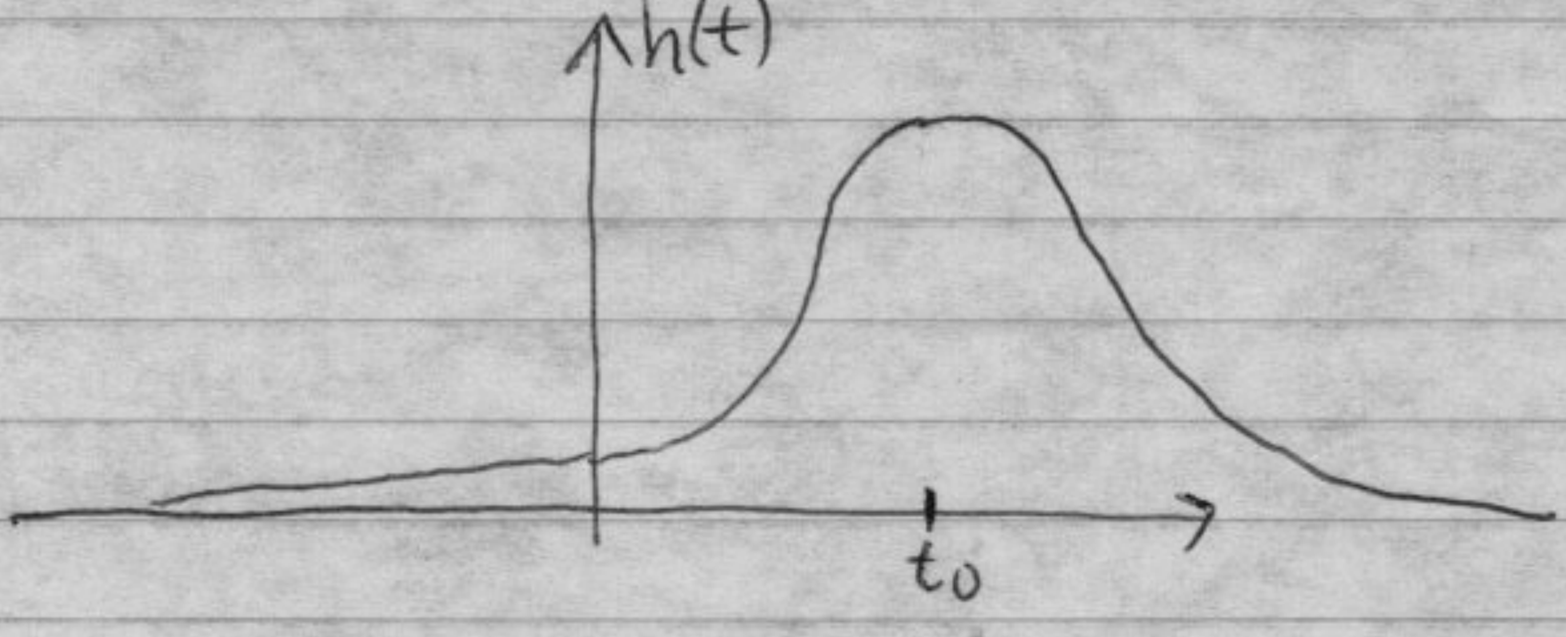
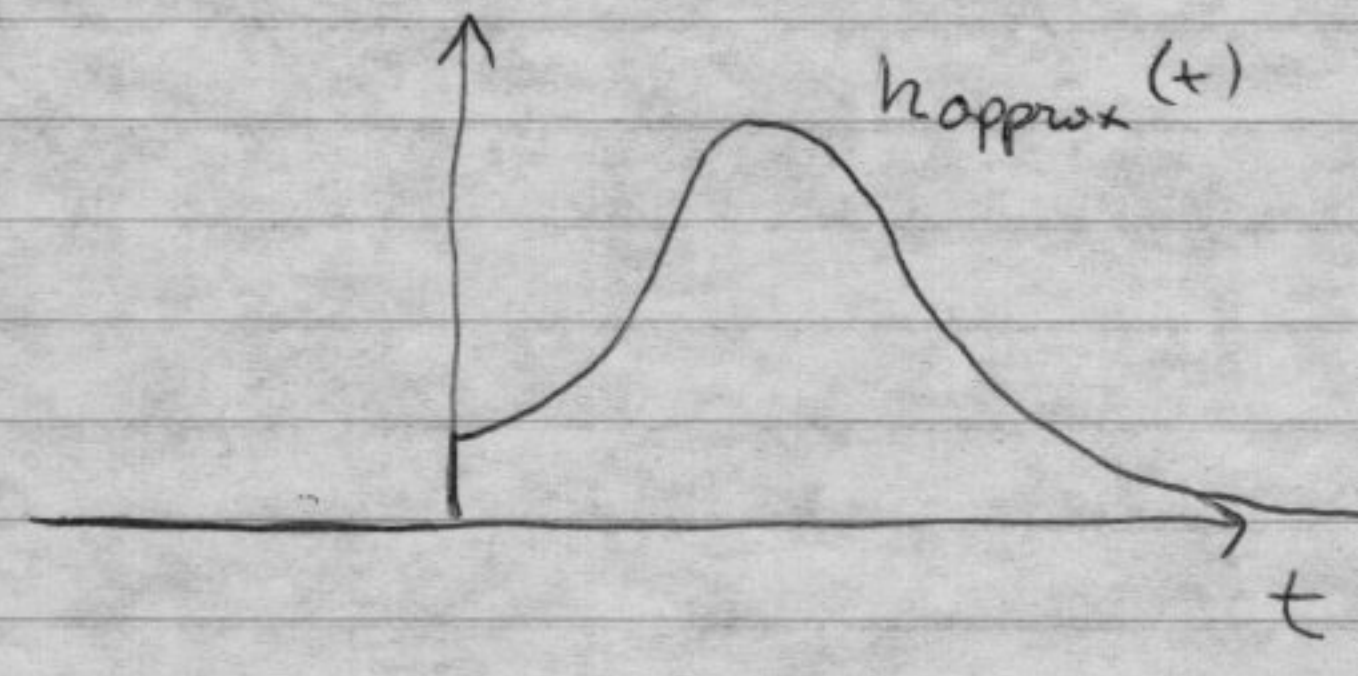
Addendum to problem 8.2

We have seen that we can approximate the non-causal filter response $h(t)$ by a causal signal $h_{approx}(t) = h(t)\,u(t)$





Clearly, the larger t_0 is the better the approximation is. How can we quantify this quality of approximation? We can look at the ratio between the energy of

$$e(t) = h(t) - h_{approx}(t)$$

and the energy of $h(t)$.

$$E_h = \int_{-\infty}^{\infty} h^2(t)\,dt = \cdots = \frac{1}{\sqrt{8\pi K}}$$

(using the fact that $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}\,dx = 1$

Now $$E_e = \int_{-\infty}^{\infty} e^2(t)\,dt = \int_{-\infty}^{0} h^2(t)\,dt$$

Therefore $$\frac{E_e}{E_h} = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi K}} e^{-\frac{(t-t_0)^2}{2K}}\,dt = \int_{-\infty}^{-t_0} \frac{1}{\sqrt{2\pi K}} e^{-\frac{t^2}{2K}}\,dt$$

This is a very well-known integral when computing probabilities* and we know for a fact that taking $t_0 > 3\sqrt{K}$ guarantees that

$$\frac{E_e}{E_h} \leq 1 - 0.997$$, therefore we capture almost all the original signal $h(t)$

*- e.g. http://en.wikipedia.org/wiki/68-95-99.7_rule