Summary:

Let \( x(t) \) be a periodic signal with period \( T_0 \) (therefore fundamental frequency \( \omega_0 = \frac{2\pi}{T_0} \))

The **exponential Fourier series** representation of \( x(t) \) is given by

\[
x(t) = \sum_{k=-\infty}^{\infty} D_k e^{j\omega_0 kt}
\]

where

\[
D_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-j\omega_0 kt} dt
\]

For real signals (i.e., \( x(t) \in \mathbb{R} \)) the **trigonometric Fourier series** representation is given by

\[
x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(\omega_0 kt) + b_k \sin(\omega_0 kt)
\]

where

\[
a_0 = \frac{1}{T_0} \int_{T_0} x(t) dt, \quad \text{and for } k \neq 0
\]

\[
a_k = \frac{2}{T_0} \int_{T_0} x(t) \cos(\omega_0 kt) dt, \quad b_k = \frac{2}{T_0} \int_{T_0} x(t) \sin(\omega_0 kt) dt
\]
We can also write the trigonometric Fourier series in a compact way, yielding the compact trigonometric Fourier series representation

\[ x(t) = C_0 + \sum_{k=1}^{\infty} C_k \cos (\omega_0 k t + \theta_k) \]

where \( C_0 = a_0 \), \( C_k = \sqrt{a_k^2 + b_k^2} \), \( \theta_k = \tan^{-1} \left( \frac{-b_k}{a_k} \right) + \pi \) if \( a_k < 0 \)

All the representations are equivalent (if \( x(t) \) is real) and the various coefficients are related in the following way:

\[ a_0 = D_0 = C_0 \]

\[ a_k = 2 \text{Re} \{ D_k \} = C_k \cos \theta_k, \quad b_k = -2 \text{Im} \{ D_k \} = C_k \sin \theta_k, \quad k \neq 0 \]

Also, \( D_0 = a_0 \), \( D_k = \frac{1}{2} (a_k - j b_k) \), \( D_{-k} = \frac{1}{2} (a_k + j b_k) \), \( k \neq 0 \)

\[ |D_k| = |D_{-k}| = \frac{1}{2} C_k \], \( k \neq 0 \)

\[ \chi D_k = - \chi D_{-k} = \theta_k \]