

ELEN3801 - Fall 2009

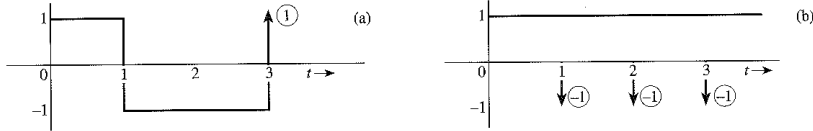
Homework 3

Due Thursday October 1st at the beginning of class

(Mudd 227 9:10am)¹

Carefully justify ALL your answers

- 3.1 - Let $f(t)$ be a signal with energy E_f . Show that the energy of the signals $-f(t)$, $f(-t)$ and $f(t - T)$ where $T \in \mathbb{R}$ is also E_f . What is the energy of the signal $g(t) = cf(at - b)$, where $a, b, c \in \mathbb{R}$ and $a \neq 0$? (*Hint: write the energy integral for the various signals, and use a change of variables to express it in terms of E_f*)
- 3.2 - Find and sketch $\int_{-\infty}^t f(x)dx$ for the signals in panels (a) and (b) depicted below. (*Hint: recall what is the area of a continuous-time impulse function*)



- 3.3 - Consider the continuous-time systems below, described in terms of an input/output relationship, where the input is $f(t)$ and the output is $y(t)$. Determine if the systems are linear or non-linear. (**Justify your answer fully, simply stating the answer will give you no credit**)

(a) $\frac{dy}{dt}(t) + 3ty(t) = t^2 f(t)$ (b) $3y(t) + 2 = f(t)$
(c) $y(t) = \int_{-\infty}^t f(\tau)d\tau$ (d) $\frac{dy}{dt}(t) + 2y(t) = f(t)\frac{df}{dt}(t)$

- 3.4 - Consider the discrete-time systems below, described in terms of an input/output relationship, where the input is $x[n]$ and the output is $y[n]$. Determine if the systems are linear or non-linear. (**Justify your answer fully, simply stating the answer will give you no credit**)

(a) $y[n] = \cos(\pi n) + x[n - 4]$ (b) $y[n] = y[n - 1] + x[n]$
(c) $y[n] = \sin(\pi n) + x[n - 4]$ (d) $y[n] = \sum_{i=-\infty}^n x[i]$

- 3.5 - Consider the systems described below, where $f(t)$ or $f[n]$ correspond to the input (respectively for continuous and discrete-time systems) and $y(t)$ or $y[n]$ correspond to the outputs (again for the CT and DT cases respectively). Determine which systems are time-invariant, and which systems

¹You can always hand-in the homework earlier if you so desire - just give it to me or to one of the TAs, or leave it in my mailbox (in the EE office on the 13th floor of Mudd).

are not. **(Justify your answer fully, simply stating the answer will give you no credit)**

$$\begin{array}{ll} \text{(a) } y(t) = f(-t) & \text{(b) } y(t) = \int_{-3}^3 f(\tau) d\tau \\ \text{(c) } y(t) = f(3t) & \text{(d) } y[n] = 2nx[n-2] \\ \text{(e) } y[n] = (x[n] - x[n-1])^2 & \text{(f) } y[n] = y[n-1] + x[n] \end{array}$$

3.6 - Consider the discrete-time system with input $x[n]$ and output $y[n]$ characterized by the equation $y[n] = y[n-1] + x[n]$. What is the system's response to a unit impulse at the origin (that is, what is the system's response when the input is $x[n] = \delta[n]$)? Write this using a single analytical expression, using the unit-step function $u[n]$.

3.7 - (*Optional, but highly recommended*) Familiarize yourself with MATLAB. There are several resources you can use to do this. For example there is a comprehensive tutorial available at <http://www.mathworks.com/access/helpdesk/help/techdoc/matlab.shtml> (in documentation choose *Getting Started*). Other resource that might be useful is the first laboratory manual of 3084, available at <http://www.ee.columbia.edu/~rmcastro/3801/files/Lab1.pdf>. In the next homework I'll assign some simple computational questions about signal manipulation, that will also help you further understand the convolution in continuous and discrete time. MATLAB is available in the computers of the Gussman lab (251 Mudd).