9.1.) $H(w) = e^{-(kw^2 + jwt_0)}$

$= e^{-kw^2} e^{-jwt_0}$

let $G(w) = e^{-kw^2}$ ... pair 22 $\sigma^2/2 = k$ $\sigma = \sqrt{2k}$

$G(w) = \frac{\sigma\sqrt{2\pi}}{\sigma\sqrt{2\pi}} e^{-\sigma^2 w^2/2}$ $\sigma = \sqrt{2k}$

$\Rightarrow g(t) = \frac{1}{\sqrt{4\pi k}} e^{-t^2/2(2k)}$

$g(t) = \frac{1}{\sqrt{4\pi k}} e^{-t^2/4k}$

$h(t) = \frac{1}{\sqrt{4\pi k}} e^{-(t-t_0)^2/4k}$

$h(t) \neq 0 \ \forall t < 0$ $\Rightarrow$ not realizable

$\int_{-\infty}^{\infty} \frac{|\ln|H(w)||}{1+w^2}$

$|H(w)| = |e^{-(kw^2 + jwt_0)}|$

$= |e^{-kw^2} e^{-jwt_0}|$

$= e^{-kw^2}$

$\ln|H(w)| = -kw^2$

$|\ln|H(w)|| = kw^2$

$\Rightarrow \int_{-\infty}^{\infty} \frac{kw^2}{1+w^2} dw$

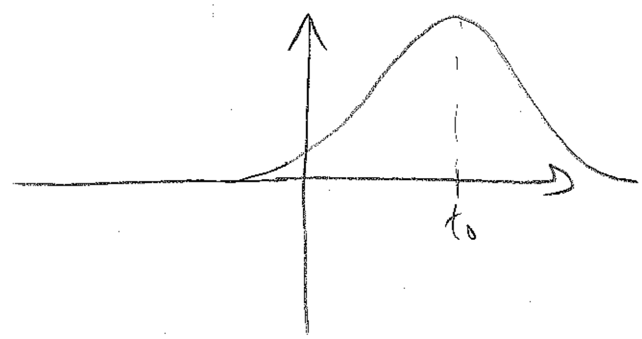
$\Rightarrow = \infty$ since we are integrating over almost a constant value k as $w \to \infty$

$\Rightarrow$ not realizable.

with increasing t_0, the equivalent time domain would be a shift in time t_0 or delay.

$\rightarrow$

$$h(t) = \frac{1}{\sqrt{4\pi k}} e^{-(t-t_0)^2/4k} \Rightarrow$$



$$\hat{h}(t) = h(t)u(t)$$

as $t_0 \to \infty$ $\quad \hat{h}(t) \cong h(t)$

σ denotes the standard deviation of the guassian curve.

The tail beyond 3σ contains only .3% of the energy.

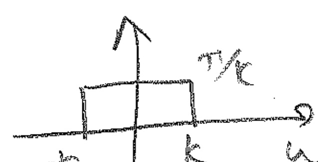
$\Rightarrow$ 3σ is a good delay $\quad 3\sigma = 3\sqrt{2k}$. let $t_0 = 3\sqrt{2k}$

9.2.)

$$\int_{-\infty}^{\infty} \mathrm{sinc}^2(kx)\,dx = \int_{-\infty}^{\infty} g^2(x)\,dx \quad \text{where } g(x) = \mathrm{sinc}(kx)$$

$$\Rightarrow \quad = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(\omega)|^2 d\omega \quad \text{Parseval's theorem.}$$

$$G(\omega) = \frac{\pi}{k}\,\mathrm{rect}\left(\frac{\omega}{2k}\right) \Rightarrow$$



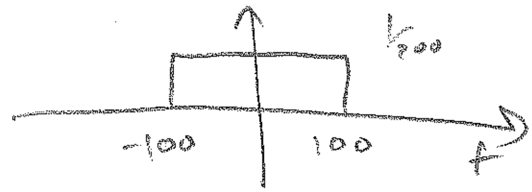
$$= \frac{1}{2\pi}\int_{-k}^{k} \left(\pi/k\right)^2 d\omega$$

$$= \frac{1}{2}\,\frac{\pi}{k^2}\,\omega\Big|_{-k}^{k}$$

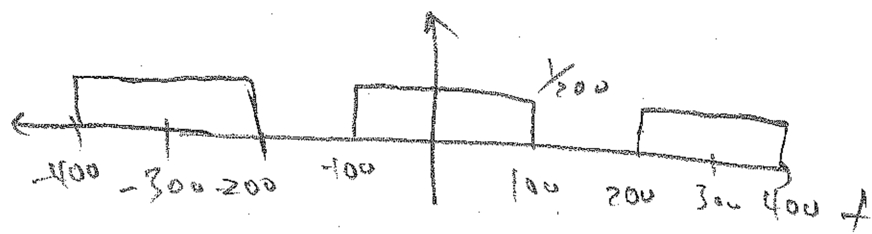
$$= \frac{1}{2}\,\frac{\pi}{k^2}(k+k) = \pi/k \qquad \square$$

ii) we can have perfect recovery, if we have an ideal lowpass filter. Since the sampling spectrum had no overlap, the integrity of the signal is kept. Also the sampling was done at Nyquist rate.

iii).

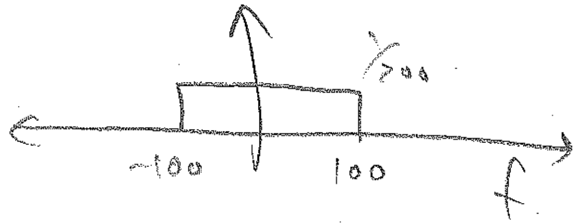


c.) 300Hz



ii.) we can have perfect recovery, if we have an ideal lowpass filter. Same reason as b) ii). Sampling rate was done at more than the Nyquist rate.

iii.)



9.4.) a) $f(t) = \text{sinc}^2(100\pi t)$ $\quad F(\omega) = \frac{2\pi}{200\pi} \Delta\left(\frac{\omega}{400\pi}\right)$

$\omega = 2\pi f$

$\omega = 200\pi$

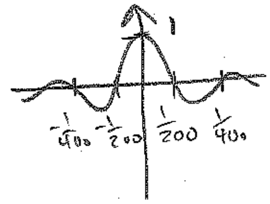
$B = \frac{200\pi}{2\pi}$

$B = 100$ Hz

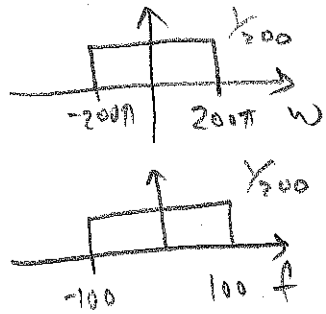
Nyquist rate $F_s = 2B$ 200Hz

$T = \frac{1}{2B} =$ 5ms

9.3) $f(t) = \text{sinc}(200\pi t)$



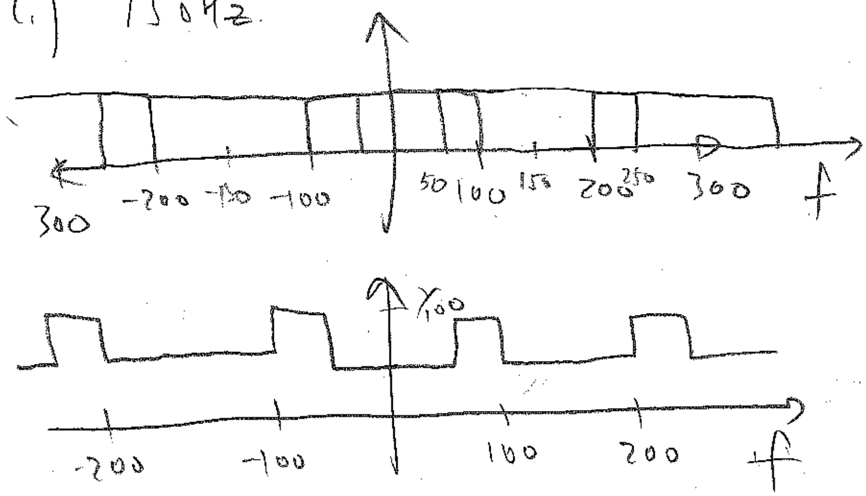
$F(\omega) = \frac{1}{200}\text{rect}\left(\frac{\omega}{400\pi}\right)$



$\omega = 2\pi f$

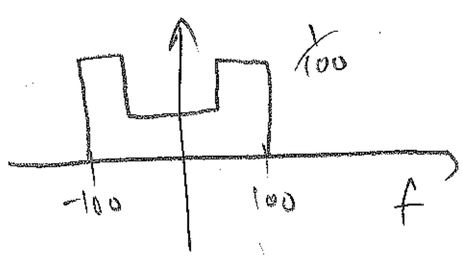
$f = \frac{\omega}{2\pi}$

a.) i.) 150 Hz

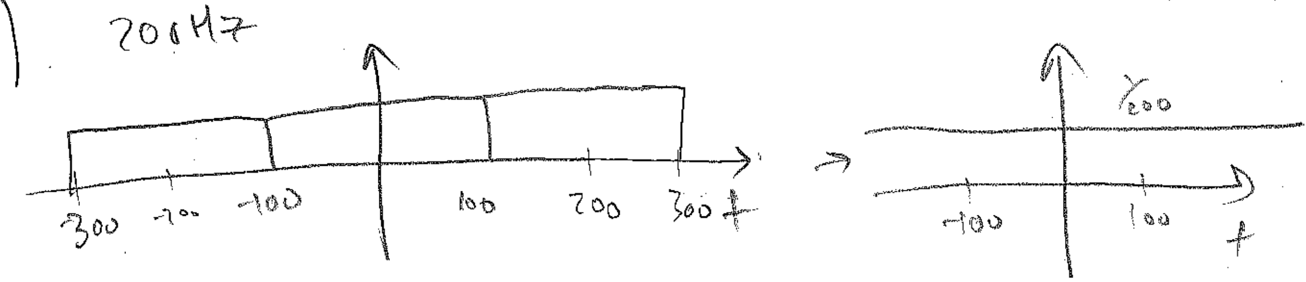


ii.) No you cannot because the sampled spectrum is overlapping. It is sampled below the Nyquist rate.

iii)



b.) i.) 200 Hz



b.) $f(t) = 0.01\,\text{sinc}^2(100\pi t)$ $\quad F(\omega) = \frac{.01}{100}\,\Delta(\omega/400\pi)$ $\quad \omega = 200\pi$, $B = 100\text{Hz}$

Nyquist rate $F_s = 2B$ $\underline{200\text{Hz}}$

$T = \frac{1}{2B} = 5\text{ms}$

c.) $\text{sinc}(100\pi t) + 3\,\text{sinc}^2(60\pi t) = f(t)$

$$F(\omega) = \frac{\pi}{100\pi}\,\text{rect}\left(\frac{\omega}{200\pi}\right) + \underbrace{\frac{3(2\pi)}{120\pi}\,\Delta\left(\frac{\omega}{240\pi}\right)}_{\text{dominates BW.}}$$

$\omega = 120\pi$

$B = \frac{120\pi}{2\pi}$

$B = 60\text{Hz}$

$\Rightarrow$ Nyquist rate $\Rightarrow F_s = 2B = 120\text{ Hz}$.

$T = \frac{1}{2B} = 8.33\text{ ms}$.

d.) $\text{sinc}(50\pi t)\,\text{sinc}(100\pi t) = f(t)$ $\Rightarrow$ let $g(t) = \text{sinc}(50\pi t)$

$m(t) = \text{sinc}(100\pi t)$

$F(\omega) = G(\omega) * M(\omega)$.

we are only interested in the band width. we can use the width property of convolution.

$G(\omega) = \frac{\pi}{50\pi}\,\text{rect}\left(\frac{\omega}{100\pi}\right)$ $\quad M(\omega) = \frac{\pi}{100\pi}\,\text{rect}\left(\frac{\omega}{200\pi}\right)$

width property total $T = T_1 + T_2$ $\quad T_1 = \text{BW of } G(\omega) = 25\text{Hz}$

$T = 75\text{Hz} = B$. $\quad T_2 = \text{BW of } M(\omega) = 50\text{Hz}$

Nyquist rate $= 2B = \underline{150\text{Hz}}$.

$T_{\text{Nyquist}} = \frac{1}{2B} = \underline{6.66\text{ms}}$

9.5.)

a.) $P_{T_s}(t) = \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t-0.004k}{0.0008}\right)$

$$P_{T_s}(t) = \sum_{k=-\infty}^{\infty} D_k e^{jk\omega_0 t} dt \qquad \omega_0 = 2\pi f \quad f = \frac{1}{T} = \frac{1}{.004}$$

$$\omega_0 = 500\pi$$

$$D_k = \frac{1}{.004} \int_{-.0004}^{.0004} e^{-jk500\pi t} dt$$

$$= \left. \frac{e^{-jk500\pi t}}{-jk500\pi(.004)} \right|_{-.0004}^{.0004}$$

$$\frac{e^{-jk(.2)\pi t}}{-jk2\pi} - \frac{e^{jk(.2)\pi t}}{-jk2\pi}$$

$$= \frac{\sin(.2\pi k)}{k\pi}$$

$$= (\text{sinc}(.2\pi k)).2$$

$$= .2 \,\text{sinc}(.2\pi k)$$

$$\Rightarrow P_{T_s}(t) = \sum_{k=-\infty}^{\infty} .2\,\text{sinc}(.2\pi k) \cdot e^{jk500\pi t}$$

b.) $\bar{f}(t) = f(t) P_{T_s}(t) \qquad f(t) = \text{sinc}(200\pi t)$

$$= \sum_{k=-\infty}^{\infty} .2\,\text{sinc}(.2\pi k) e^{jk500\pi t} \cdot \text{sinc}(200\pi t)$$

$$= \sum_{k=-\infty}^{\infty} .2\,\text{sinc}(.2\pi k)\,\text{sinc}(200\pi t) e^{jk500\pi t}$$

⇒ let $g(t) = \sum_{k=-\infty}^{\infty} .2\,\text{sinc}(.2\pi k)\,\text{sinc}(200\pi t)$

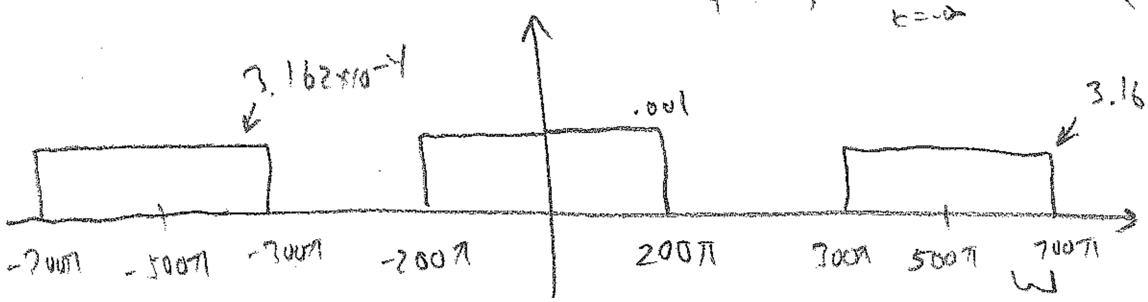
$f(t) = g(t)e^{jk500\pi t}$

⇒ $F(\omega) = G(\omega - 500\pi k)$

$$G(\omega) = \sum_{k=-\infty}^{\infty} .2\,\text{sinc}(.2\pi k)\,\text{rect}\left(\frac{\omega}{400\pi}\right)\frac{\pi}{200\pi}$$

$$= \sum_{k=-\infty}^{\infty} .001\,\text{sinc}(.2\pi k)\cdot\text{rect}\left(\frac{\omega}{400\pi}\right)$$

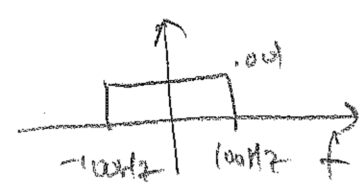
$$F(\omega) = \sum_{k=-\infty}^{\infty} .001\,\text{sinc}(.2\pi k)\,\text{rect}\left(\frac{\omega - 500\pi k}{400\pi}\right)$$



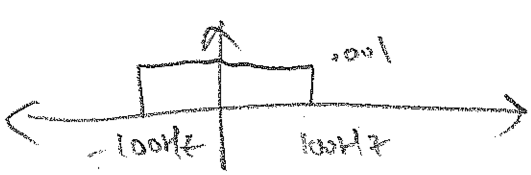
c.) Yes we can. We need an ideal low pass filter to get the spectrum at $\omega=0$. Since there is no overlap and $F(\omega) = \text{rect}(\omega/400\pi)\frac{1}{200}$ The low pass needs a gain of 5.

d.) $\omega = 2\pi f$ $\omega = 200\pi$ $f = 100$ Hz. $B = 100$Hz.

If $f(t)$ is passed the ideal low pass filter and unit gain, we get $\hat{f}(t) = \frac{1}{5}f(t) = \frac{1}{5}\text{sinc}(200\pi t)$ or $\hat{F}(\omega) =$



If B is $100 < B < 150$ we get



same as the 100Hz BW filter.

If B>150 we get the other spectrum residue caused by a non ideal pulse train.