10.1) a) $\bar{f}(t) = f(t)\delta_{T_s}(t)$ $\qquad f(t) = \text{sinc}(200\pi t)$

$$\bar{f}(t) = f(t)\sum_{k=-\infty}^{\infty}\delta(t-kT_s) \qquad T_s = \frac{1}{150}$$

$$\text{F.S. of } \delta_{T_s}(t) = \sum_{k=-\infty}^{\infty} D_k e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T} = 300\pi$$

$$D_k = \frac{1}{T_0}\int_{T_0} \delta(t)\, e^{-jk\omega_0 t}\, dt$$

$$= 150\, e^{-jk\omega_0 0}\, dt$$

$$= 150$$

$$\Rightarrow \delta_{T_s}(t) = \sum_{k=-\infty}^{\infty} 150\, e^{jk300\pi t}$$

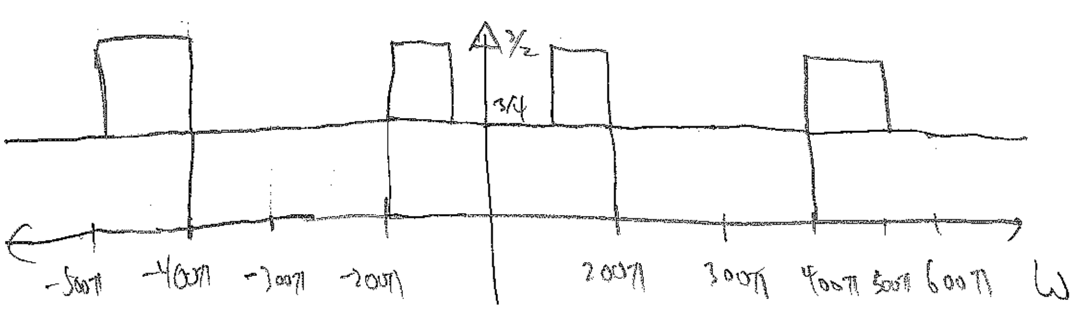
$$\bar{f}(t) = f(t)\sum_{k=-\infty}^{\infty} 150\, e^{jk300\pi t}$$

$$\bar{f}(t) = \text{sinc}(200\pi t)\sum_{k=-\infty}^{\infty} 150\, e^{jk300\pi t}$$

let $g(t) = \text{sinc}(200\pi t)$ $\qquad \bar{f}(t) = \sum_{k=-\infty}^{\infty} 150\, g(t)\, e^{jk300\pi t}$

$$\Rightarrow \bar{F}(\omega) = G(\omega - 300\pi k) \qquad G(\omega) = \frac{1}{200}\,\text{rect}\left(\frac{\omega}{400\pi}\right)$$

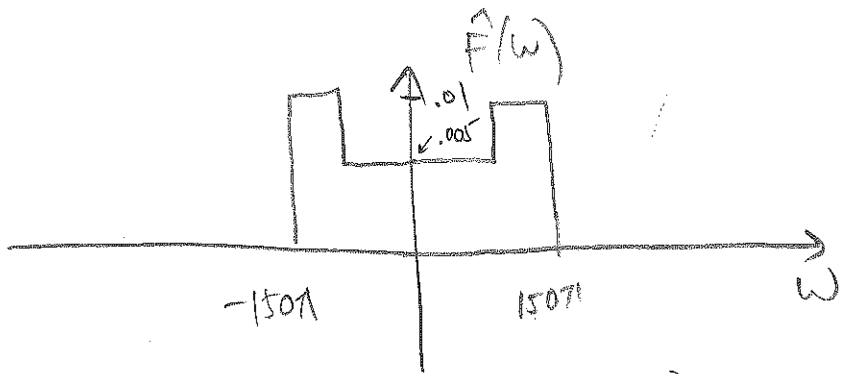
$$\bar{F}(\omega) = \sum_{k=-\infty}^{\infty} \frac{3}{4}\,\text{rect}\left(\frac{\omega - 300\pi k}{400\pi}\right)$$

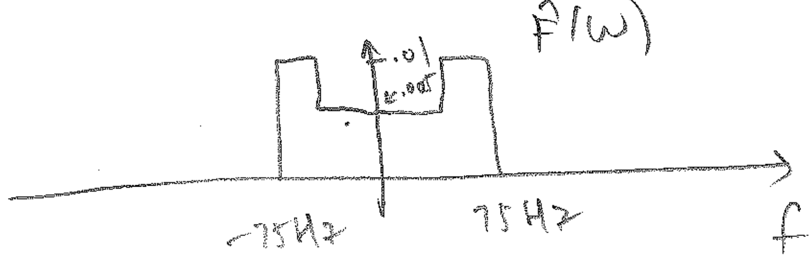


b)

$\hat{f}(t) = f(t) * h(t)$

$\hat{F}(\omega) = F(\omega) H(\omega) \qquad H(\omega) = \frac{1}{150} \text{rect}\left(\frac{\omega}{300\pi}\right)$

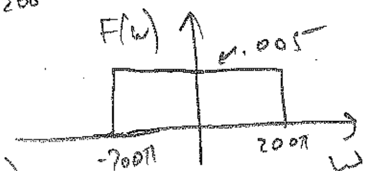




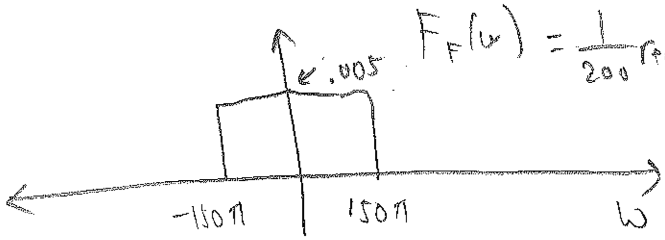
c.) $f_F(t) = f(t) * h_a(t) \qquad h_a(t) =$ Anti-aliasing filter

$F(\omega) = \frac{1}{200} \text{rect}\left(\frac{\omega}{400\pi}\right)$

$F_F(\omega) = F(\omega) H_a(\omega)$

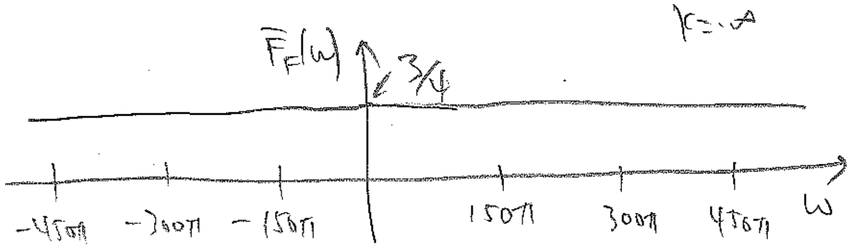


$F_F(\omega) = \frac{1}{200} \text{rect}\left(\frac{\omega}{300\pi}\right)$



$\bar{f}_F(t) = f_F(t)\, \delta_{T_s}(t)$

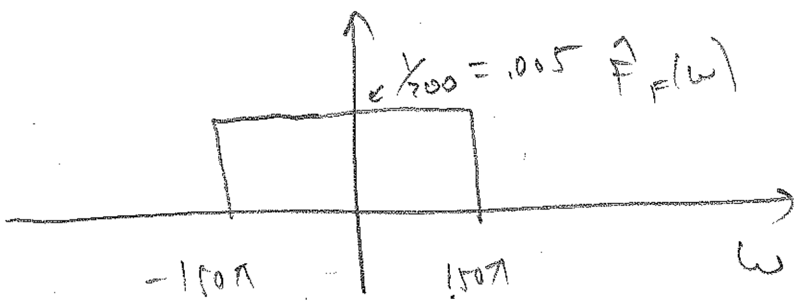
$\bar{F}_F(\omega) = F_s(\omega - 300\pi k) = \sum_{k=-\infty}^{\infty} \frac{3}{4} \text{rect}\left(\frac{\omega - 300\pi k}{300\pi}\right)$

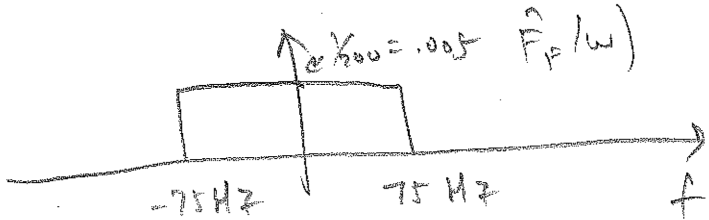


d.) $\hat{f}_F(t) = f_F(t) * h(t)$

$\hat{F}_F(\omega) = F_F(\omega)\, H(\omega) \qquad H(\omega) = \frac{1}{150}\, rect\left(\frac{\omega}{300\pi}\right)$

$\hat{F}_F(\omega) = \frac{1}{200}\, rect\left(\frac{\omega}{300\pi}\right)$

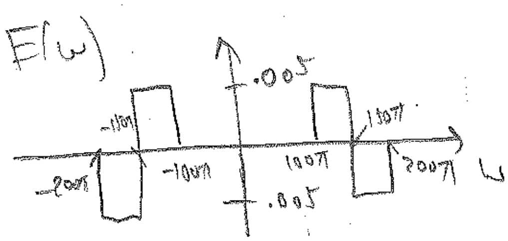


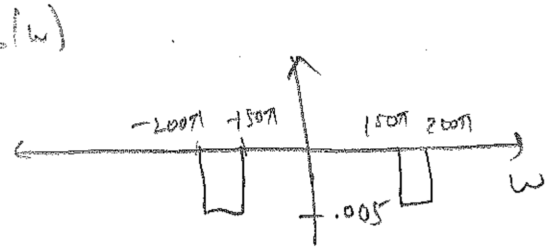


e.) let $e(t) = \hat{f}(t) - f(t) \Rightarrow E(\omega) = \hat{F}(\omega) - F(\omega)$

and $e_F(t) = \hat{f}_F(t) - f(t) \Rightarrow E_F(\omega) = \hat{F}_F(\omega) - F(\omega)$

$E_f(e(t)) = \int_{-\infty}^{\infty} |e(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |E(\omega)|^2 d\omega$



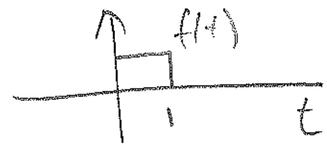


$E_f(e(t)) = \frac{1}{2\pi}(2.5\times10^{-5})(200\pi - 100\pi)\times 2 = .0025$

$E_f(e_F(t)) = \frac{1}{2\pi}(2.5\times10^{-5})(200\pi - 150\pi)\times 2 = .00125$

$e_F(t) < e(t) \Rightarrow \hat{f}_F(t)$ is a better reconstruction.

10.2) a.) $u(t) - u(t-1) = f(t)$



$$F(s) = \int_0^\infty f(t)\, e^{-st} dt$$

$$= \int_0^1 e^{-st} dt$$

$$= \frac{e^{-st}}{-s}\Big|_0^1 = \frac{e^{-s} - 1}{-s} = \frac{1 - e^{-s}}{s}$$ ROC $s \neq 0, \infty, -\infty$

b.) $f(t) = t e^{-t} u(t)$

$$F(s) = \int_0^\infty t e^{-t} e^{-st} dt$$

$$= \int_0^\infty t e^{-(s+1)t} dt$$

$u = t \quad dv = e^{-(s+1)t} dt$

$du = dt \quad v = \frac{e^{-(s+1)t}}{-(s+1)}$

$$= \frac{t e^{-(s+1)t}}{-(s+1)}\Big|_0^\infty - \int_0^\infty \frac{e^{-(s+1)t}}{-(s+1)} dt$$

ROC: $s+1 > 0$

$s > -1$

$$= 0 - 0 \quad - \frac{e^{-(s+1)t}}{(s+1)^2}\Big|_0^\infty$$

$$= \frac{1}{(s+1)^2}$$ ROC: $s > -1$

d.) $\left(e^{2t}-2e^{-t}\right)u(t) = f(t)$

$$F(s) = \int_0^\infty \left(e^{2t}-2e^{-t}\right)e^{-st}\,dt$$

$$= \int_0^\infty e^{(2-s)t} - 2e^{-(s+1)t}\,dt$$

$$= \underbrace{\left.\frac{e^{(2-s)t}}{2-s}\right|_0^\infty}_{\begin{array}{c}\text{ROC: } 2-s<0 \\ s>2\end{array}} - \underbrace{\left.\frac{2e^{-(s+1)t}}{-(s+1)}\right|_0^\infty}_{\begin{array}{c}\text{ROC: } s+1>0 \\ s>-1\end{array}}$$

$$\Rightarrow = 0 - \frac{1}{2-s} - \left(0 - \frac{2}{-(s+1)}\right)$$

$$= \frac{1}{s-2} - \frac{2}{s+1} \quad \text{ROC: } s>2$$

10.3) a.) $u(t) - u(t-1) = f(t)$ let $g(t) = u(t)$

$h(t) = u(t-1) = g(t-1)$

$G(s) = \frac{1}{s}$ $H(s) = G(s)e^{-s(1)}$

$$\Rightarrow F(s) = \frac{1}{s} - \frac{e^{-s}}{s} = \frac{1-e^{-s}}{s}$$

b.) $e^{-(t-\tau)}u(t-\tau) = f(t)$

let $g(t) = e^{-t}u(t)$ $\quad f(t) = g(t-\tau)$

$F(s) = G(s)e^{-s(\tau)}$ $\qquad G(s) = \frac{1}{s+1}$

$$F(s) = \frac{e^{-s\tau}}{s+1}$$

c.) $e^{-(t-\tau)}u(t) = f(t)$

let $g(t) = e^{-t}u(t)$

$f(t) = e^{\tau}g(t)$ $\qquad e^{\tau}$ constant

$\Rightarrow F(s) = e^{\tau}G(s)$ $\qquad G(s) = \frac{1}{s+1}$

$$F(s) = \frac{e^{\tau}}{s+1}$$

f.) $f(t) = \sin(\omega_0(t-\tau))u(t-\tau)$

let $g(t) = \sin(\omega_0 t)u(t)$ $\quad f(t) = g(t-\tau)$

$F(s) = G(s)e^{-s\tau}$ $\qquad G(s) = \frac{\omega_0}{s^2+\omega_0^2}$

$$F(s) = \frac{e^{-s\tau}\omega_0}{s^2+\omega_0^2}$$

10.4) a) $f(t) = t(u(t) - u(t-1))$

$= tu(t) - tu(t-1)$

$= tu(t) - tu(t-1) + u(t-1) - u(t-1)$

$= tu(t) - (t-1)u(t-1) - u(t-1)$

$F(s) = \frac{1}{s^2} - \frac{1e^{-s}}{s^2} - \frac{1e^{-s}}{s}$

$= \frac{1}{s^2}\left(1 - e^{-s} - se^{-s}\right)$

b.) $f(t) = \sin t\,(u(t) - u(t-\pi))$

$= \sin t\, u(t) - (\sin t)u(t-\pi)$

$= \sin t\, u(t) + \sin(t-\pi)\, u(t-\pi)$

$\Rightarrow F(s) = \frac{1}{s^2+1} + \frac{e^{-s\pi}}{s^2+1}$

$= \frac{1+e^{-s\pi}}{s^2+1}$

c.) $f(t) = \frac{t}{e^1}(u(t) - u(t-1)) + e^{-t}u(t-1)$

$$f(t) = \frac{1}{e}\, t\,u(t) - t\,u(t-1) + u(t-1) - u(t-1) + e^{-t}u(t-1)\frac{e}{e}$$

$$= \frac{1}{e}\Big(t\,u(t) - (t-1)u(t-1) - u(t-1)\Big) + \frac{e^{-(t-1)}u(t-1)}{e}$$

$$= \frac{1}{e^{1}s^{2}}\left(1 - e^{-s} - s e^{-s}\right) + \frac{e^{-s}e^{-1}}{s+1}$$

$$= \frac{1}{e^{1}s^{2}}\left(1 - e^{-s} - s e^{-s}\right) + \frac{e^{-(s+1)}}{s+1}$$