

# ADAPTIVE SPECTRUM SENSING FOR AGILE COGNITIVE RADIOS

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## ABSTRACT

Vast segments of the frequency spectrum are licensed to specific users for particular applications. These legacy users, however, often under-utilize their designated spectrum segments. Unlicensed (secondary) users can benefit from this fact and opportunistically exploit the vacant spectrum segments (spectral holes). Due to the transient nature of the spectrum occupancy it becomes imperative for secondary users to quickly identify such spectral holes. To accomplish this, we propose a novel sequential and adaptive spectrum sensing procedure. The underlying notion of this procedure is to progressively allocate the sensing resources to only the most promising areas of the spectrum. This translates in a reduction of sensing resources and time needed to accurately identify spectrum holes, in contrast with more conventional approaches that allocate the sensing budget over the entire spectrum uniformly. The proposed method is theoretically sound and further supported by simulation results.

*Index Terms*— Adaptive spectrum sensing, agility, cognitive radio, reliability.

## 1. INTRODUCTION

The notion of cognitive radio has emerged as a way to cope with the fact that vast segments of the frequency spectrum are often under-utilized, despite being licensed to legacy (primary) users. Unlicensed (secondary) users can therefore monitor the frequency spectrum and identify and opportunistically exploit under-utilized spectrum segments (spectrum holes). This monitoring task, often referred to as spectrum sensing, is a major aspect of cognitive radio and has received a considerable amount of research interest [1–4]. The detection of a spectrum hole in a wideband spectrum involves two major challenges. First, the spectrum holes are spread across the wideband spectrum and their availability status changes rapidly. Therefore, the secondary users should be *agile* enough to detect the holes within a period considerably shorter than the entire duration of its vacancy. Secondly, in order to avoid harming the communication of the primary users, the secondary users must distinguish the holes from the channels occupied by the primary users *reliably*, irrespective of how weak the transmissions of the primary users are.

In this paper we propose a novel adaptive spectrum sensing procedure. The underlying premise is that secondary users can adaptively decide how to spend a given sensing budget in the course of the measurement process, based on earlier observations. Such premise facilitates focusing the sensing resources in more promising segments of the spectrum. This is in contrast with conventional spectrum sensing schemes that have a pre-defined sensing strategy. Our proposed adaptive procedure consists of two phases, namely *refinement* and *detection*. The task of the refinement phase is to eliminate a considerable portion of the occupied channels while retaining most of the holes. This phase is an iterative process, where in each iteration a large number of less promising segments of the spectrum are eliminated. A key fact is that this task can be accomplished even if the measurements are very rough and noisy. In each iteration we

further sequentially monitor the spectrum and improve on the refinement of the previous iteration. Therefore, the output of the refinement phase has a significantly larger ratio of spectrum holes to non-holes. The refinement phase is followed by the detection phase, where a spectrum hole is accurately identified among the channels retained in the refinement phase.

A method for sequential refinement for signal detection and estimation was formalized in [5,6], where the authors develop the idea of *distilled sensing*. It is shown that given a fixed sensing budget some signals that are detectable/estimable using adaptive measurements cannot be recovered using non-adaptive strategies. The results show that closing the loop between the data analysis and collection processes can yield significant gains. The goal of hole detection, however, is different than the problems considered in [5,6], as our desire is to identify a *single* hole, and not estimate the location of *all* holes or verify whether a hole is present. Furthermore the statistical observation model is significantly different than the ones previously considered. Nevertheless the distillation philosophy is still applicable here, and is at the core of the refinement phase described above.

## 2. DESCRIPTIONS

We consider a *wideband* spectrum shared by primary and secondary users with interference-avoiding spectrum access. The primary users have the right to use the spectrum whenever desired. On the other hand, the secondary users are allowed to opportunistically seek for and exploit the portions of the spectrum unused by the primary users. In this work our objective is to provide an *agile* and *reliable* mechanism for identifying such transmission opportunities.

We assume the available wideband spectrum consists of  $n$  channels, indexed by  $\{1, \dots, n\}$ . At a given instant some of these channels are being used by the primary users, which we refer to as *occupied channels*. Also, a reduced number of channels, which we hereinafter call *spectrum holes*, are not being utilized by any user and thereof are available to be exploited by secondary users.

We consider a simple probabilistic model for the occupancy of the spectrum. Define the binary random variable  $Z_i \in \{0, 1\}$ , for  $i = 1, \dots, n$ , where  $Z_i = 1$  conveys that channel  $i$  is occupied by a primary user and  $Z_i = 0$  means that it is a spectrum hole. We assume that each channel is a spectrum hole with probability  $\epsilon(n)$  and the occupancies of the different channels are statistically independent, i.e.,  $Z_i \stackrel{\text{i.i.d.}}{\sim} \text{Ber}(1 - \epsilon(n))$  for  $i = 1, \dots, n$ . We also assume that the occupancy status of the spectrum modeled by  $\{Z_i\}$  remains unchanged during the spectral scanning process. Let us define

$$\mathcal{H}_0 \triangleq \{i \in \{1, \dots, n\} : Z_i = 0\}, \quad (1)$$

which contains the indices of the spectrum holes, and let  $\mathcal{H}_1 \triangleq \{1, \dots, n\} \setminus \mathcal{H}_0$  contain the indices of the occupied channels. In order to capitalize on the availability of spectral holes, the secondary users monitor the spectrum by performing channel measurements. Each measurement of channel  $i$ , denoted by  $X_i$ , is of the form

$$X_i \triangleq \sqrt{p_i} H_i \cdot S_i \cdot Z_i + W_i \quad \text{for } i = 1, \dots, n. \quad (2)$$

Let  $\mathcal{CN}(0, 1)$  denote a complex Gaussian distribution with mean 0 and variance 1. For  $i \in \mathcal{H}_1$ ,  $p_i$  accounts for the combined effect of the transmission power of the primary user active on the  $i^{\text{th}}$  channel and its associated path-loss signal attenuation;  $H_i \sim \mathcal{CN}(0, 1)$  represents the flat-fading channel between the primary user active on the  $i^{\text{th}}$  channel and the secondary user, and  $S_i$  denotes the unit-power signal ( $|S_i|^2 = 1$ ) of the primary user active on the  $i^{\text{th}}$  channel. Finally  $W_i$  denotes the additive white channel noise distributed as  $\mathcal{CN}(0, 1)$ . The measurements are statistically independent in time and location.

We further assume that the power of the primary users is lower-bounded by  $\gamma(n) > 0$ , i.e.,  $p_i \geq \gamma(n) > 0$  and define  $\Psi(\gamma(n))$  as the class of primary user networks satisfying this lower-bound. Clearly the probability  $\epsilon(n)$  and the lower bound  $\gamma(n)$  affect the capability of the secondary users in detecting spectrum holes (the larger each one of these quantities is the easier it will be to find a hole). Our goal is to use measurements of the form (2) and identify a *single* spectrum hole, i.e., one element of  $\mathcal{H}_0$ .

For convenience we will use the following notation: let  $a_n, b_n$  be two *positive* sequences. We say that  $a_n = \omega(b_n)$  if  $\liminf_{n \rightarrow \infty} b_n/a_n = \infty$  and  $a_n = o(b_n)$  if  $\limsup_{n \rightarrow \infty} a_n/b_n = 0$ . Also  $a_n \doteq b_n$  indicates  $\lim_{n \rightarrow \infty} a_n/b_n = 1$ .  $\lesssim$  and  $\gtrsim$  are defined accordingly.

### 3. NON-ADAPTIVE HOLE DETECTION

In Section 4 we propose an adaptive procedure for spectrum sensing, consisting of two phases, respectively *refinement* and *detection*. The refinement phase aims at reducing the set of channels to be considered in the detection phase, the latter being a non-adaptive procedure. In this section we analyze an optimal non-adaptive detection scheme for spectral sensing, having a two-fold purpose: (i) providing a solid and optimal framework for the detection phase; and (ii) creating a baseline for comparison and assessment of the proposed adaptive procedure performance.

For the construction of an optimal non-adaptive spectral sensing procedure special care must be taken on how the measurement resources are allocated to the different channels (that is, the experimental design). This allocation depends on the expected proportion of holes  $\epsilon(n)$ . In this work we assume the number of holes is unknown but small when compared to the total number of channels. In that scenario the optimal procedure must “probe” each candidate channel equally. This intuition can be formalized by noting that, for small  $\epsilon(n)$ , the probability of successfully locating a hole increases almost linearly with the number of channels sensed and sub-linearly with the number of measurements per individual channel, indicating that the number of channels sensed plays a dominant role. In the remainder of the paper we assume  $\epsilon(n) \rightarrow 0$  as  $n \rightarrow \infty$ , as this is the most interesting scenario.

Given the model of Section 2 and the considerations above an optimal detector, maximizing the probability of successfully detecting a hole, is simply the maximum *a-posteriori* (MAP) detector. Suppose for a moment that the power of the various primary users  $\{p_i\}$  is known. Assume our measurement budget is  $Mn$ , where  $M \in \mathbb{N}$ . The optimal measurement scheme entails  $M$  measurements of the form (2) per channel. Denote the  $j^{\text{th}}$  measurement of the  $i^{\text{th}}$  channel by  $X_i(j)$ , and define the collection of all observations

$$\mathcal{D}_n \triangleq \{X_i(j) : X_i(j) = \sqrt{p_i} H_i(j) \cdot S_i(j) \cdot Z_i + W_i(j)\}, \quad (3)$$

where  $i = 1, \dots, n$  and  $j = 1, \dots, M$ . From (2) we have

$$X_i(j) | Z_i \stackrel{\text{i.i.d.}}{\sim} \mathcal{CN}(0, 1 + p_i Z_i), \quad \text{for } j = 1, \dots, M. \quad (4)$$

In other words, each measurement  $X_i(j)$  is a sample from a mixture distribution. Recalling that  $\epsilon(n) = o(1)$  we have, for a fixed channel  $i$  and large  $n$

$$P(i \in \mathcal{H}_0 | \mathcal{D}_n) = \left[ 1 + \frac{\frac{1}{\epsilon(n)} - 1}{(1 + p_i)^M} e^{\left( \frac{p_i}{1 + p_i} \sum_{j=1}^M |X_i(j)|^2 \right)} \right]^{-1},$$

and therefore the MAP detector, which we denote by  $\hat{m}_{\text{MAP}}$ , maximizes the probability of finding a hole given the data  $\mathcal{D}_n$ , and is

$$\arg \min_{i \in \{1, \dots, n\}} \left\{ \log(1 + p_i)^{-M} + \frac{1}{1 + 1/p_i} \sum_{j=1}^M |X_i(j)|^2 \right\} \quad (5)$$

The MAP detector above depends on the parameters  $\{p_i\}$ , which are generally not known. All that is known is that  $p_i \geq \gamma(n) > 0$  for all the occupied channels. Given this fact we define instead a *robust* MAP criterion, which optimizes the detection performance for the worst-case realization of  $\{p_i\}$  which can be readily argued to be  $p_i = \gamma(n) \forall i \in \mathcal{H}_1$ . The performance of the MAP criterion for the worst case scenario is given by  $\inf_{\{p_i\} \subseteq \Psi(\gamma(n))} P(\hat{m}_{\text{MAP}} \in \mathcal{H}_0)$ . Simple manipulations along with the assumption that  $\epsilon(n) = o(1)$  show that the infimum value is attained when  $p_i = \gamma(n)$  for all  $i$ . Therefore the robust MAP detector is asymptotically given by (5). By replacing  $p_i$  with  $\gamma(n)$ , yielding a simple minimum energy detector. In summary, for the class of networks  $\Psi(\gamma(n))$  and  $\epsilon(n) = o(1)$ , the **robust MAP hole detector** is asymptotically given by

$$\hat{m}_{\text{NA}} \doteq \arg \max_{m \in \{1, \dots, n\}} \sum_{j=1}^M |X_m(j)|^2. \quad (6)$$

From (6) it is clear that all that is needed are the sufficient statistics  $Y_i \triangleq \sum_{j=1}^M |X_i(j)|^2$  for  $i = 1, \dots, n$ . Taking into account (4) we have  $Y_i | Z_i \sim \text{Gamma}(M, 1 + p_i Z_i)$  for  $i = 1, \dots, n$ , where  $\text{Gamma}(a, b)$  denotes a Gamma distribution with parameters  $a$  and  $b$ <sup>1</sup>. The following theorem characterizes the asymptotic performance robust MAP hole detector.

**Theorem 1 (Non-Adaptive Tradeoff)** *For the class  $\Psi(\gamma(n))$  of primary user networks, when  $\epsilon(n) = o(1)$ , the error probability of robust MAP hole detection is given by*

$$P_{\text{NA}}(n) \triangleq \inf_{\{p_i\}} P(\hat{m}_{\text{NA}} \notin \mathcal{H}_0) \doteq \left[ 1 + (1 + \gamma(n))^M \epsilon(n) \right]^{-1}.$$

This result is proved in Appendix A. As expected there exists a trade-off between reliability and agility since increasing the sampling budget per channel  $M$  favors reliability, improving the probability of success, and on the other hand imposes more delay in spectrum sensing (recall that the total number of measurements is  $Mn$ , and this translates linearly in the time it takes to complete the acquisition of data). Moreover an increase in the minimum power of primary users  $\gamma(n)$  improves the reliability. The following corollary further clarifies those issues and offers a necessary and sufficient condition on the scaling of the power of the primary users to ensure the successful identification of a spectral hole.

**Corollary 1 (Non-Adaptive Power Scaling)** *For the class  $\Psi(\gamma(n))$  of primary user networks, when  $\epsilon(n) = o(1)$ , the necessary and sufficient condition for  $P_{\text{NA}}(n) \rightarrow 0$  as  $n \rightarrow \infty$  is that*

$$\gamma(n) = \omega \left( \epsilon(n)^{-\frac{1}{M}} \right). \quad (7)$$

In other words, in the worst-case scenario  $P_{\text{NA}}(n) \doteq 0$  if and only if the power of the “faintest” user is strictly larger than  $\epsilon(n)^{-\frac{1}{M}}$  can a secondary user reliably identify a hole for opportunistic transmission by employing a non-adaptive procedure.

<sup>1</sup>Recall that for random variable  $G$  with distribution  $\text{Gamma}(a, b)$ , where  $a, b > 0$ , has a density with respect to the Lebesgue measure of the form  $f_G(t) = t^{a-1} \frac{\exp(-t/b)}{b^a \Gamma(a)} \mathbf{1}\{t \geq 0\}$ .

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1: Input  $\{M_1, \dots, M_{K+1}\}$  where  $M_k \in \{1, \dots, \infty\}$ .
2: Initialize the index set  $\mathcal{J}_1 \leftarrow \{1, \dots, n\}$ .
3: for  $k = 1, \dots, K$  do
4:   Set  $Y_i^k = \begin{cases} \sum_{j=1}^{M_k} |X_i^k(j)|^2 & \text{for } i \in \mathcal{J}_k \\ +\infty & \text{for } i \notin \mathcal{J}_k \end{cases}$ 
5:   Obtain  $\mathcal{J}_{k+1} \leftarrow \{i \in \mathcal{J}_k \mid Y_i^k < \lambda_k(1 + \gamma(n))\}$ 
   where  $\lambda_k$  is the median of  $\text{Gamma}(M_k, 1)$ .
6: end for
7: Identify the index of a spectrum hole as
 $\hat{m}_A = \arg \min_{i \in \mathcal{J}_{K+1}} \sum_{j=1}^{M_{K+1}} |X_i(j)|^2$ .
8: Output  $\hat{m}_A$ .

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**Fig. 1.** Adaptive Robust MAP Hole Detection Algorithm

#### 4. ADAPTIVE HOLE DETECTION

The proposed adaptive procedure consists of two phases; (i) the refinement phase, which is the crucial part of this procedure and is intended to adaptively refine the set of channels to be sensed for detecting a hole, and (ii) the detection phase which is similar to the robust MAP detection scheme of Section 3.

The refinement phase proceeds in an iterative way, where in each iteration a group of channels that are most likely occupied are eliminated from further consideration. The key idea is that it is relatively easy to identify occupied channels with low-quality measurements, since there are few holes available (recall that  $\epsilon(n)$  is small). Each iteration involves thresholding the observed energy on each channel. The threshold depends only on  $\gamma(n)$ , and is designed such that at each iteration roughly *half* of the occupied channels are eliminated, while *almost all* of the spectrum holes are retained. Therefore, the output of the refinement process will have a more condensed proportion of spectrum holes to occupied channels. Subsequently the detector developed in Section 3 is applied to identify a hole.

Let us denote the number of iterations of the refinement phase by  $K$  and the number of sampling budget per channel in the  $k^{\text{th}}$  iteration by  $M_k$ . The algorithm is initialized by including *all* channels for sensing and the refinement procedure is carried out as follows. In the first iteration all channels are allocated identical sampling budgets of  $M_1$ . The energy level of each channel is compared against  $\lambda_1(1 + \gamma(n))$ , where  $\lambda_1$  is the *median* of the distribution  $\text{Gamma}(M_1, 1)$ . Those exceeding this threshold are discarded and the rest are carried over to the second iteration for further sensing. The same procedure is repeated throughout all  $K$  iterations such that in the  $k^{\text{th}}$  iteration all the channels retained by the  $(k-1)^{\text{th}}$  iteration are allocated identical sampling budget of  $M_k$ . The energy level of these channels is compared with  $\lambda_k(1 + \gamma(n))$  and the refinement is performed via thresholding as in the first iteration. Finally, after the refinement phase, each of the remaining channels is allocated the sampling budget  $M_{K+1}$  and the robust MAP hole detection scheme provided in Section 3 is applied in order to detect a hole. Similar to (3) we define the set of measurements for  $k = 1, \dots, K+1$  as

$$\mathcal{D}_n^k \triangleq \{X_i^k(j) : X_i^k(j) = \sqrt{p_i} H_i^k(j) \cdot S_i^k(j) \cdot Z_i + W_i^k(j)\},$$

The algorithm is formally described in Fig. 1. The performance of the adaptive robust MAP hole detection in the following theorem.

**Theorem 2 (Adaptive Tradeoff)** *For the class  $\Psi(\gamma(n))$  of primary user networks, when  $\epsilon(n) = o(1)$  and  $n\epsilon(n) = \omega(1)$ , the adaptive robust MAP hole detection algorithm has an error probability given by*

$$P_A(n) \triangleq \inf_{\{p_i\}} P(\hat{m}_A \notin \mathcal{H}_0) \leq \left[1 + 2^K(1 + \gamma(n))^{M_{K+1}} \epsilon(n)\right]^{-1},$$

with asymptotic equality if and only if  $p_i = \gamma(n)$  for all  $i$ .

The theorem is proved using the ideas and techniques presented in [5], and then applying Theorem 1. The above result suggests that for achieving the best *asymptotic* performance we have to maximize  $M_{K+1}$ . For this purpose, given a fixed sampling budget, the best asymptotic strategy is to allocate as much as possible sensing budget for the detection phase. This also implies that allocating as low as one sample per channel in the  $k^{\text{th}}$  iteration of the refinement phase ensures that for sufficiently large  $n$ , almost surely all the holes are retained and roughly half of the occupied channels are eliminated.

An analogue of Corollary 1 can be derived for the adaptive procedure, providing a sufficient improvement in the scaling of  $\gamma(n)$  required for guaranteeing a reliable hole detection. We consider the case that both schemes use the same sampling budget.

**Corollary 2 (Adaptive Power Scaling)** *For the class  $\Psi(\gamma(n))$  of primary user networks, when  $\epsilon(n) = o(1)$ ,  $n\epsilon(n) = \omega(1)$ , and the sampling budget is  $Mn$ , a sufficient condition for  $P_A(n) \rightarrow 0$  as  $n \rightarrow \infty$  is*

$$\gamma(n) = \omega\left((2^K \epsilon(n))^{-\frac{1}{M'}}\right), \quad (8)$$

where  $M'$  is some constant that satisfies  $M' \geq 2^K(M-2) + 2$ .

Comparing the result above with that of Corollary 1 shows that an adaptive scheme can cope with much weaker primary users than the optimal non-adaptive scheme. Next we characterize the *agility factor*, which we define as the ratio of the expected time for scanning the spectrum in the adaptive scheme to that of the non-adaptive scheme under the same accuracy constraint. For finding the agility factor, we equate the error probabilities  $P_A(n) = P_{\text{NA}}(n)$  and quantify the required sampling budget by each scheme to achieve this error probability.

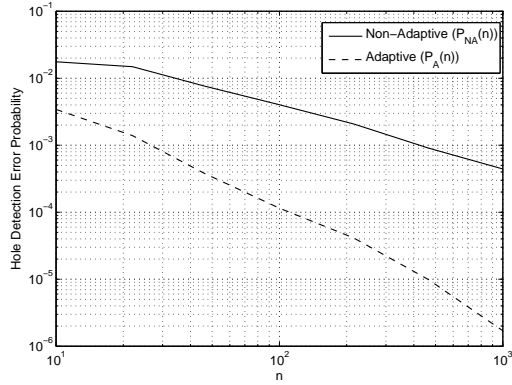
**Corollary 3 (Agility)** *For the  $\Psi(\gamma(n))$  class of networks, when  $\epsilon(n) = o(1)$  and  $n\epsilon(n) = \omega(1)$ , the agility factor of the adaptive robust MAP hole detection algorithm is asymptotically upper bounded by  $(\frac{1}{2^K} + \frac{2}{M})$ , where  $K$  is the number of iterations and  $Mn$  is the sampling budget.*

We can further find a precise value for the agility factor. This indicates that after some point the agility factor is very insensitive to increasing  $K$  which states that there is a limit on the agility gain of the proposed adaptive procedure.

#### 5. SIMULATIONS

In Fig. 2 we compare the reliability of the proposed adaptive procedure with two refinement iterations  $K = 2$  with that of the non-adaptive scheme over the range of  $n = 10 - 1000$  channels. The sampling budget in both schemes is set  $5n$  which means that in the non-adaptive scheme each channel is measured  $M = 5$  times. In the non-adaptive scheme in each cycle of the refinement phase each channel is measured once. The sampling resources not used in the refinement phase are equally divided among the remaining channels retained by the refinement phase. We set the channel occupancy probability  $\epsilon(n) = n^{-2/3}$ , which clearly satisfies the conditions  $\epsilon(n) = o(1)$  and  $n\epsilon(n) = \omega(1)$ , and finally assume that the power of primary users are  $p_i = \gamma(n) = n^{1/5}$ ,  $\forall i \in \mathcal{H}_1$ . As expected by the analysis, for large values of  $n$  the adaptive procedure demonstrates a significant improvement upon the non-adaptive scheme, e.g., for  $n = 100$  we gain two orders of magnitude in error probability. The improvement attained for small values of  $n$  is also considerable, e.g., for  $n = 20$  it is one order of magnitude. It is noteworthy that the choices of  $\epsilon(n) = n^{-2/3}$  and  $p_i = \gamma(n) = n^{1/5}$  have been arbitrary and extensive simulations show that the gains are not very sensitive to these choices.

In Fig. 3 we investigate the agility factor. In particular, we aim at achieving the error probability  $10^{-4}$  in spectrum hole detection and



**Fig. 2.** Hole detection error probabilities  $P_A(n)$  and  $P_{NA}(n)$  versus the number of channels  $n$ .

look the sampling budget required by each scheme. For the adaptive scheme we consider the performance with  $K = 1, \dots, 5$  cycles of refinement. Again we consider the choices of  $\epsilon(n) = n^{-2/3}$  and  $p_i = \gamma(n) = n^{1/5}$ . It is seen that the agility is improved by increasing  $K$  and for  $K = 5$ , compared to the non-adaptive scheme, the adaptive procedure requires about 80% less sampling budget. In other words the adaptive procedure is 5 times faster than the non-adaptive scheme for detecting a hole with error probability is  $P_{NA} = 10^{-4}$ .

## 6. CONCLUSIONS

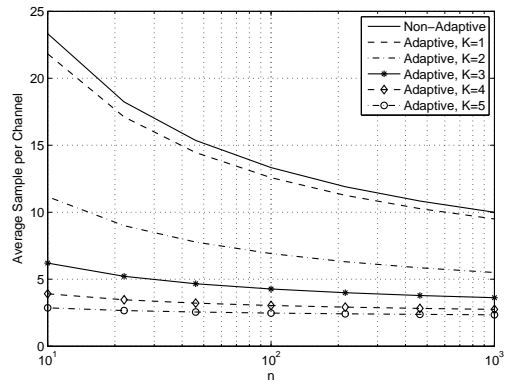
In this paper we present an adaptive sensing methodology for spectral monitoring. By gradually focusing the measurement process using information gleaned from the previous measurements we are able to greatly improve the probability of correctly detecting a spectral hole. This dramatic gain is patent both in the theoretical analysis and in the simulation results. More importantly, in the cognitive radio setting this improvement translates into a significant increase in agility, allowing spectral holes to be identified quickly which widens the window of time for opportunistic transmission. Such dramatic improvements are not possible without the use of adaptive sampling techniques, as demonstrated.

### A. PROOF OF THEOREM 1

The proof of this theorem relies on the properties of the minimum value of a set of random variables. This is a well studied problem in the extreme value theory [7], but in the present setup the number and distribution of the involved random variables changes simultaneously, and the available results are not directly applicable. The following lemma addresses this issue.

**Lemma 1** Let  $\{Y_m\}_{m=1}^n$  be a sequence of i.i.d. random variable distributed as  $\text{Gamma}(M, \alpha_n)$ . Let  $b_n \triangleq \alpha_n [\Gamma(M+1)/n]^{1/M}$  and define the random variable  $W_n \triangleq \frac{\min_m \{Y_m\}_{m=1}^n}{b_n}$ . Then  $W_n$  converges in distribution to a random variable  $W$  with cumulative density function (CDF)  $P(W < w) = 1 - \exp(-w^M)$ .

The above lemma is the key piece needed to assess the performance of the robust detection scheme. Consider the worst case scenario, where all the occupied channels meet the lower bound in the power, i.e.,  $\forall i \in \mathcal{H}_1, p_i = \gamma(n)$ . It is clear that the spectrum hole detector will make a mistake if the minimum energy measured in all the hole



**Fig. 3.** Average number of sample(s) taken per channel versus the number of channels  $n$  for achieving  $P_A(n) = P_{NA}(n) = 10^{-4}$ .

sites is larger than the minimum energy measured in the occupied channels. Hence,

$$P_{NA}(n) \triangleq P(\hat{m}_{NA} \notin \mathcal{H}_0) = 1 - P(\min_{i \in \mathcal{H}_0} Y_i < \min_{i \in \mathcal{H}_1} Y_i).$$

Consider the two sequences of Gamma random variables  $\{Y_i\}_{i \in \mathcal{H}_0}$  and  $\{Y_i\}_{i \in \mathcal{H}_1}$  distributed as  $\text{Gamma}(M, 1)$  and  $\text{Gamma}(M, \gamma(n))$ , respectively. Define  $n_0 \triangleq |\mathcal{H}_0|$  and  $n_1 \triangleq |\mathcal{H}_1| = n - n_0$  and let  $b_0 \triangleq [\Gamma(M+1)/n_0]^{1/M}$  and  $b_1 \triangleq (1+\gamma(n)) [\Gamma(M+1)/n_1]^{1/M}$ . Also define  $W_0 \triangleq \frac{1}{b_0} \min_{i \in \mathcal{H}_0} Y_i$  and  $W_1 \triangleq \frac{1}{b_1} \min_{i \in \mathcal{H}_1} Y_i$ . Notice that these two sets of random variables are independent, and assume for the time being that  $n_0, n_1 \rightarrow \infty$  as  $n \rightarrow \infty$ . Therefore,

$$P(\min_{i \in \mathcal{H}_0} Y_i < \min_{i \in \mathcal{H}_1} Y_i) = 1 - \left(1 + (b_1/b_0)^M\right)^{-1}.$$

The above result is conditional on the sequence  $\{Z_i\}_{i=1}^n$ . The final result follows by plugging the values of  $b_0$  and  $b_1$  and noting that the law of large numbers guarantees that, with probability 1 as  $n \rightarrow \infty$  we have  $n_0, n_1 \rightarrow \infty$ , and  $\frac{n_0}{n_1} \rightarrow \epsilon(n)$ .

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