ELEN3801 - Fall 2009
Homework 5

Due Thursday October 15th at the **beginning** of class
(Mudd 227 9:10am)[1]
Carefully justify ALL your answers

5.1 - *The diving board system*: Consider the physical system of Figure 1.

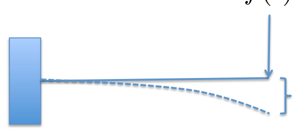

Figure 1: The diving board system: the input is the force applied to the board, and the output is the vertical displacement of the tip of the board.

Although the true system is generally non-linear (and it is also a distributed-parameter system) it can be reasonably approximated by a linear and time-invariant system described by the differential equation

$$(D^2 + 2D + 10)y(t) = f(t) \ ,$$

where $D = \frac{d}{dt}$.

(a) - Verify that the zero-state impulse response of the system is $h(t) = \frac{1}{3}e^{-t}\sin(3t)u(t)$. Sketch the impulse response. (*Hint:* you simply have the check that $h(t)$ satisfies the system equation when the input is $\delta(t)$. Also recall that $du(t)/dt = \delta(t)$)

(b) - Suppose that the system is at rest at time 0, that is the board is not moving and is leveled. In other words $y(0^-) = 0$ and $\dot{y}(0^-) = \left.\frac{dy(t)}{dt}\right|_{t=0^-} = 0$. Suppose that at time zero we place a weight on the tip of the board. This corresponds to applying an input $f(t) = mgu(t)$. What is the system's response $y(t)$? Sketch your answer and comment on the physical plausibility.

Now suppose the board that the board was leveled, but moving upward at time zero. In particular suppose that $\dot{y}(0^-) = -6$ and $y(0^-) = 0$. Let's compute the system's response in that case

[1] You can always hand-in the homework earlier if you so desire - just give it to me or to one of the TAs, or leave it in my mailbox (in the EE office on the 13th floor of Mudd).

(c) - Let $c_1, c_2 \in \mathbb{R}$ be arbitrary. Check that $y(t) = c_1 e^{-t}\sin(3t) + c_2 e^{-t}\cos(3t)$ satisfies the system equation when the input is zero.

(d) - Find the zero-input response for initial conditions $\dot{y}(0^-) = -6$ and $y(0^-) = 0$. This amounts to using (c) and find the right values for c_1 and c_2.

(e) - Compute and sketch the total system response when the input is $f(t) = mgu(t)$ and the initial system conditions are $\dot{y}(0^-) = -6$ and $y(0^-) = 0$. Comment on your results.

4.2 - Consider the periodic signal $f(t)$, with period $T = 2$ and such that

$$f(t) = t^2, \quad \text{for } t \in [0, 2) \ .$$

(a) - Sketch $f(t)$.

(b) - Find the trigonometric Fourier series representation $\varphi(t)$ of this signal.

(c) - Sketch the $\varphi(t)$ for all values of t.

4.3 - *MATLAB question:* Use MATLAB to check the answers of problem 4.2.

(a) - Plot the truncated Fourier series for $t \in [-5, 5]$ using N harmonics. Attach the plots for the cases $N =$ 4,10 and 100. Comment on the results.

(b) - What is the value of the truncated Fourier series at the point $t = 2$ for the values of N in (b). What can you say about that value as N increases?

4.4 - Consider the signal

$$x(t) = 2\cos(3\pi t - \pi/3) - \sin(2\pi t) \ .$$

(a) - Justify that $x(t)$ is periodic. What is the period T_0 and fundamental frequency ω_0?

(b) - Write the signal as a trigonometric Fourier series. Explicitly compute the trigonometric Fourier coefficients a_0 and a_k, b_k for $k = 1, 2, \ldots$.

(c) - Write $x(t)$ as an exponential Fourier series. What are the Fourier coefficients D_k?