2.1) $E_f = \int_{-\infty}^{\infty} |f(t)|^2 dt$

In full generality, time shifting or time reversing can be represented as $f(\alpha t + \beta)$ for some scalars α, β. Likewise, this can be extended to multiplied by a scalar

$$g(t) = a f(\alpha t + \beta) \quad \text{where} \quad \alpha, \beta, a \in \mathbb{R} \quad \text{and} \quad f(t) \in \mathbb{R} \ \forall t$$

$$E_g = \int_{-\infty}^{\infty} |a f(\alpha t + \beta)|^2 dt = a^2 \int_{t=-\infty}^{\infty} f(\alpha t + \beta)^2 dt \qquad \begin{array}{l} u = \alpha t + \beta \\ du = \alpha dt \end{array}$$

$$= \frac{a^2}{|\alpha|} \int_{u=-\infty}^{\infty} f(u)^2 du$$

Key of functions

a	b	
c	d	e

a) $f(t) = t\,(u(t) - u(t-1))$

$$E_f = \int_0^1 t^2 dt = \frac{1}{3} t^3 \Big|_0^1 = \frac{1}{3}$$

b) $f(t) = -t\,(u(t+1) - u(t))$

$$E_f = \int_{-1}^{0} t^2 dt = \frac{1}{3} t^3 \Big|_0^1 = \frac{1}{3}$$

c) $f(t) = -t\,(u(t) - u(t-1))$

$$E_f = \int_0^1 t^2 dt = \frac{1}{3} t^3 \Big|_0^1 = \frac{1}{3}$$

d) $f(t) = t - 1\,(u(t-1) - u(t-2))$

$$E_f = \int_1^2 (t-1)^2 dt$$

$$= \int_1^2 (t^2 - 2t + 1) dt$$

$$= \left(\frac{1}{3} t^3 - t^2 + t\right)\Big|_{t=1}^{2} = \left(\frac{8}{3} - 4 + 2\right) - \left(\frac{1}{3} - 1 + 1\right)$$

$$= \frac{1}{3}$$

e) $f(t) = 2t\,(u(t) - u(t-1))$

$$E_f = \int_0^1 4t^2 dt = \frac{4}{3}$$

Note that this is consistent w/ the derived expression.

2.2. $f_1(t) = f(-t)$

$f_2(t) = f(t-1) + f(1-t)$

$f_3(t) = f(t-1) + \underbrace{f(-t-1)}$

time reverse and shift left by 1

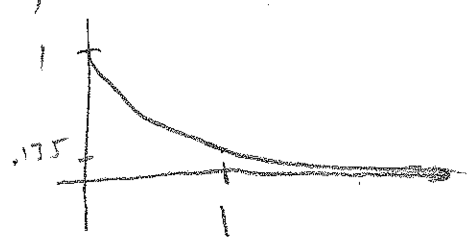
$f(-(t+1))$

$f_4(t) = f(t - \tfrac{1}{2}) + f(-(t + \tfrac{1}{2}))$

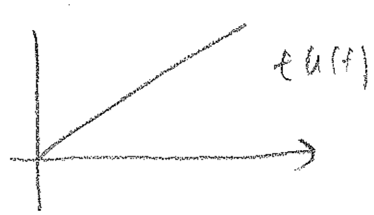
$f_5(t) = \tfrac{3}{2} f\left(\frac{t-2}{2}\right) = \tfrac{3}{2} f(t/2 - 1)$

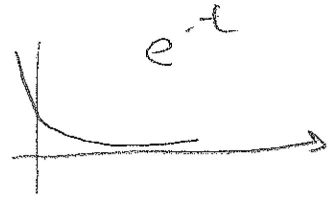
2.3. $f(t) = e^{-2t} u(t)$ causal $f(t) = 0 \quad t < 0$

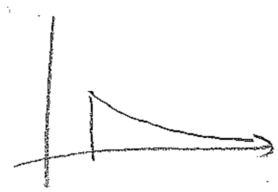
a.)

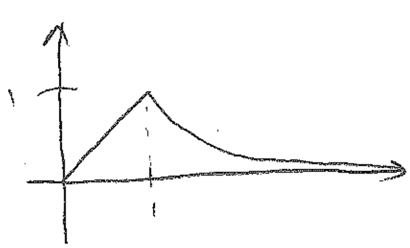


b.) $f(t) = t\,u(t) + (e^{(1-t)} - t)\,u(t-1)$ causal $f(t) = 0 \quad t < 0$









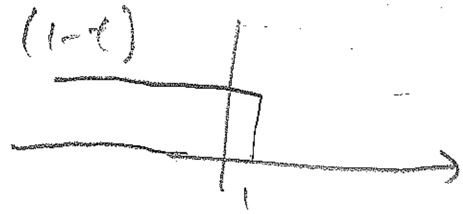
c, $\mathrm{Re}\{e^{(1+2\pi j)t}\}\, u(1-t)$

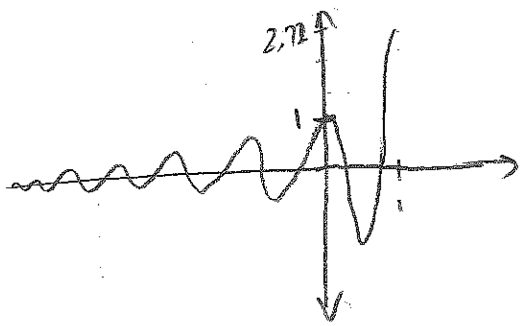
$\mathrm{Re}\{e^{t} e^{2\pi j t}\}\, u(1-t)$

$e^{t}\,\mathrm{Re}\{e^{2\pi j t}\}\, u(1-t)$

$= e^{t} \cos(2\pi t)\, u(1-t)$

$u(1-t)$





non-causal $f(t) \neq 0 \quad t<0$

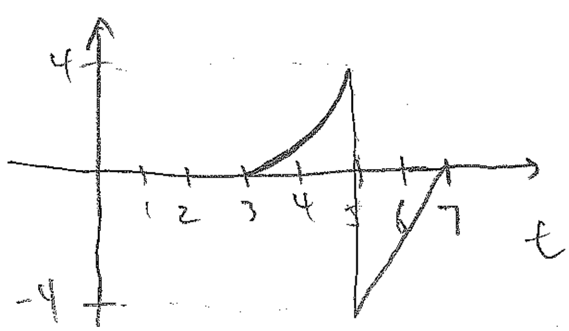
2.4 a. i.) $f_2(t) = t^2(u(t)-u(t-2)) + 2(t-4)(u(t-2)-u(t-4))$

ii.) $f(t) = -t(u(t+4)-u(t)) + t(u(t)-u(t-2))$

$= -t\,u(t+4) + t\,u(t) + t\,u(t) - t\,u(t-2)$

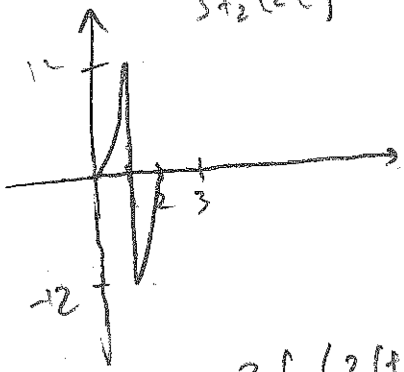
$= -t(u(t+4)+u(t-2)) + 2t\,u(t)$

b.) $f_2(t-3)$

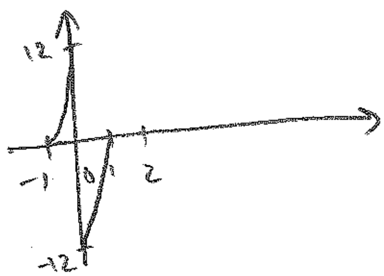


$3f_2(2t+2)$

$3f_2(2t)$

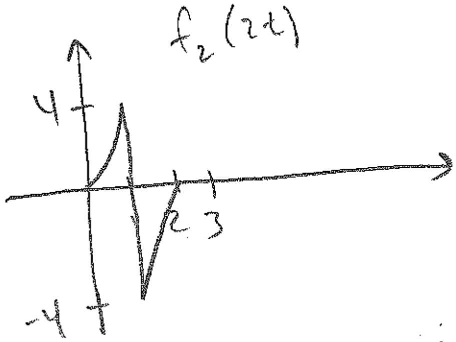


$3f_2(2(t+1)) = 3f_2(2t+2)$

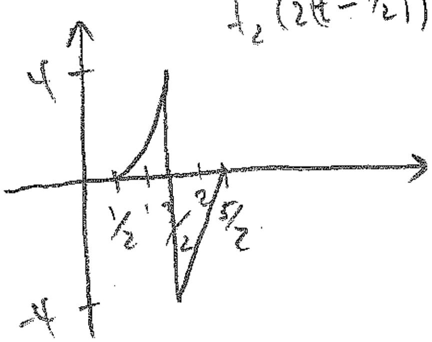


$f_2(2t-1)$

$f_2(2t)$



$f_2(2(t-\frac{1}{2})) = f_2(2t-1)$



2.5, a.) $\left(\frac{\sin(t)}{t^3+2}\right)\delta(t) = \frac{\sin(0)}{0+2}\delta(t)$

$= 0$

b.) $\left(\frac{5+jt}{2-jt}\right)\delta(t-2) = \left(\frac{5+2j}{2-2j}\right)\delta(t-2)$

c.) $\int_{-\infty}^{\infty}(2(2)-5)\delta(t-2)dt$

$= \int_{-\infty}^{\infty} -1\delta(t-2)dt$

$= -1$

d.) $e^{3t}\cos(401\pi t)\delta(t+1)$

$e^{-3}\cos(-401\pi)\delta(t+1)$

$= -e^{-3}\delta(t+1)$

e.) $\int_{-\infty}^{x+1}(2t-5)\delta(t-x)dt$

$= \int_{-\infty}^{x+1}(2x-5)\delta(t-x)dt$ because $x+1$ will integrate over the impulse $\delta(t-x)$

$= 2x-5$

f.) $\int_{-\infty}^{\infty} e^{3x}\cos(401\pi x)\delta(x+1)dx$

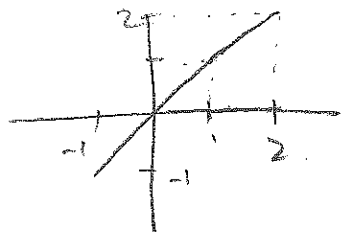
$= e^{-3}\cos(401\pi(-1))$

$= -e^{-3}$

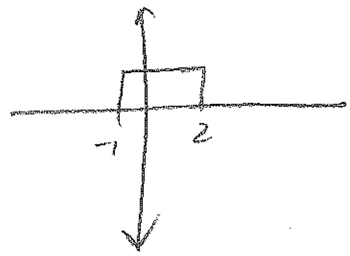
2.6.

$f(t) = t\left(u(t+1) - u(t-2)\right)$

$\Rightarrow$

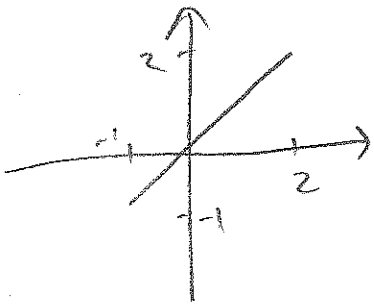


$u(t+1) - u(t-2)$

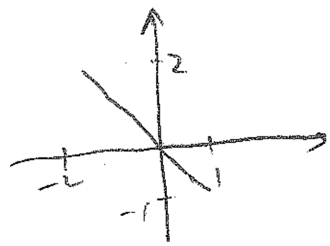


$Ev\{x(t)\} = \frac{1}{2}\left[x(t) + x(-t)\right]$

$x(t) = t\left(u(t+1) - u(t-2)\right)$



$x(-t) = -t\left(u(-t+1) - u(-t-2)\right)$

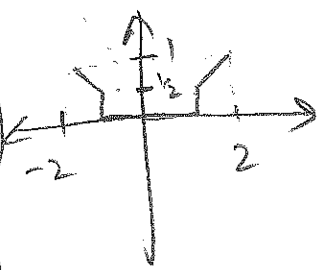


$\frac{1}{2}\left(x(t) + x(-t)\right) =$

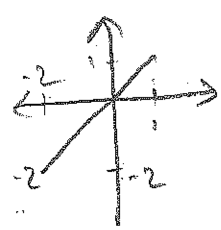
$\frac{1}{2}\left(t\,u(t+1) - t\,u(t-2) - t\left(u(1-t)\right) + t\,u(-t-2)\right)$

$= \frac{1}{2}t\left(u(t+1) - u(t-2) - u(1-t) + u(-t-2)\right)$

$= \frac{1}{2}t\left(\left(u(t+1) + u(-(t+2))\right) - \left(u(t-2) + u(1-t)\right)\right)$



$Odd\{x(t)\} = \frac{1}{2}\left[x(t) - x(-t)\right]$ $-x(-t) = t\left(u(-t+1) - u(-t-2)\right)$



$\frac{1}{2}\left(x(t) - x(-t)\right) = \frac{1}{2}\left(t\left(u(t+1) - u(t-2)\right) + t\left(u(1-t) - u(-t-2)\right)\right)$

$= \frac{1}{2}t\left(u(t+1) + u(1-t) - \left(u(t-2) + u(-t-2)\right)\right)$