

ELEN3801 - Fall 2009

Homework 10

Due Thursday December 3rd at the **beginning** of class

(Mudd 227 9:10am)¹

Carefully justify ALL your answers

10.1 - We have seen in class how the use of an anti-aliasing filter can mitigate some of the difficulties when trying to reconstruct a signal that was sampled below the Nyquist rate (either because the original signal itself is not band-limited or because it is sampled at an inadequate rate).

Let $f(t) = \text{sinc}(200\pi t)$, and suppose we sample this signal at a rate 150 Hz using a train of impulses $\delta_{T_s}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$, where $T_s = 1/150$ sec.

- Sketch the Fourier transform of $\bar{f}(t) = f(t)\delta_{T_s}(t)$.
- To reconstruct (imperfectly) the signal we will filter $\bar{f}(t)$ using an ideal filter, with response $H(\omega) = \frac{1}{150} \text{rect}\left(\frac{\omega}{300\pi}\right)$ (note that this corresponds to an ideal low-pass filter with cut-off frequency 75 Hz and gain 1/150). The result of this operation is $\hat{f}(t)$. Plot the Fourier transform of the reconstructed signal $\hat{f}(t)$ (note that $\hat{F}(\omega) = H(\omega)\bar{F}(\omega)$).
- Now suppose we use an anti-aliasing filter. The overall procedure is depicted in Figure 1. Sketch the Fourier transform of the sampled signal $\bar{f}_F(t)$.

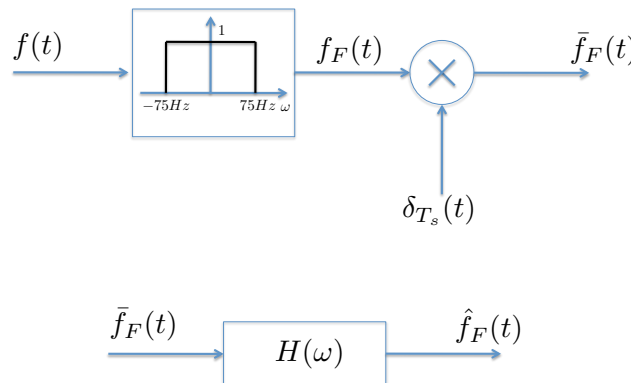


Figure 1: Sampling a signal using an anti-aliasing filter.

¹You can always hand-in the homework earlier if you so desire - just give it **in hand** to me or to one of the TAs, or leave it in **my** mailbox (in the EE office on the 13th floor of Mudd).

- d) Plot the Fourier transform of the reconstructed signal $\hat{f}_F(t)$.
- e) From your answers you should note that neither reconstruction is perfect, but we would like to make sure the use of the anti-aliasing filter is actually helping us. To assess the quality of the reconstruction we look at the energy of the difference between the reconstructed signal and the original signal in both cases. Compute the energy of $\hat{f}(t) - f(t)$ and $\hat{f}_F(t) - f(t)$. Which one is smaller, indicating a better reconstruction? (**Hint:** remember Parseval's relation).

10.2 - Solve problem 6.1-1 (a), (b) and (d) from the textbook (NOTE: you should use equation 6.18 - unilateral Laplace transform), instead of the one in the question).

10.3 - Problem 6.2.1 (a), (b), (c) and (f) from the textbook.

10.4 - Problem 6.2-2 from the textbook.