E85.2607: Lecture 10 – Modulation
Modulation

Use an audio signal to vary the parameters of a sinusoid

\[ y_{\text{mod}}[n] = m[n] \cos(2\pi f_c n + \phi[n]) \]

- \( m[n], \phi[n] \) modulating signals
- \( \cos(f_c n) \) carrier signal with carrier freq. \( f_c \)

Used for:
- Transmitting radio signals
- Tremolo, vibrato, other effects
- Synthesizing complex harmonic series

Tremolo?
- When the modulating frequency is less than 20Hz this produces a well known music effect called tremolo
- In musical terms: A regular and repetitive variation in amplitude for the duration of a single note.
- However when the modulation frequency is in the audible range, new sound textures can be created

FM synthesis
- Like in the case of AM, when the frequency variation is in the audible range a timbre change is produced.
- This modulation may be controlled to produce varied dynamic spectra with relative little computational overheads.
- FM was well developed for radio applications in the 1930's, and is nowadays the most widely used broadcast signal format for radio
- Introduced as a tool for sound synthesis by John Chowning (Stanford U.) in the early 1970's
- 1980s: Used by Yamaha to develop its DX series (The DX7 was to become one of the most popular synthesisers of all times) and the OPL chip series (soundblaster sound cards, mobile phones)
Ring modulation

\[ y_{RM}[n] = m[n] \cos(2\pi f_c n) \]

- Shifts spectrum of modulating signal to be centered around \( f_c \)
- e.g. let \( m[n] = \cos(2\pi f_m n) \):

\[
y_{RM}[n] = \cos(2\pi f_m n) \cos(2\pi f_c n) \\
= \frac{1}{2} \cos (2\pi(f_c - f_m)) + \cos (2\pi(f_c + f_m))
\]

Using trigonometric identity: \( \cos(a \pm b) = \cos(a) \cos(b) \mp \sin(a) \sin(b) \)
Ring modulation

\[ f_c - f_m \]
\[ f_c + f_m \]

amp

freq

250 Hz
500 Hz
Amplitude modulation

Like ring modulation, but with DC offset added to modulating signal

\[ y_{AM}[n] = (1 + \alpha m[n]) \cos(2\pi f_c n) \]

Receiver (demodulator) is easier to build

e.g. let \( m[n] = \cos(2\pi f_m n) \):

\[ y_{AM}[n] = (1 + \alpha \cos(2\pi f_m n)) \cos(2\pi f_c n) \]
\[ = \cos(2\pi f_c n) + \frac{\alpha}{2} \cos(2\pi(f_c - f_m)) + \cos(2\pi(f_c + f_m)) \]

Thus its Fourier Transform is defined as:

\[ Y(\omega) \]
\[ \frac{1}{2} M(\omega + \omega_c) \]
\[ -\omega_c \]
\[ \omega_c \]
\[ \frac{1}{2} M(\omega - \omega_c) \]
Amplitude modulation

\[ \omega_c - \omega_m, \omega_c + \omega_m \]

\[ \text{freq} \]

\[ \text{amp} \]

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Amplitude modulation in the time domain

- Demodulate using an **envelope detector**
  - rectifier + LPF
- or **product detector**
  - coherent ring modulation + LPF

\[ y_{AM}[t] \cos(2\pi f_c) \]
\[ = (1 + \alpha m[n]) \cos(2\pi f_c n) \cos(2\pi f_c n) \]
\[ = (1 + \alpha m[n]) \left( \frac{1}{2} + \frac{1}{2} \cos(2\pi 2f_c n) \right) \]

- Also works for ring modulation
Effect of modulation index ($\alpha$)
Single Sideband (SSB) modulation

AM and RM waste bandwidth (and power) in redundant sidelobes

$H(j\omega) = \begin{cases} 
-j & \omega > 0 \\
+j & \omega < 0 
\end{cases}$

With changes of $\omega$, the spectrum of $m(t)$ will be shifted accordingly, so SSB modulation is also known as frequency shifting.
Angle modulation

\[ y_{PM/FM}[n] = \cos(2\pi f_c n + \beta \phi_{PM/FM}[n]) \]

\[ \phi_{PM}[n] = m[n] \]

\[ \phi_{FM}[n] = 2\pi \int_{-\infty}^{n} m[\tau] d\tau \]

- Looks like phase is being modulated, but they’re really the same
  - instantaneous frequency = \( \frac{\partial}{\partial n} (2\pi f_c n + \beta \phi[n]) \)
  - (“FM” often used to refer to phase modulation)

![Graphs showing FM and PM modulation](image-url)
FM vs PM

(c) FM

(d) PM

(e) FM

(f) PM

$X_{FM}(n)$ and $m(n)$

$X_{PM}(n)$ and $m(n)$
Implementing angle modulation

- Just index into carrier using time-varying delay
- Interpolate as necessary
Effects: Tremolo

Modulate amplitude of audio signal with low frequency sinusoid

Low-frequency Amplitude Modulation (f_c=20 Hz)
**Vibrato** modulate phase of audio signal with low frequency sinusoid

\[ m(n) = M + \text{DEPTH} \cdot \sin(\omega nT) \]

**Detuning** SSB modulation to shift spectrum up or down in frequency
Applications: synthesizing notes

**AM synthesis** change carrier frequency to change pitch
- e.g. simple synthesizer with 3 harmonics by modulating sinusoidal carrier with sinusoidal signal:
  \[(1 + \cos(2\pi f_m n)) \cos(2\pi f_c n)\]
- easy to implement
- but, limited timbral possibilities . . .

**FM synthesis** produce spectrally rich sounds with minimal effort
- \[\cos(2\pi f_c n + \beta \sin(2\pi f_m n))\]
- need integer \(\frac{f_c}{f_m}\) to make harmonic sounds
- sidebands at \(f_c \pm k f_m\)
- introduced by John Chowning at Stanford in early 1970s
- commercialized by Yamaha in the 1980s (DX7)
FM modulation index

\[ y[n] = \cos(2\pi \cdot 220 \cdot n + \beta \sin(2\pi \cdot 440 \cdot n)) \]

- FM signals theoretically have infinite bandwidth
- \( \sim 2(\beta + 1) \) audible sidebands
Note dynamics

- Real notes are time-limited
  - struck/plucked vs. bowed/blown

Simulate using **ADSR envelope**

- 4-parameter classic envelope model
  - **Attack**: initial rise time
  - **Decay**: fall time immediately following initial attack
  - **Sustain**: amplitude of asymptote of decay while key is held down
  - **Release**: decay from sustain to zero after key released
Amplitude modulation alone is not enough
- real instruments have time-varying spectra
- e.g. plucked string

Model using LPF
- high frequencies die away after initial transient
- Or just model the physics...

Or just model the physics...
DAFX  Chapter 4 - Modulators and Demodulators