2.11 \[ E_x = \int_{-\infty}^{\infty} |f(t)|^2 dt \]

In full generality, time shifting or time reversing can be represented as \( f(\alpha t + \beta) \) for some scalars \( \alpha, \beta \). Likewise, this can be extended to multiplied by a scalar \( g(t) = \alpha f(\alpha t + \beta) \) where \( \alpha, \beta, \alpha \in \mathbb{R} \) and \( 0 \leq t \in \mathbb{R} \).

\[ E_y = \int_{-\infty}^{\infty} |\alpha f(\alpha t + \beta)|^2 dt = a^2 \int_{-\infty}^{\infty} f(\alpha t + \beta)^2 dt \]

\[ u = \alpha t + \beta \]

\[ du = \alpha dt \]

\[ = \frac{a^2}{|\alpha|} \int_{-\infty}^{\infty} f(u)^2 du \]

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a) \( f(t) = t(u(t) - u(t-1)) \)

\[ E_x = \int_{0}^{1} t^2 dt = \frac{1}{3} t^3 \bigg|_0^1 = \frac{1}{3} \]

b) \( f(t) = -t(u(t+1) - u(t)) \)

\[ E_x = \int_{-1}^{0} t^2 dt = \frac{1}{3} t^3 \bigg|_{-1}^0 = \frac{1}{3} \]

c) \( f(t) = -t(u(t) - u(t-1)) \)

\[ E_x = \int_{0}^{1} t^2 dt = \frac{1}{3} t^3 \bigg|_0^1 = \frac{1}{3} \]

d) \( f(t) = t-1(u(t-1) - u(t-2)) \)

\[ E_x = \int_{1}^{2} (t-1)^2 dt \]

\[ = \int_{1}^{2} (t^2 - 2t + 1) dt \]

\[ = \int_{\frac{8}{3} - 4 + 2}^{\frac{1}{3} - 1 + 1} \left( \frac{3t^3}{3} - t^2 + t \right) dt \]

\[ = \frac{1}{3} \]

e) \( f(t) = 2t(u(t) - u(t-1)) \)

\[ E_x = \int_{0}^{1} 4t^2 dt = \frac{4}{3} \]

Note that this is consistent with the derived expression.
2.2. \[ f(t) = f(-t) \]
\[ f_0(t) = f(t-1) + f(1-t) \]
\[ f_2(t) = f(t-1) + f(-t-1) \]
Shift 1st \( t \) by 1
\[ f((-t+1)) \]
\[ f_{01}(t) = f(t-\frac{3}{2}) + f(-t+\frac{3}{2}) \]
\[ f_{05}(t) = \frac{3}{2}f\left(\frac{t-\frac{3}{2}}{2}\right) = \frac{3}{2}f\left(\frac{3}{4} - t\right) \]

2.3. \( f(t) = e^{-2t}u(t) \) causal \( f(t) = 0 \ t < 0 \)

a)

\[ \begin{array}{c}
\text{1} \\
\text{.75} \\
\text{1}
\end{array} \]

b) \( f(t) = tu(t) + (e^{(-2)} - t)u(t-1) \) causal \( f(t) = 0 \ t < 0 \)

\[ \begin{array}{c}
\text{tu(t)} \\
\text{e}^{-t}
\end{array} \]
\[ C \cdot \text{Re}\{e^{(12\pi j)t^2}\} u(1-t) \]

\[ \text{Re}\{e^{t e^{2\pi j t^2}} u(1-t) \} \]

\[ e^t \text{Re}\{e^{2\pi j t^2}\} u(1-t) = e^t \cos(2\pi t) u(1-t) \]

\[ u(1-t) \]

\[ \begin{array}{c}
\text{non-causal} \\
f(t) \neq 0 \quad t < 0
\end{array} \]

\[ 2.4. a. \]

\[ f_2(t) = \frac{t^2(u(t) - u(t-2)) + 2(t-4)(u(t-2) - u(t-4))}{t(u(t+4) - u(t+4)) + t(u(t) - u(t-2))} \]

\[ f(t) = -t(u(t+4) - u(t+4)) + t(u(t) - u(t-2)) \]

\[ = -t u(t+4) + tu(t) + tu(t) - tu(t-2) \]

\[ = -t(u(t+4) + u(t-4)) + 2tu(t) \]

\[ b. \]

\[ f_2(t-3) \]
\[3 f_2(2\tau+2) = 3 f_2(2\tau+1)\]

\[f_2(2\tau + 1) = f_2(2\tau - 1)\]
2.5, a.) \[
\left( \frac{\sin(t)}{t^3 + 2} \right) \delta(t) = \frac{\sin(0)}{0+2} \delta(t)
\]
\[= 0\]

b.) \[
\left( \frac{\tau + j \tau}{2 - j \tau} \right) \delta(t-2) = \left( \frac{\tau + 2j}{2 - 2j} \right) \delta(t-2)
\]

c.) \[
\int_{-\infty}^{\infty} (2(t-2) - 5) \delta(t-2) dt
\]
\[= \int_{-\infty}^{\infty} -1 \delta(t-2) dt
\]
\[= -1
\]

d.) \[
e^{3t} \cos(401\pi t) \delta(t+1)
\]
\[= -e^{3} \delta(t+1)
\]

e.) \[
\int_{-\infty}^{x+1} (2t - 5) \delta(t-x) dt
\]
\[= \int_{-\infty}^{x+1} (2x - 5) \delta(t-x) dt
\]
\[= 2x - 5
\]

f.) \[
\int_{-\infty}^{\infty} e^{2x} \cos(401\pi x) \delta(x+1) dx
\]
\[= e^{3} \cos(401\pi (-1))
\]
\[= -e^{3}
\]
\[ f(t) = t(u(t+1) - u(t-2)) \]

\[ \Rightarrow \]

\[ \exists \sum \Delta x(t) = \frac{1}{3} [\Delta x(t) + x(t-1)] \]

\[ x(t) = t(u(t+1) - u(t-2)) \]

\[ x(t-1) = -t(u(t-1) - u(t-2)) \]

\[ \exists \left( x(t) + x(t-1) \right) = \frac{1}{2} \Delta x(t) - \Delta x(t-1) \]

\[ \Delta x(t) = \frac{1}{2} (u(t+1) - u(t-2)) - \Delta x(t-1) \]

\[ \Delta x(t) = \frac{1}{2} (u(t+1) - u(t-2)) - \Delta x(t-1) \]

\[ \exists \left( x(t) - x(t-1) \right) = \frac{1}{2} \left( u(t+1) - u(t-2) + u(t-1) - u(t-3) \right) \]

\[ \exists \left( x(t) - x(t-1) \right) = \frac{1}{2} \left( u(t+1) - u(t-2) + u(t-1) - u(t-3) \right) \]