Summary:

Let $x(t)$ be a periodic signal with period T_0
(therefore fundamental frequency $\omega_0 = \frac{2\pi}{T_0}$)

The exponential Fourier series representation of $x(t)$ is given by

$$x(t) = \sum_{k=-\infty}^{\infty} D_k e^{j\omega_0 k t}$$

where $$D_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-j\omega_0 k t} dt$$

For real signals (i.e. $x(t) \in \mathbb{R}$) the trigonometric Fourier series representation is given by

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(\omega_0 k t) + b_k \sin(\omega_0 k t)$$

where $a_0 = \frac{1}{T_0} \int_{T_0} x(t) dt$, and for $k \neq 0$

$$a_k = \frac{2}{T_0} \int_{T_0} x(t) \cos(\omega_0 k t) dt \quad , \quad b_k = \frac{2}{T_0} \int_{T_0} x(t) \sin(\omega_0 k t) dt$$

We can also write the trigonometric Fourier series in a compact way, yielding the <u>compact trigonometric Fourier series</u> representation

$$x(t) = C_0 + \sum_{k=1}^{\infty} C_k \cos(w_0 k t + \theta_k)$$

where $C_0 = a_0$, $C_k = \sqrt{a_k^2 + b_k^2}$, $\theta_k = \tan^{-1}\left(-\frac{b_k}{a_k}\right) + \pi$ if $a_k < 0$

All the representations are equivalent (if $x(t)$ is real) and the various coef. are related in the following way:

$$a_0 = D_0 = C_0$$

$$a_k = 2\,\mathrm{Re}\{D_k\} = C_k \cos\theta_k, \quad b_k = -2\,\mathrm{Im}\{D_k\} = -C_k \sin(\theta_k), \quad k \neq 0$$

also $D_0 = a_0$, $D_k = \frac{1}{2}(a_k - j b_k)$, $D_{-k} = \frac{1}{2}(a_k + j b_k)$, $k \neq 0$

$$|D_k| = |D_{-k}| = \frac{1}{2} C_k \quad , \quad k \neq 0$$

$$\measuredangle D_k = -\measuredangle D_{-k} = \theta_k$$