ELEN3801 - Fall 2009
Midterm 1
October 22, 2009

INSTRUCTIONS:

- Carry only notes on one side of a $8\frac{1}{2}'' \times 11''$ piece of paper, and a pencil and/or pen with you.
- The exam is closed-book, closed-notes. No calculator or other electronic devices are allowed, apart from a dedicated timing device (cell-phones in particular are not allowed).
- You have 75 minutes to complete the exam.
- A correct answer does not guarantee full credit, and a wrong answer does not guarantee loss of credit. You should clearly and concisely indicate your reasoning and **show all relevant work**. Your grade on each problem will be based on our best assessment of your level of understanding as reflected by what you have written. **JUSTIFY** your answers and be **CRITICAL** of your results.
- Please be organized in your write-up – we can't grade what we can't decipher!
- Write all your answers in the blue booklet provided - Ask if you need extra paper. Hand in your notes sheet with the exam. Remember to **IDENTIFY YOUR HANDOUT**.

Problem	Points
I	/10
II	/10
III	/10
IV	/10
V	/20
VI	/10
VII	/20
VIII	/10
Total	/100

Facts that might be useful:

$$\cos(\pi/6) = \sin(\pi/3) = \sqrt{3}/2 \; ; \; \cos(\pi/4) = \sin(\pi/4) = \sqrt{2}/2 \; ; \; \cos(\pi/3) = \sin(\pi/6) = 1/2$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \; ; \; \sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

Problem I [10pts]

State whether each of the following statements is TRUE or FALSE.

(2) a) If a signal $f(t)$ is odd then $-f(-t)$ is an even signal.

(2) b) A time-invariant system must also be linear.

(2) c) A periodic signal $f(t)$ is equal to its Fourier series representation for all $t \in \mathbb{R}$.

(2) d) The signal $x(t) = \cos(\sqrt{2}\pi t) + \sin(2\sqrt{2}\pi t)$ is periodic.

(2) e) Periodic signals are always finite energy signals.

Problem II [10pts]

Express $f(t) = -2\cos(\omega_0 t) + 2\sqrt{3}\sin(\omega_0 t + \pi/3)$ in the form

$$C\cos(\omega_0 t + \theta)\ ,$$

where $C \geq 0$.

Problem III [10pts]

Simplify the following expressions.

(5) a) $\delta(t-3)\frac{4-jt^2}{2t}$.

(5) b) $\int_{-\infty}^{\infty} \sqrt{t}\cos(3\pi t)\delta(1-t)dt$.

Problem IV [10pts]

Consider the signal $f(t)$ represented in the Figure 1.

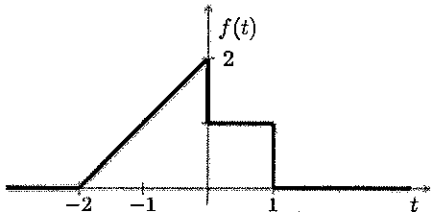

Figure 1: Illustration of Problem IV

(4) a) Write the signal using a single analytical expression, with the aid of the unit step function $u(t)$ defined in class.

(6) b) Plot $f(3-2t)$.

Problem V [20pts]

Let H be a continuous-time Linear Time-Invariant system (LTI), such that the system's response to a pulse input $p(t) = u(t) - u(t-1)$ is $H\{p(t)\} = y_p(t)$. Both signals are depicted in Figure 2(a).

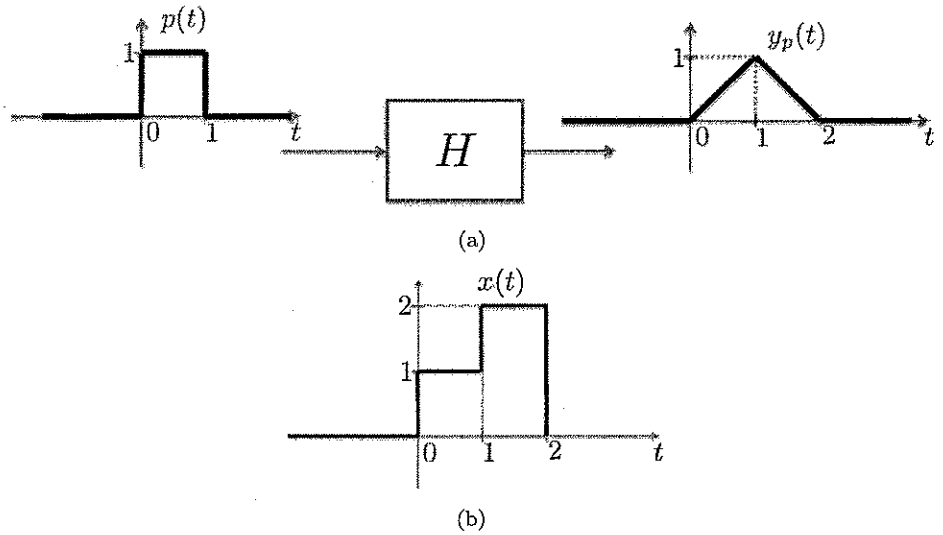

Figure 2: Illustration of Problem V

Given only the information above we want to calculate the system's response input $x(t)$ depicted in Figure 2(b). Let's break down the problem into two parts.

(10) a) Note that $x(t)$ can be written as a sum of scaled and time-shifted versions of $p(t)$. In particular

$$x(t) = ap(t) + bp(t - t_0) \ .$$

Find the adequate values of a, b and t_0.

(10) b) Use what you know about the system and the result of part (a) to plot the system's response to input $x(t)$.

Problem VI [10pts]

Let H denote a continuous-time system such that the relationship between the input $f(t)$ and output $y(t) = H\{f(t)\}$ is given by the equation

$$y(t) - \frac{1}{2}y(t-1) = tf(t) \ .$$

Is this system linear? **Carefully justify your answer.**

Problem VII [20pts]

Let $f(t) = 2(u(t) - u(t-1))$ and $g(t) = e^t(u(t) - u(t-2))$.

(6) a) Sketch $f(t)$ and $g(t)$.

(14) b) Compute the convolution $(f * g)(t)$. You must do the computations either using the graphical or analytical method - **the use of convolution tables is NOT considered for credit.**

Problem VIII [10pts]
Consider the periodic signal $x(t)$ given by the expression

$$x(t) = (2+2j)e^{-j3t} - 3je^{-j2t} + 5 + 3je^{j2t} + (2-2j)e^{j3t} \ .$$

(2) a) What is the period and fundamental frequency of $x(t)$?

(3) b) Justify that $x(t)$ is a real signal and write the corresponding compact trigonometric Fourier series representation.

(3) c) Sketch both the exponential Fourier spectra and the trigonometric Fourier spectra of the signal.

(2) d) What is the power of $x(t)$? **Hint:** Remember the result/theorem proved in class relating the Fourier coefficients to the power of the signal.