.
  - Alignment is only as good as model that created it.

Things Can Get Pretty Hairy



Recap: Acoustic Model Training for LVCSR

- Take-home messages.
  - Hidden model training is fraught with local minima.
  - Seeding more complex models with simpler models helps avoid terrible local minima.
  - People have developed many recipes/heuristics to try to improve the minimum you end up in.
  - Training is insanely complicated for state-of-the-art research models.
- The good news . . .
  - I just saved a bunch on money on my car insurance by switching to GEICO.

Part III

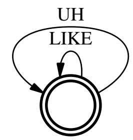
Decoding for LVCSR (Inefficient)

Decoding for LVCSR (Inefficient)

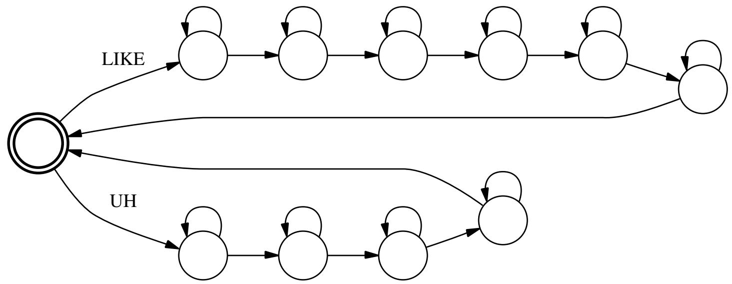
$$\begin{aligned}
\text{class}(\mathbf{x}) &= \arg\max_{\omega} P(\omega|\mathbf{x}) \\
&= \arg\max_{\omega} \frac{P(\omega)P(\mathbf{x}|\omega)}{P(\mathbf{x})} \\
&= \arg\max_{\omega} P(\omega)P(\mathbf{x}|\omega)
\end{aligned}$$

- Now that we know how to build models for LVCSR . . .
  - CD acoustic models via complex recipes.
  - n-gram models via counting and smoothing.
- How can we use them for decoding?
  - Let's ignore memory and speed constraints for now.

Decoding: Small Vocabulary



- Take graph/WFSA representing language model
  - *i.e.*, all allowable word sequences.
- Expand to underlying HMM

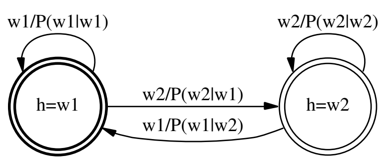


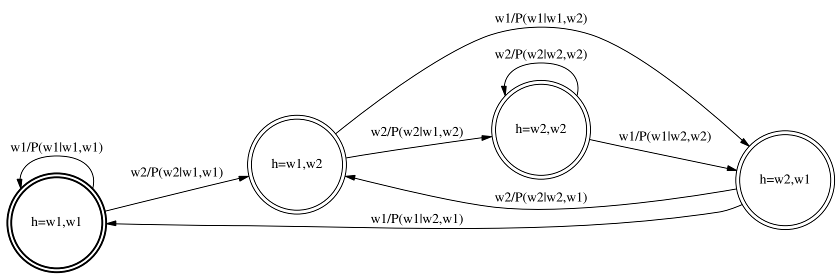
- Run the Viterbi algorithm!

Issue 1: Are N-Gram Models WFSA's?

- Yup.
- Invariants.
  - One state for each $(n-1)$-gram history.
  - All paths ending in state for $(n-1)$-gram ω . . .
  - Are labeled with word sequence ending in ω.
  - State for $(n-1)$-gram ω has outgoing arc for each word w . . .
  - With arc probability $P(w|\omega)$.

Bigram, Trigram LM's Over Two Word Vocab



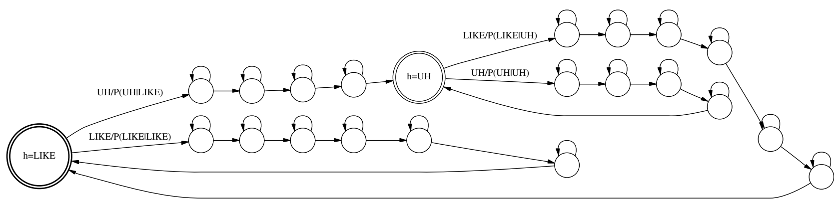


Pop Quiz

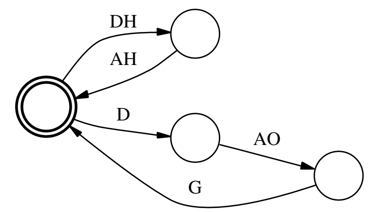
- How many states in FSA representing n-gram model . . .
  - With vocabulary size $|V|$?
- How many arcs?

Issue 2: Graph Expansion

- Word models.
  - Replace each word with its HMM.
- CI phone models.
  - Replace each word with its phone sequence(s)
  - Replace each phone with its HMM.

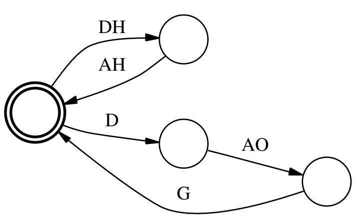


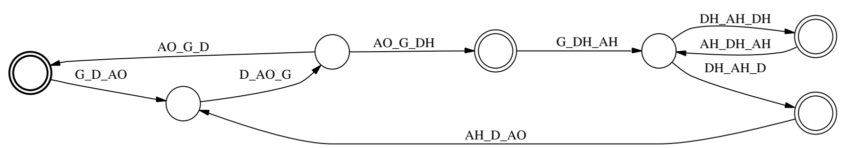
Context-Dependent Graph Expansion



- How can we do context-dependent expansion?
  - Handling branch points is tricky.
- Other tricky cases.
  - Words consisting of a single phone.
  - Quinphone models.

Triphone Graph Expansion Example





Word-Internal Acoustic Models

- Simplify acoustic model to simplify graph expansion.
- *Word-internal* models.
  - Don't let decision trees ask questions across word boundaries.
  - Pad contexts with the *unknown phone*.
  - Hurts performance (*e.g.*, coarticulation across words).
- As with word models, just replace each word with its HMM.

Context-Dependent Graph Expansion

- Is there some elegant theoretical framework . . .
- That makes it easy to do this type of expansion . . .
- And also makes it easy to do lots of other graph operations useful in ASR?
- $\Rightarrow$ Finite-state transducers (FST's)! (Part IV)

Recap: Decoding for LVCSR (Inefficient)

- In theory, do same thing as we did for small vocabularies.
  - Start with LM represented as word graph.
  - Expand to underlying HMM.
  - Viterbi.
- In practice, computation and memory issues abound.
- How to do the graph expansion? FST's (Part IV)
- How to make decoding efficient? search (Part V)

Part IV

Introduction to Finite-State Transducers

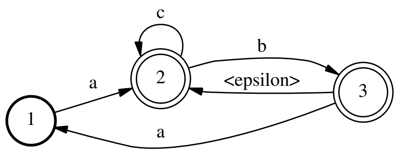
Introduction to Finite-State Transducers

Overview

- FST's are closely related to finite-state automata (FSA).
  - An FSA is a graph.
  - An FST . . .
  - Takes an FSA as input . . .
  - And produces a new FSA.
- Natural technology for graph expansion . . .
  - And much, much more.
- FST's for ASR pioneered by AT&T in late 1990's

Review: What is a Finite-State Acceptor?

- It has states.
  - Exactly one initial state; one or more final states.
- It has arcs.
  - Each arc has a label, which may be empty (ϵ).
- Ignore probabilities for now.
- Meaning: a (possibly infinite) list of strings.

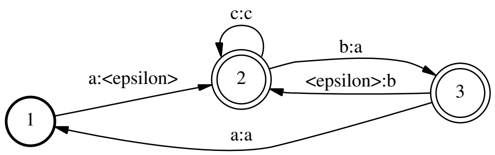


Review: Pop Quiz

- What are the differences between the following:
  - HMM's with discrete output distributions.
  - FSA's with arc probabilities.

What is a Finite-State Transducer?

- It's like a finite-state acceptor, except ...
- Each arc has two labels instead of one.
  - An *input* label (possibly empty)
  - An *output* label (possibly empty)
- Meaning: a (possibly infinite) list of pairs of strings ...
  - An input string and an output string.



Terminology

- *finite-state acceptor* (FSA): one label on each arc.
- *finite-state transducer* (FST): input and output label on each arc.
- *finite-state machine* (FSM): FSA or FST.
  - Also, *finite-state automaton*
- Incidentally, an FSA can act like an FST.
  - Pretend input label is both input and output label.

Transforming a Single String

- Let's say you have a string, *e.g.*,

THE DOG

- Let's say we want to apply a transformation.
  - *e.g.*, map words to their baseforms.

DH AH D AO G

- This is easy, *e.g.*, use `sed` or `perl` or ...

Transforming Lots of Strings At Once

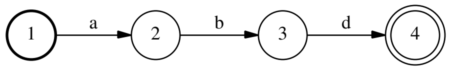
- Let's say you have a (possibly infinite) list of strings ...
  - Expressed as an FSA, as this is compact.
- Let's say we want to apply a transformation.
  - *e.g.*, map words to their baseforms.
- On all of these strings.
- And have the (possibly infinite) list of output strings ...
  - Expressed as an FSA, as this is compact.
- Efficiently.

The Composition Operation

- FSA: represents a list of strings $\{i_1 \cdots i_N\}$.
- FST: represents a list of strings pairs $\{(i_1 \cdots i_N, o_1 \cdots o_M)\}$.
  - A compact way of representing string transformations.
- Composing FSA A with FST T to get FSA $A \circ T$.
  - If string $i_1 \cdots i_N \in A$ and ...
  - Input/output string pair $(i_1 \cdots i_N, o_1 \cdots o_M) \in T$, ...
  - Then, string $o_1 \cdots o_M \in A \circ T$.

Rewriting a Single String

A



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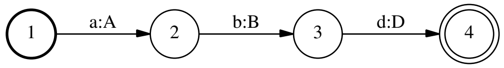
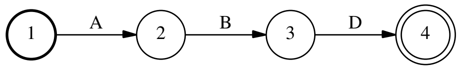
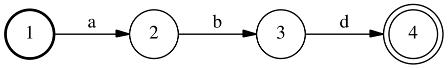


$A \circ T$



Rewriting a Single String

A



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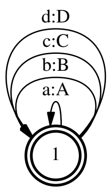
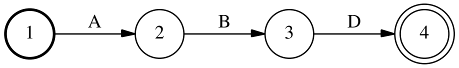
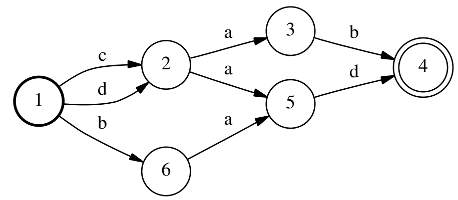


$A \circ T$



Rewriting Many Strings At Once

A



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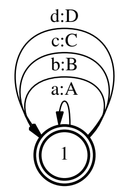
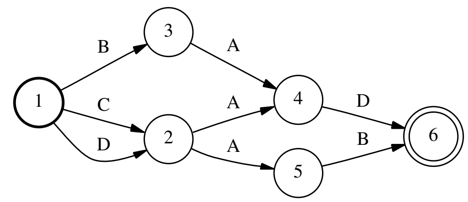
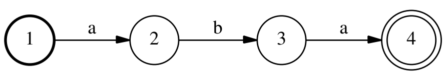


$A \circ T$



Rewriting A Single String Many Ways

A



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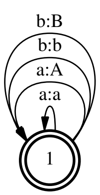
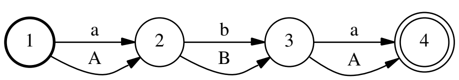
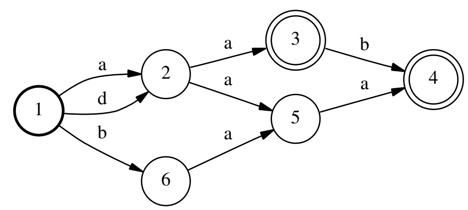


$A \circ T$



Rewriting Some Strings Zero Ways

A



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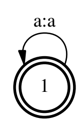
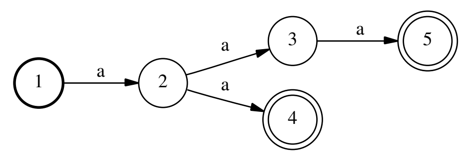


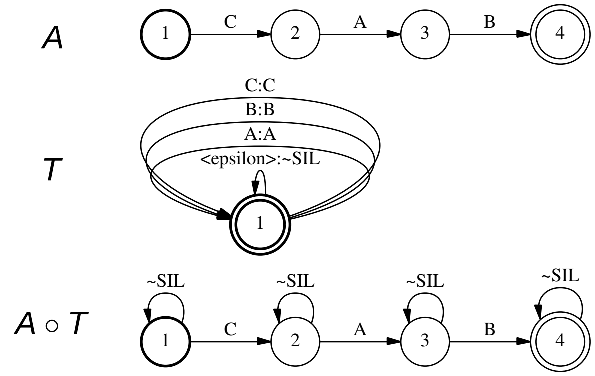
$A \circ T$



And a Dessert Topping!

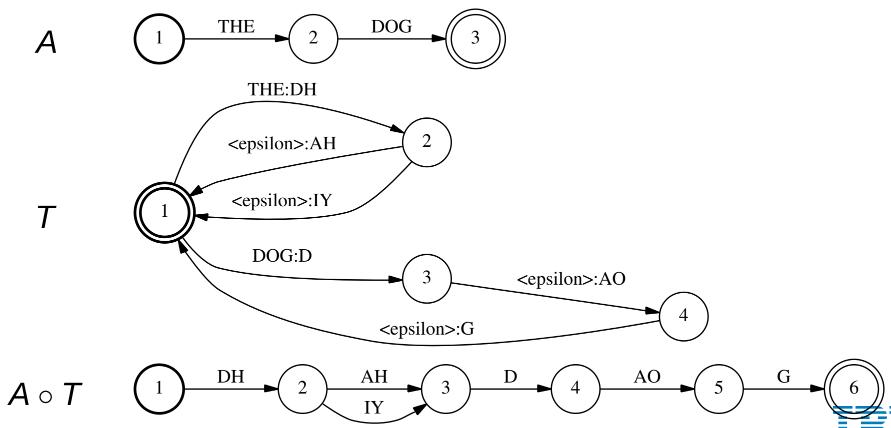
- Composition seems pretty versatile.
- Can it help us build decoding graphs?

Example: Inserting Optional Silences

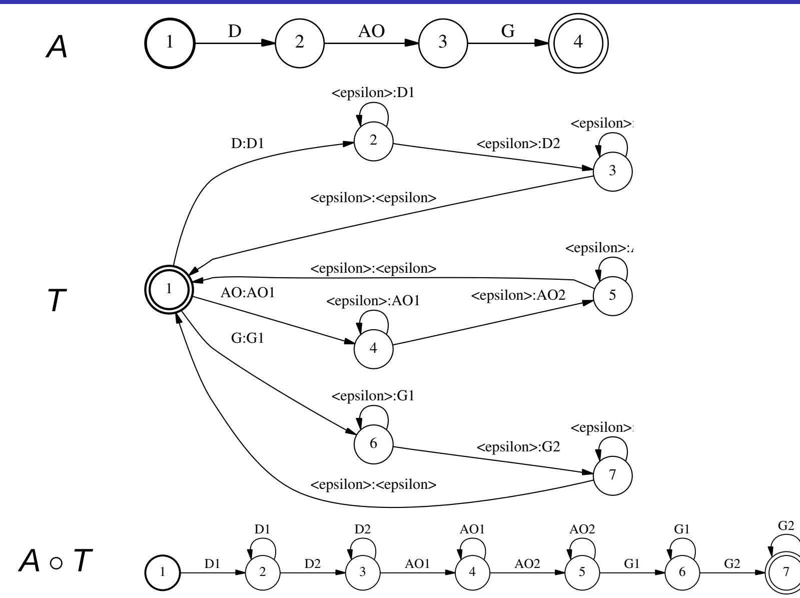


Example: Mapping Words To Phones

THE(01) DH AH
THE(02) DH IY



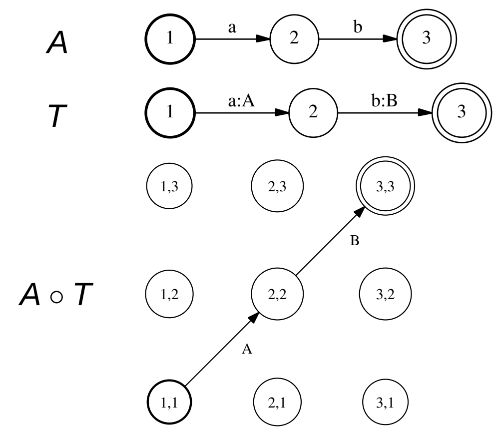
Example: Rewriting CI Phones as HMM's



Computing Composition

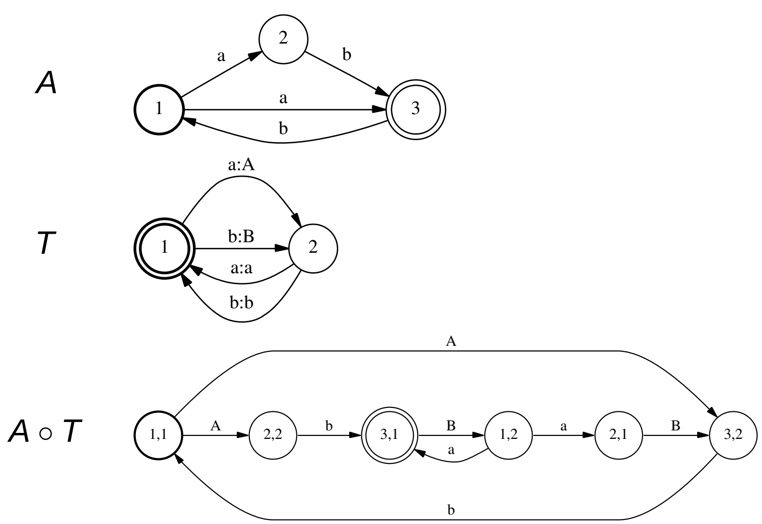
- For now, pretend no ϵ-labels
- For every state $s \in A$, $t \in T$, create state $(s, t) \in A \circ T$
- Create arc from (s_1, t_1) to (s_2, t_2) with label o iff ...
  - There is an arc from s_1 to s_2 in A with label i
  - There is an arc from t_1 to t_2 in T with input label i and output label o
- (s, t) is initial iff s and t are initial; similarly for final states.
- (Remove arcs and states that cannot reach both an initial and final state.)
- What is time complexity?

Example: Computing Composition



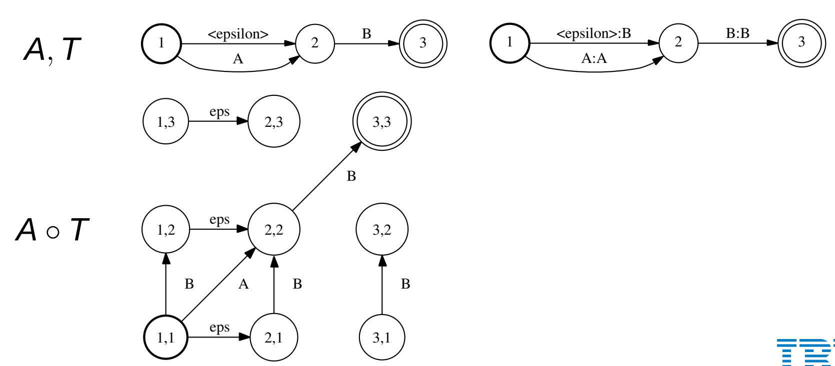
- Optimization: start from initial state, build outward.

Another Example



Composition and ϵ-Transitions

- Basic idea: can take ϵ-transition in one FSM without moving in other FSM.
  - A little tricky to do exactly right.
  - Do the readings if you care: (Pereira, Riley, 1997)

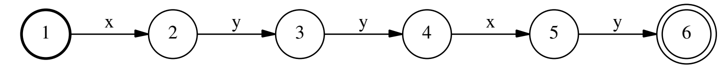


How to Express CD Expansion via FST's?

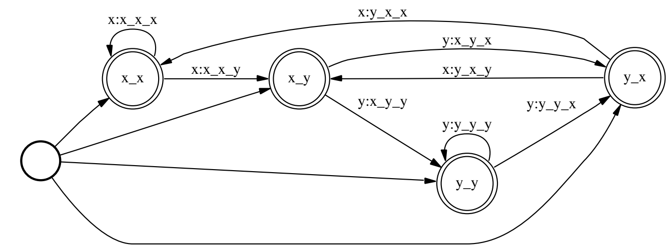
- Step 1: Rewrite each phone as a triphone.
  - Rewrite AX as `DH_AX_R` if `DH` to left, `R` to right.
- Step 2: Rewrite each triphone with correct context-dependent HMM for center phone.
  - Just like rewriting a CI phone as its HMM.
  - Need to precompute HMM for each possible triphone ($\sim 50^3$).

How to Express CD Expansion via FST's?

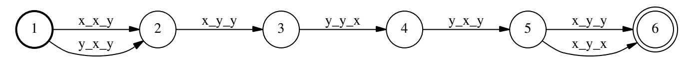
A



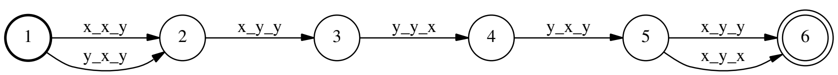
T



$A \circ T$



How to Express CD Expansion via FST's?



- Point: composition automatically expands FSA to correctly handle context!
  - Makes multiple copies of states in original FSA . . .
  - That can exist in different triphone contexts.
  - (And makes multiple copies of *only* these states.)

Recap: Finite-State Transducers

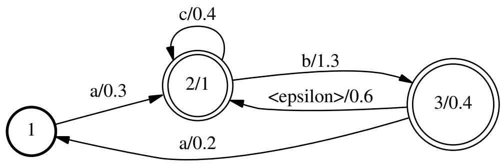
- Graph expansion can be expressed as series of composition operations.
  - Need to build FST to represent each expansion step, *e.g.*,

```
1   2   THE
2   3   DOG
3
```

  - With composition operation, we're done!
- Composition is efficient.
- Context-dependent expansion can be handled effortlessly.

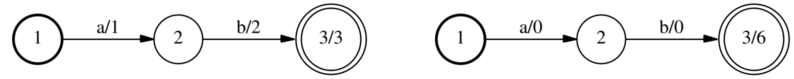
What About Those Probability Thingies?

- *e.g.*, to hold language model probs, transition probs, etc.
- FSM's $\Rightarrow$ *weighted* FSM's
  - WFSA's, WFST's
- Each arc has a score or *cost*.
  - So do final states.

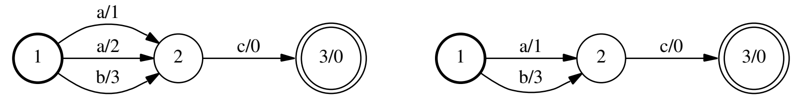


Arc Costs *vs.* Probabilities

- Typically, we take costs to be negative log probabilities.
  - Costs can move back and forth along a path.
  - The cost of a path is sum of arc costs plus final cost.



- If two paths have same labels, can be combined into one.
  - Typically, use min operator to compute new cost.

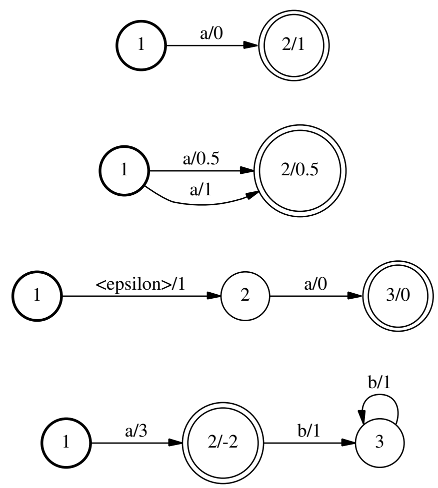


- Operations $(+, \min)$ form a *semiring* (the *tropical* semiring).
  - Other semirings are possible.

The Meaning of Life

- WFSA: a list of (unique) string and cost pairs $\{(i_1 \cdots i_N, c)\}$.
- WFST: a list of triples $\{(i_1 \cdots i_N, o_1 \cdots o_M, c')\}$.

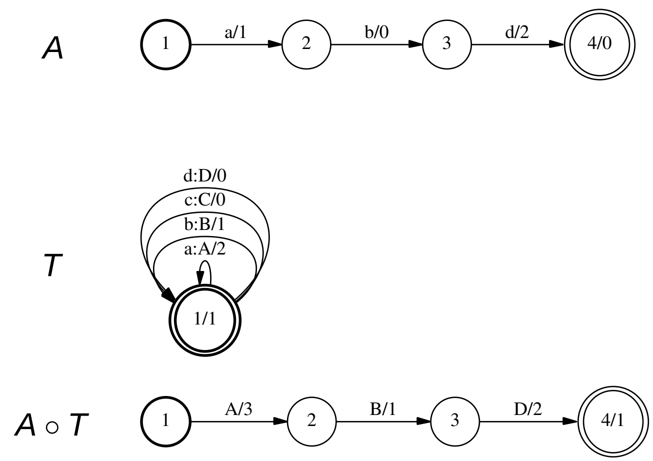
Which Is Different From the Others?



Weighted Composition

- Composing WFSA A with WFST T to get WFSA $A \circ T$.
- If $(i_1 \cdots i_N, c) \in A$ and ...
- $(i_1 \cdots i_N, o_1 \cdots o_M, c') \in T$, ...
- Then, $(o_1 \cdots o_M, c + c') \in A \circ T$.
- Combine costs for all different ways to produce same $o_1 \cdots o_M$.

Weighted Composition



Weighted Graph Expansion

- Start with weighted FSA representing language model.
- Use composition to apply weighted FST for each level of expansion.
  - Scores/logprobs will be accumulated.
  - Log probs may move around along paths.
  - All that matters for Viterbi is total score of paths.

Recap: Composition

- Like `sed`, but can operate on all paths in a lattice simultaneously.
- Rewrite symbols as other symbols.
  - *e.g.*, rewrite words as phone sequences (or vice versa).
- Context-dependent rewriting of symbols.
  - *e.g.*, rewrite CI phones as their CD variants.
- Add in new scores.
  - *e.g.*, language model lattice rescoring.
- Restrict the set of allowed paths/intersection.
  - *e.g.*, find all paths in lattice containing word NOODGE.
- Or all of the above at once.

Road Map

- Part I: The LVCSR acoustic model.
- Part II: Acoustic model training for LVCSR.
- Part III: Decoding for LVCSR (inefficient).
  - Part IV: Introduction to finite-state transducers.
- Part V: Search (Lecture 8).
  - Making decoding for LVCSR efficient.

Course Feedback

1. Was this lecture mostly clear or unclear? What was the muddiest topic?
2. Other feedback (pace, content, atmosphere)?