1.1 $3+4j$

a. $r=\sqrt{3^2+4^2}$
$=5$
$\theta=\tan^{-1}\left(\frac{4}{3}\right)$
$=.927$
$5e^{j(.927)}$

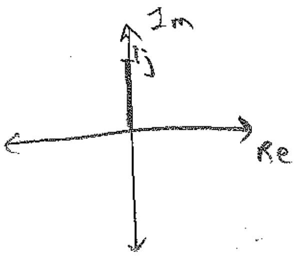
b. $7-5j$
$r=\sqrt{7^2+5^2}$
$=\sqrt{74}$
$\theta=\tan^{-1}\left(\frac{-5}{7}\right)$
$=-.620$
$\sqrt{74}\,e^{j(-.620)}$

c. $23-7j$
$r=\sqrt{23^2+7^2}$
$=\sqrt{578}$
$\theta=\tan^{-1}\left(\frac{-7}{23}\right)$
$=-.295$
$\sqrt{578}\,e^{j(-0.295)}$

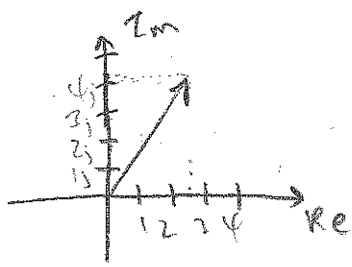
d. $-100-46j$
$r=\sqrt{100^2+46^2}$
$=2\sqrt{3029}$
$\theta=\tan^{-1}\left(\frac{-46}{-100}\right)$
$=.431-\pi$ ← due to being in third quadrant
$2\sqrt{3029}\,e^{j(-2.71)}$

1.2. $e^{j\pi/2}$

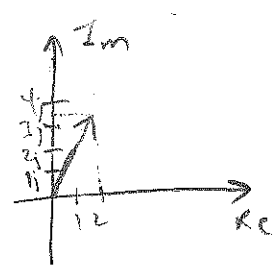
a. $r=1$
$a=\cos\pi/2$
$b=\sin\pi/2$
j



b. $5e^{\pi/3 j}$
$r=5$
$a=5\cos\pi/3$
$b=5\sin\pi/3$
$2.5+\frac{5}{2}\sqrt{3}j$



c. $-4e^{-2\pi/3 j}=4e^{-2\pi/3 j}e^{\pi j}$
$=4e^{\pi/3 j}$
$a=4\cos\pi/3$
$b=4\sin\pi/3$
$2+2\sqrt{3}$



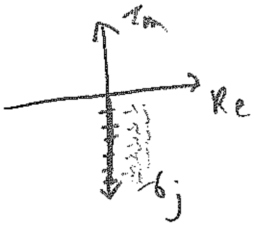
d. $6e^{j(3\pi+\pi/2)}$

$=6e^{j(\pi+\pi/2)}$

$a=6\cos(\pi+\pi/2)$

$b=6\sin(\pi+\pi/2)$

$-6j$

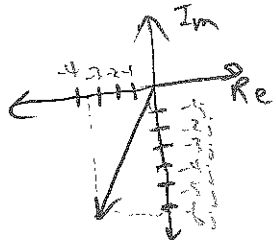


e. $7e^{j4\pi/3}$

$a=7\cos(4\pi/3)$

$b=7\sin(4\pi/3)$

$-3.5-\frac{7\sqrt{3}}{2}j$



1.3. $z_1=7-5j \quad z_2=-3+4j$

a. $(7-5j)(-3+4j) \quad z_1 z_2$

$-21+28j+15j+20$

$-1+43j$

$r=\sqrt{1+43^2}$

$r=\sqrt{1850}$

$=5\sqrt{74}$

$\theta=\tan^{-1}(43/-1)$

$\underline{5\sqrt{74}\,e^{j(1.59)}}$

b. $z_1/z_2 \quad \frac{7-5j}{-3+4j}\,\frac{(-3-4j)}{(-3-4j)} = \frac{-21-28j+15j-20}{9+16}$

$=\frac{-41-13j}{25}$

$=\frac{-41}{25}-\frac{13}{25}j$

$r=\sqrt{\left(\frac{-41}{25}\right)^2+\left(\frac{-13}{25}\right)^2}$

$=1.72$

$\theta=\tan^{-1}\left(\frac{-13}{-41}\right)$

$=.307-\pi$

$\underline{1.72\,e^{j(-2.83)}}$

c. z_2/z_1 $\frac{-3+4j}{7-5j}\cdot\frac{7+5j}{7+5j}=\frac{-21+13j-20}{49+25}=\frac{-41+13j}{74}=\underline{\frac{-41}{74}+\frac{13}{74}j}$

$r=\sqrt{\left(\frac{-41}{74}\right)^2+\left(\frac{13}{74}\right)^2}=.581$ $\quad \theta=\tan^{-1}\left(\frac{13}{-41}\right)$

$\theta=-.307+\pi$

$.581e^{j(2.835)}$

1.4.

$f(t)=6\cos(wt)-\sqrt{5}\sin(wt)$ $\quad a=6 \quad b=\sqrt{5}$

$C=\sqrt{a^2+b^2}$ $\quad C=\sqrt{36+5}=\sqrt{41}$

$\theta=\tan^{-1}\left(\sqrt{5}/6\right)$

$\theta=.357$

$f(t)=\sqrt{41}\cos(wt+.357)$

b. $f(t)=-18\cos(wt)+2\sin(wt)$ $\quad a=-18 \quad b=2$

$C=\sqrt{324+4}=\sqrt{328}$

$\theta=\tan^{-1}\left(\frac{-2}{-18}\right)=.111+\pi$

$f(t)=\sqrt{328}\cos(wt+3.252)$ or $f(t)=-\sqrt{328}\cos(wt+.11)$

1.5. $e^{j\alpha}\cdot e^{j\beta}$

$=(\cos\alpha+j\sin\alpha)(\cos\beta+j\sin\beta)$ $\quad =e^{j\alpha}\cdot e^{j\beta}$

$=e^{j(\alpha+\beta)}$

$\cos\alpha\cos\beta-\sin\beta\sin\alpha+2\cos\alpha\sin\beta j$ $\quad =\cos(\alpha+\beta)+j\sin(\alpha+\beta)$

Imaginary part equal each other
Real part equal each other

$\Rightarrow$ $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\beta\sin\alpha$

$\underline{\sin(\alpha+\beta)=2\cos\alpha\sin\beta}$

1.6.

a. $f(t) = \cos\left(\frac{2\pi}{3}t\right) + 3\sin\left(\frac{\pi}{2}t\right)$ Use Superposition

Definition $f(t) = f(t+T)$ periodic

$f(t+T) = \cos\left(\frac{2\pi}{3}(t+T)\right)$

$= \cos\left(\frac{2\pi}{3}t + \frac{2\pi}{3}T\right)$

Cos is 2π periodic

$\Rightarrow \cos(t) = \cos(t + 2\pi n)$

$\Rightarrow \frac{2\pi}{3}T = 2\pi$

$T = 3$

$f(t+T) = \sin\left(\frac{\pi}{2}(t+T)\right)$

$= \sin\left(\frac{\pi}{2}t + \frac{\pi}{2}T\right)$

$\Rightarrow \sin(t) = \sin(t + 2\pi n)$

$\Rightarrow \frac{\pi}{2}T = 2\pi$

$T = 4$

LCM $3, 4 = \underline{12}$ $\underline{\text{periodic}}$

b. $f(t) = \cos(2\pi t) + \cos(\sqrt{2}\pi t)$ use super position

$f(t) = f(t+T)$ periodic

$f(t+T) = \cos(2\pi(t+T))$

$\Rightarrow \cos(2\pi t + 2\pi T)$

$\Rightarrow 2\pi T = 2\pi$

$T = 1$

$f(t+T) = \cos(\sqrt{2}\pi(t+T))$

$= \cos(\sqrt{2}\pi t + \sqrt{2}\pi T)$

$\Rightarrow \sqrt{2}\pi T = 2\pi$

$T = \frac{2}{\sqrt{2}} \; \frac{2\sqrt{2}}{2} = \sqrt{2}$

LCM $1, \sqrt{2} = \emptyset$ not periodic

1.7.

$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2 dt$ For the power to be zero and energy to be ∞, it is only necessary the denominator grows faster than numerator.

If $f(t) = \begin{cases} \frac{1}{\sqrt{t}} & t \geq 1 \\ 0 & \text{otherwise} \end{cases}$

then $P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left|\frac{1}{\sqrt{t}}\right|^2 dt = \lim_{T\to\infty} \frac{1}{T}\int_1^T \frac{1}{t}\, dt = \lim_{T\to\infty} \frac{\ln(T)}{T} = 0$

$E_f = \int_{\infty}^{\infty} \frac{1}{t}\, dt = \infty$

L'Hôpital's Rule