ELEN3801 - Fall 2009
Midterm 2

November 24, 2009

INSTRUCTIONS:

- Carry only notes on one side of a $8\frac{1}{2}'' \times 11''$ piece of paper, and a pencil and/or pen with you.
- The exam is closed-book, closed-notes. No calculator or other electronic devices are allowed, apart from a dedicated timing device (cell-phones in particular are not allowed).
- You have 75 minutes to complete the exam.
- A correct answer does not guarantee full credit, and a wrong answer does not guarantee loss of credit. You should clearly and concisely indicate your reasoning and **show all relevant work**. Your grade on each problem will be based on our best assessment of your level of understanding as reflected by what you have written. **JUSTIFY** your answers and be **CRITICAL** of your results.
- Please be organized in your write-up – we can't grade what we can't decipher!
- Write all your answers in the question sheet provided - Ask if you need extra paper. Hand in your notes sheet with the exam. Remember to **IDENTIFY YOUR HANDOUT**.

Problem	Points
I	/15
II	/15
III	/25
IV	/20
V	/25
Total	/100

Name:

Problem I [15pts]

State whether each of the following statements is TRUE or FALSE.

a) A band-limited signal with positive energy is also time-limited.

b) Let $h(t)$ be a real signal with finite energy. The convolution of $h(t)$ with $\cos(2t)$ is always equal to $A\cos(4t-\theta)$ for some A and θ. **Hint:** the answer is straightforward if you recall what is the response of an LTI system to a sinusoidal input.

c) Let $f(t)$ be a bandlimited signal with bandwidth 300Hz. The signal $f(2t)$ is band-limited with bandwidth 150Hz.

d) If $x(t)$ is a real signal then the magnitude of the Fourier transform $\mathcal{F}\{x(t)\}$ is an even function.

e) The Fourier transform of $f(t) = e^{-2t}u(t)$ is given by $F(\omega) = \frac{1}{2+j\omega}$.

Problem II [15pts]

Let $f(t)$ be a signal, and let $F(\omega) = \mathcal{F}\{f(t)\}$ be its Fourier transform. Find the Fourier transform of the signal

$$g(t) = f(a(t-b)) \ ,$$

where $a \neq 0$ and $b \in \mathbb{R}$.

Problem III [25pts]

Let $\mathcal{H}$ be a Linear Time-Invariant (LTI) system with impulse response $h(t)$ and such that the transfer function of the system is

$$H(\omega) = \mathcal{F}\{h(t)\} = \frac{2}{2 + j2\omega} e^{j\omega} \ .$$

a) Find and sketch the system's impulse response $h(t)$.

b) Is the system practically realizable? Why or why not?

c) Let $g(t) = \cos(t)$ be the input of the system. What is $G(\omega) = \mathcal{F}\{g(t)\}$? What is the system output $y(t) = (h * g)(t)$? **Hint:** we have derived the answer to this question in class, and you can find it very easily by using the convolution property of the Fourier transform.

Problem IV [20pts]

Using the definition of the Fourier transform compute the the following:

a) $\mathcal{F}\{e^{at}u(-t))\}$, where $a > 0$.

b) $\mathcal{F}\{-u(t+1) + 2u(t) - u(t-1)\}$.

Problem V [25pts]

Let $f(t)$ be a signal with spectrum depicted below.

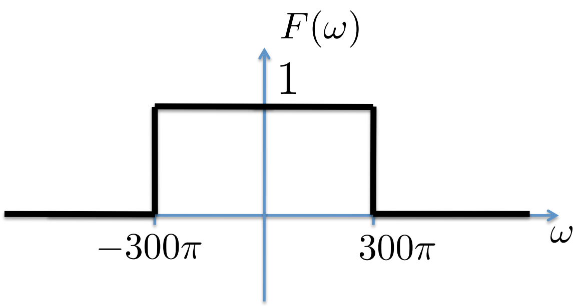

Figure 1: The spectrum of signal $f(t)$.

a) What is the Nyquist sampling frequency (in Hz) of signal $f(t)$?

b) Suppose the signal is sampled using a Dirac train as we did in class. The procedure is depicted in Figure 2.

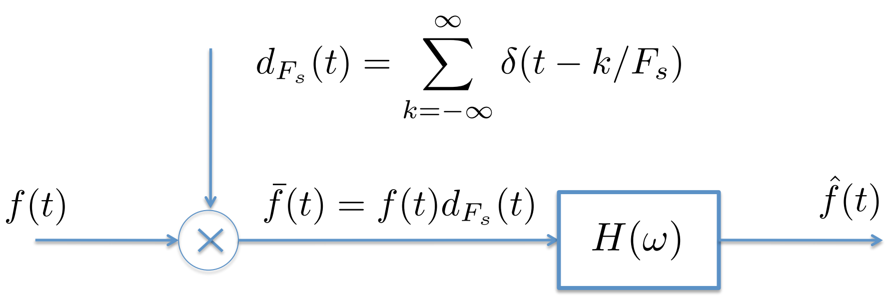

Figure 2: Sampling and reconstruction of signal $f(t)$.

Consider two different scenarios for the sampling frequency: (i) $F_s = 200$Hz and (ii) $F_s = 400$Hz. The sampled signal $\bar{f}(t)$ is then passed through a **unit gain** ideal low-pass filter with bandwidth $F_s/2$ Hz. For each of the cases sketch the spectrum of the output signal $\hat{f}(t)$. Comment your results, and state if you can recover the original signal from the samples in each case.

c) For case (i) in question b) compute the energy of the reconstructed signal $\hat{f}(t)$.