11.1) a.) $\frac{2s+5}{s^2+5s+6} = \frac{K_1}{(s+2)} + \frac{K_2}{(s+3)}$

$K_1 = \frac{2(-2)+5}{1} = 1$

$K_2 = \frac{2(-3)+5}{-1} = 1$

$\Rightarrow \frac{1}{s+2} + \frac{1}{s+3} = F(s)$

$\Rightarrow f(t) = e^{-2t}u(t) + e^{-3t}u(t)$

c.) $\frac{(s+1)^2}{s^2-s-6} = \frac{s^2+2s+1}{s^2-s-6} = F(s)$

$F(s) = 1 + \frac{K_1}{(s-3)} + \frac{K_2}{(s+2)}$

$K_1 = 3.2$

$K_2 = -.2$

$F(s) = 1 + \frac{3.2}{s-3} + \frac{-.2}{s+2}$

$f(t) = \delta(t) + 3.2e^{3t}u(t) - .2e^{-2t}u(t)$

e.) $\frac{2s+1}{(s+1)(s^2+2s+2)} = F(s) = \frac{k_1}{s+1} + \frac{As+B}{s^2+2s+2}$

$$k_1 = \frac{2(-1)+1}{-1^2+1(-2)+2} = -1$$

$$F(s) = \frac{-1}{s+1} + \frac{As+B}{s^2+2s+2}$$

$$2s+1 = -1(s^2+2s+2) + (As+B)(s+1)$$

$$2s+1+s^2+2s+2 = As^2 + As + Bs + B$$

$$As^2 = s^2 \Rightarrow A = 1$$

$$4s = Bs + As \Rightarrow B = 3$$

$$F(s) = \frac{-1}{s+1} + \frac{s+3}{s^2+2s+2}$$

$$\Rightarrow f(t) = -e^{-t}u(t) + r\,e^{-\alpha t}\cos(bt+\theta)u(t)$$

$$r = \sqrt{\frac{1(2)+(3)^2-2(3)(1)}{2-1}} = \sqrt{5}$$

$$\alpha = 1 \quad b = \sqrt{2-1} = 1$$

$$\theta = \tan^{-1}\frac{1-3}{\sqrt{2-1}} = \tan^{-1} -2$$

$$= f(t) = -e^{-t}u(t) + \sqrt{5}\,e^{-t}\cos(t+(-1.107))u(t)$$

g.) $$\frac{1}{(s+1)(s+2)^4} = \frac{K_1}{(s+1)} + \frac{K_2}{(s+2)} + \frac{K_3}{(s+2)^2} + \frac{K_4}{(s+2)^3} + \frac{K_5}{(s+2)^4}$$

$$K_1 = \frac{1}{(-1+2)^4} = 1 \quad K_2 = \frac{1}{-2+1} = -1$$

$$K_3 = \frac{d}{ds}(s+1)^{-1}\Big|_{s=-2} = -(s+1)^{-2}\Big|_{s=-2} = -1$$

$$K_4 = \frac{1}{2}\frac{d^2}{ds^2}(s+1)^{-1}\Big|_{s=-2} = 2(s+1)^{-3}\Big|_{s=-2} = -1$$

$$K_5 = \frac{1}{2\cdot 3}\frac{d^3}{ds^3}(s+1)^{-1}\Big|_{s=-2} = -6(s+1)^{-4}\Big|_{s=-2} = -1$$

$$\Rightarrow F(s) = \frac{1}{s+1} + \frac{-1}{s+2} + \frac{-1}{(s+2)^2} + \frac{-1}{(s+2)^3} + \frac{-1}{(s+2)^4}$$

$$f(t) = \left(e^{-t} - e^{-2t} - te^{-2t} - \frac{1}{2}t^2e^{-2t} - \frac{1}{6}t^3e^{-2t}\right)u(t)$$

i.) $$F(s) = \frac{s^3}{(s+1)^2(s^2+2s+5)} = \frac{K_1}{s+1} + \frac{K_2}{(s+1)^2} + \frac{As+B}{s^2+2s+5}$$

$$s^3 = K_1(s+1)(s^2+2s+5) + K_2(s^2+2s+5) + (As+B)(s+1)^2$$

$$s^3 = K_1(s^3+2s^2+5s+s^2+2s+5) + K_2s^2 + 2K_2s + 5K_2 + (As+B)(s^2+2s+1)$$

$$\Rightarrow s^3 = k_1(s^3+3s^2+7s+5) + k_2(s^2+2s+5) + \underbrace{(As+B)(s^2+2s+1)}_{=As^3+2As^2+As+Bs^2+2Bs+B}$$

(1) $\Rightarrow k_1 s^3 + As^3 = s^3 \quad \Rightarrow k_1 + A = 1$

(2) $\Rightarrow 2As^2 + Bs^2 + ks^2 + 3k_1 s^2 = 0 \Rightarrow 2A + B + k_2 + 3k_1 = 0$

(3) $\Rightarrow 7k_1 s + 2k_2 s + As + 2Bs = 0 \Rightarrow 7k_1 + 2k_2 + A + 2B = 0$

(4) $\Rightarrow 5k_1 + 5k_2 + B = 0$

$\hookrightarrow$ (3) $- 2\times$(2) $= k_1 - 3A = 0 \quad k_1 = 3A$

(1) $\Rightarrow k_1 + A = 1 \quad \Rightarrow 4A = 1 \quad A = 1/4, \; k_1 = 3/4$

(4) $-$ (2) $= 15/4 + 5k_2 + B - 1/2 - B - k_2 - 9/4 = 0$

$\Rightarrow \quad 1 + 4k_2 = 0, \quad k_2 = -1/4, \quad B = -5/2$

$$F(s) = \frac{3/4}{(s+1)} + \frac{-1/4}{(s+1)^2} + \frac{1/4 s - 5/2}{s^2+2s+5}$$

$$f(t) = \left(3/4\, e^{-t} - \frac{t}{4} e^{-t} + r e^{-t}\cos(bt+\theta)\right)u(t)$$

$$r = \sqrt{\frac{(1/4)^2(5) + (-5/2)^2 - 2(1/4)(-5/2)}{5-1}} = \sqrt{\frac{125}{64}} = \frac{5\sqrt{5}}{8}$$

$$b = \sqrt{5-1} = 2 \quad \theta = \tan^{-1}\left(\frac{1/4 - (-5/2)}{1/4\sqrt{5-1}}\right) = \tan^{-1}\left(\frac{11}{2}\right)$$

$f(t) = \left[\frac{3}{4} e^{-t} - \frac{t}{4} e^{-t} + \frac{5\sqrt{5}}{8} e^{-t} \cos(2t + 1.39) \right] u(t)$

(11.2) b.) $\frac{s}{s^2+2s+2} e^{-3s} + \frac{2}{s^2+2s+2}$

$r = \sqrt{\frac{2+0-0}{1}} = \sqrt{2}$

$\theta = \tan^{-1} \frac{1}{1(1)} = \pi/4 \quad b = 1$

first part $\sqrt{2} e^{-t} \cos(t + \pi/4) u(t)$

↳ time shifted $\sqrt{2} e^{-(t-3)} \cos(t - 3 + \pi/4) u(t-3)$

for $\frac{2}{s^2+2s+2}$

$b = \sqrt{2 - 1} = 1$

$e^{-t} \, 2 \sin(t) \, u(t)$

$\Rightarrow f(t) = \sqrt{2} e^{-(t-3)} \cos(t - 3 + \pi/4) u(t-3) +$

$2 e^{-t} \sin(t) u(t)$

d.) $\frac{e^{-s}}{s^2+3s+2} + \frac{e^{-2s}}{s^2+3s+2} + \frac{1}{s^2+3s+2} = F(s)$

$\Rightarrow \frac{1}{s^2+3s+2} = \frac{k_1}{(s+2)} + \frac{k_2}{(s+1)} = \frac{-1}{s+2} + \frac{1}{s+1}$

$k_1 = -1$

$k_2 = 1$

$\Rightarrow f(t) = -e^{-2(t-1)}u(t-1) + e^{-(t-1)}u(t-1)$

$-e^{-2(t-2)}u(t-2) + e^{-(t-2)}u(t-2)$

$-e^{-2t}u(t) + e^{-t}u(t)$

11.3) a.) $(D^2+3D+2)y(t) = Df(t) \quad y(0^-) = \dot{y}(0^-) = 0$

$f(t) = u(t)$

$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{df(t)}{dt}$

$s^2Y(s) + 3sY(s) + 2Y(s) = s\frac{1}{s}$

$(s^2+3s+2)Y(s) = 1$

$Y(s) = \frac{1}{s^2+3s+2} = \frac{k_1}{(s+1)} + \frac{k_2}{(s+2)}$

$k_1 = -1 \quad k_2 = 1$

$\Rightarrow y(t) = \left(e^{-2t} - e^{-t}\right)u(t)$

b.) $\left(D^2+4D+4\right)y(t) = (D+1)f(t) \quad y(0^-)=2 \quad \dot{y}(0^-)=1$

$f(t) = e^{-t}u(t)$

$s^2Y(s) - s2 - 1 + 4\left(sY(s) - 2\right) + 4Y(s) = \frac{1}{s+1} + \frac{s}{s+1}$

$\left(s^2+4s+4\right) - 2s - 9 = \frac{1}{s+1} + \frac{s}{s+1}$

$s^2+4s+4 = \frac{1}{s+1} + \frac{s}{s+1} + 2s + 9$

$Y(s) = \frac{2s+10}{s^2+4s+4}$

$Y(s) = \frac{k_1}{s+2} + \frac{k_2}{(s+2)^2}$

$2s+10 = (s+2)k_1 + k_2 \qquad k_1 = 2$

$2s+10 = k_1 s + 2k_1 + k_2 \qquad 4 + k_2 = 10$

$k_2 = 6$

$Y(s) = \frac{2}{s+2} + \frac{6}{(s+2)^2}$

$y(t) = \left(2e^{-2t} + 6te^{-2t}\right)u(t)$

$= (2+6t)e^{-2t}u(t)$

c.) $\left(D^2+6D+25\right) y(t) = (D+2) f(t)$ $\quad y(0^-) = \dot{y}(0^-) = 1$

$f(t) = 25u(t)$

$s^2 Y(s) - s1 - 1 + 6(sY(s) - 1) + 25Y(s) = 25 + \frac{50}{s}$

$(s^2 + 6s + 25)Y(s) - s - 7 = 25 + 50/s$

$(s^2+6s+25)Y(s) = 32 + \frac{50}{s} + s$

$$Y(s) = \frac{s^2+32s+50}{s(s^2+6s+25)}$$

$$= \frac{K_1}{s} + \frac{As+B}{s^2+6s+25}$$

$$K_1 = \frac{50}{25} = 2$$

$$= \frac{2}{s} + \frac{As+B}{s^2+6s+25}$$

$s^2+32s+50 = 2(s^2+6s+25) + (As+B)(s)$

$s^2+32s+50 = 2s^2+12s+50 + As^2+Bs$

$s^2 = 2s^2 + As^2$ $\qquad$ $Bs + 12s = 32s$

$A = -1$ $\qquad$ $B = 20$

$$Y(s) = \frac{2}{s} + \frac{-s+20}{s^2+6s+25}$$

$$y(t) = 2u(t) + re^{at}\cos(bt+\theta)u(t))$$

$$r = \sqrt{\frac{(-1)^2(25)+(20)^2-2(-1)(20)(3)}{25-3^2}}$$

$$r = \frac{\sqrt{545}}{4} = 5.836$$

$$b = \sqrt{25-3^2} = 4$$

$$\theta = \tan^{-1}\left(\frac{-1(3)-20}{-1\sqrt{25-3^2}}\right) = \left(\tan^{-1} 5.75\right) - \pi$$

$$\theta = -1.7429$$

because $\tan^{-1}\left(\frac{-23}{-4}\right)$

$$y(t) = (2 + 5.836e^{-3t}(4t-1.7429))u(t)$$

11.4.) i, a.)

$$H(s) = \frac{s+5}{s^2+5s+6} \qquad f(t) = e^{-3t}u(t)$$

$$F(s) = \frac{1}{s+3}$$

$$Y(s) = H(s)F(s)$$

$$Y(s) = \frac{s+5}{(s^2+5s+6)(s+3)}$$

$$= \frac{s+5}{(s+3)(s+2)(s+3)} = \frac{k_1}{s+2} + \frac{k_2}{s+3} + \frac{k_3}{(s+3)^2}$$

$$k_1 = 3 \qquad k_3 = -2$$

$$\frac{s+5}{(s+3)^2(s+2)} = \frac{3}{s+2} + \frac{K_3}{s+3} + \frac{-2}{(s+3)^2}$$

$$s+5 = 3(s+3)^2 + K_3(s+2)(s+3) - 2(s+2)$$

$$\Rightarrow 6K_3 = 5 - 27 + 4$$

$$K_3 = -3$$

$$y(t) = \left(3e^{-2t} - 3e^{-3t} - 2te^{-3t}\right)u(t)$$

b.) $Y(s)(s^2+5s+6) = (s+5)F(s)$

$(D^2+5D+6)y(t) = (D+5)f(t)$

for all parts i, iii, and iv have the same transfer function therefore this differential equation satisfy the system

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(iii) a.) $$Y(s) = \frac{s+5}{(s^2+5s+6)}\frac{1}{s+4}$$

input is shifted by $t_0 = 5$
output is shifted also by $t_0 = 5$

$$= \frac{s+5}{(s+3)(s+2)(s+4)}$$

$$= \frac{K_1}{s+3} + \frac{K_2}{s+2} + \frac{K_3}{s+4}$$

$k_1 = -2 \quad k_2 = 3/2 \quad k_3 = 1/2$

$$= \frac{-2}{s+3} + \frac{3/2}{s+2} + \frac{1/2}{s+4}$$

$$= \left(-2e^{-3t} + \tfrac{3}{2}e^{-2t} + \tfrac{1}{2}e^{-4t}\right)u(t)$$

need to shift output by $t_0 = 5$

$$\Rightarrow y(t) = \left(-2e^{-3(t-5)} + \tfrac{3}{2}e^{-2(t-5)} + \tfrac{1}{2}e^{-4(t-5)}\right)u(t-5)$$

(v.) a.) $f(t) = e^{-4(t-5)}u(t)$

$$= e^{-4t+20}u(t)$$

$$= e^{-4t}e^{20}u(t)$$

Since the system is linear we know the response

$\underbrace{e^{-4t}u(t)}_{\text{previous part}}$ then we just need to scale it.

$$y(t) = e^{20}\left(-2e^{-3t} + \tfrac{3}{2}e^{-2t} + \tfrac{1}{2}e^{-4t}\right)u(t)$$

11.5) c.) $\frac{d^4y}{dt^4} + 4\frac{dy}{dt} = 3\frac{df}{dt} + 2f(t)$

$$(s^4 + 4s)Y(s) = (3s+2)F(s)$$

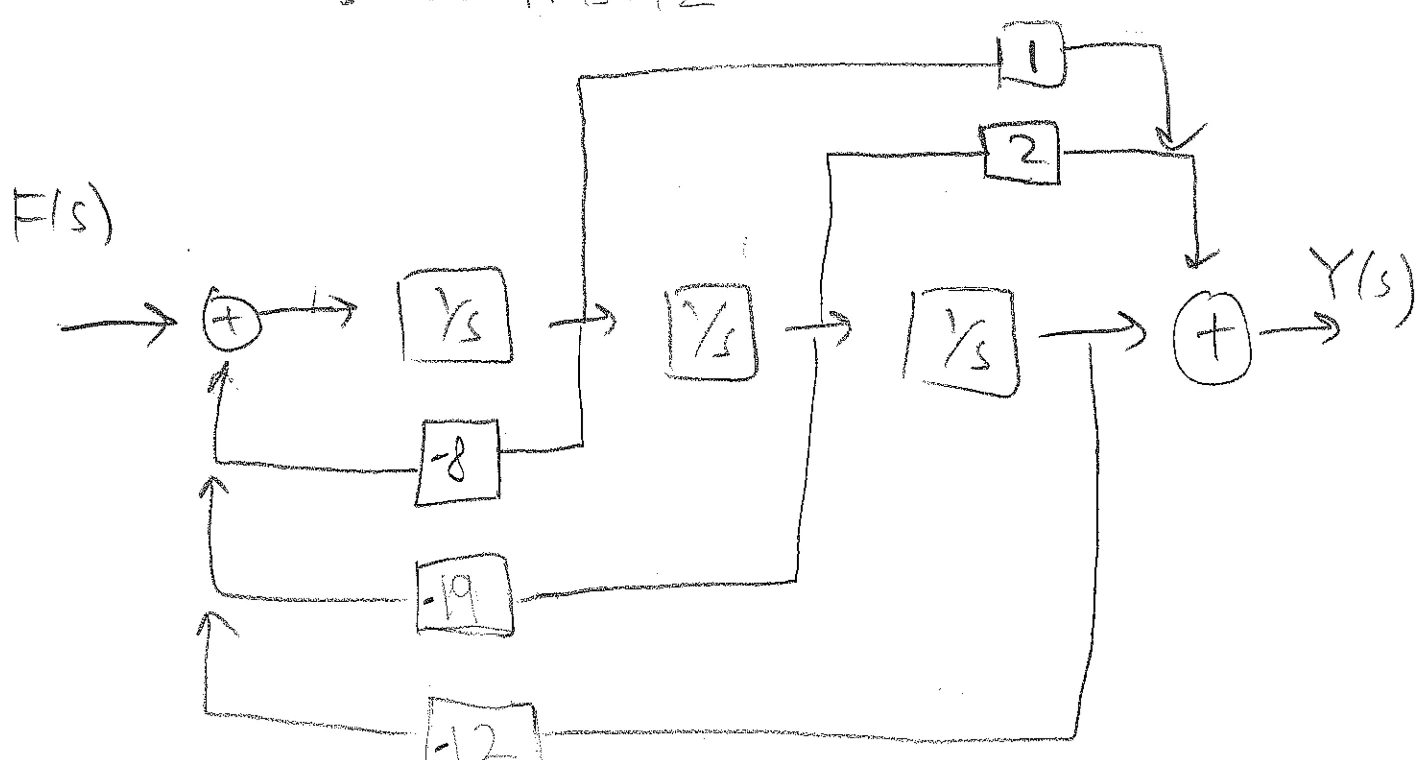
$$F(s)\,H(s) = Y(s) \qquad H(s) = \frac{Y(s)}{F(s)} = \frac{3s+2}{s^4+4s}$$

11.6.) $$H(s) = \frac{s(s+2)}{(s+1)(s+3)(s+4)}$$

canonical

$$H(s) = \frac{s^2+2s}{(s^2+3s+1s+3)(s+4)} = \frac{s^2+2s}{s^3+4s^2+4s^2+16s+3s+12}$$

$$= \frac{s^2+2s}{s^3+8s^2+19s+12}$$

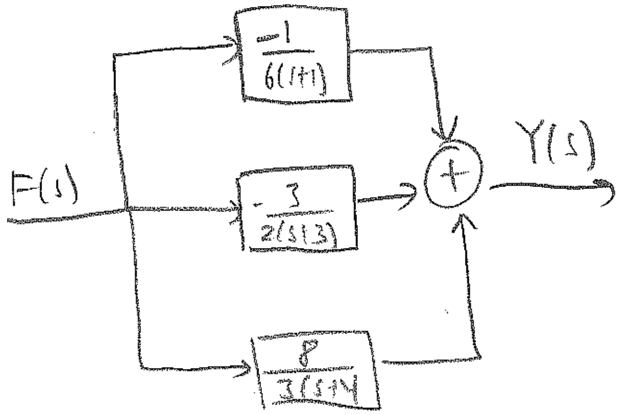


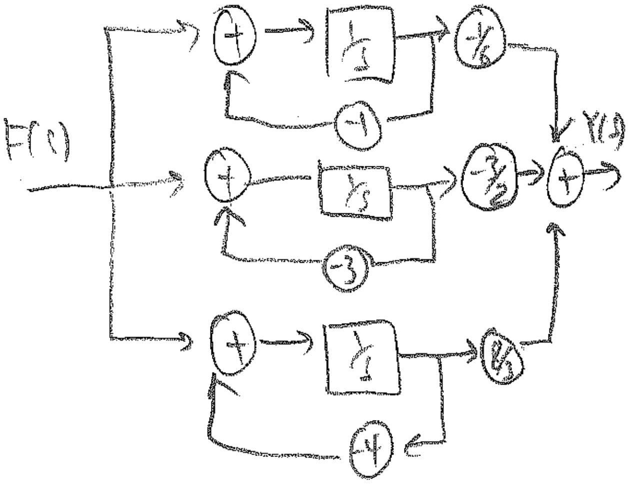
Parallel form

$$H(s) = \frac{k_1}{s+1} + \frac{k_2}{s+3} + \frac{k_3}{s+4} \qquad k_1 = \frac{-1}{6} \quad k_2 = \frac{3}{-2}$$

$$k_3 = \frac{8}{3}$$

$$H(s) = -\frac{1}{6(s+1)} - \frac{3}{2(s+3)} + \frac{8}{3(s+4)}$$





Cascade or series form $\qquad H(s) = \frac{s(s+2)}{(s+1)(s+3)(s+4)}$