Uniform Approximation of the Distribution for the Number of Retransmissions of Bounded Documents*

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ABSTRACT

Retransmission-based failure recovery represents a primary approach in existing communication networks, on all protocol layers, that guarantees data delivery in the presence of channel failures. Contrary to the traditional belief that the number of retransmissions is geometrically distributed, a new phenomenon was discovered recently, which shows that retransmissions can cause long (-tailed) delays and instabilities even if all traffic and network characteristics are light-tailed, e.g., exponential or Gaussian. Since the preceding finding holds under the assumption that data sizes have infinite support, in this paper we investigate the practically important case of bounded data units $0 \le L_b \le b$. To this end, we provide an explicit and uniform characterization of the entire body of the retransmission distribution $\mathbb{P}[N_b > n]$ in both n and b. This rigorous approximation clearly demonstrates the previously observed transition from power law distributions in the main body to exponential tails. The accuracy of our approximation is validated with a number of simulation experiments. Furthermore, the results highlight the importance of wisely determining the size of data units in order to accommodate the performance needs in retransmission-based systems. From a broader perspective, this study applies to any other system, e.g., computing, where restart mechanisms are employed after a job processing failure.

Categories and Subject Descriptors

G.3 [**Probability And Statistics**]: Probabilistic algorithms; C.4 [**Performance of Systems**]: Performance Attributes; H.4 [**Information Systems Applications**]: Miscellaneous

General Terms

Algorithms, Performance, Theory

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Keywords

Retransmissions, restarts, channel with failures, truncated distributions, power laws, Gamma distributions, heavy-tailed distributions, light-tailed distributions.

1. INTRODUCTION

Failure recovery mechanisms are employed in almost all engineering systems since complex systems of any kind are often prone to failures. One of the most straightforward and widely used failure recovery mechanism is to simply restart the system and all of the interrupted jobs from the beginning after a failure occurs. It was first recognized in [5, 13] that such mechanisms may result in long-tailed (power law) delays even if the job sizes and failure rates are exponential. In [7], it was noted that the same mechanism is at the core of modern communication networks where retransmissions are used on all protocol layers to guarantee data delivery in the presence of channel failures. Furthermore, [7] shows that the power law number of retransmissions and delay occur whenever the hazard functions of the data and failure distributions are linearly proportional. Hence, power laws may arise even if the data and channel failure distributions are both Gaussian. In particular, retransmission phenomena can lead to zero throughput and system instabilities, and therefore need to be carefully considered for the design of fault tolerant systems.

More specifically, in communication networks, retransmissions represent the basic building blocks for failure recovery in all network protocols that guarantee data delivery in the presence of channel failures. These types of mechanisms have been employed on all networking layers, including, for example, Automatic Repeat reQuest (ARQ) protocol (e.g., see Section 2.4 of [3]) in the data link layer where a packet is resent automatically in case of an error; contention based ALOHA type protocols in the medium access control (MAC) layer that use random backoff and retransmission mechanism to recover data from collisions; end-to-end acknowledgement for multi-hop transmissions in the transport layer; HTTP downloading scheme in the application layer, etc. It has been shown that several well-known retransmission based protocols in different layers of networking architecture can lead to power law delays, e.g., ALOHA type protocols in MAC layer [8] and end-to-end acknowledgements in transport layer [6, 9] as well as in other layers [7].

Traditionally, retransmissions were thought to follow light-tailed distributions (with rapidly decaying tails), namely geometric, which requires the further assumption of independence between data (packet) sizes and transmission error probability. However, these two are often highly correlated in most communication systems, meaning that longer data units have higher probability of error, thus violating the independence assumption. Recent work [7, 8] has shown that, when the data size distribution has infinite support, all retransmission-based protocols could cause heavy-tailed behavior and possibly result in zero throughput, regardless of how light-tailed the distributions of data sizes and channel failures are. Nevertheless, in reality, packet sizes are upper bounded by the maximum transmission unit. For example, WaveLAN's maximum transfer unit is 1500 bytes. This fact motivates us to investigate the transmission of bounded data and approximate uniformly the entire body of the resulting retransmission distribution as it transits from the power law to the exponential tail.

We use the following generic channel with failures [7] to model the preceding situations. This model was first introduced (for $U_i = 0$) in [5] in a different application context. The channel dynamics is described as an on-off process $\{(A, U), (A_i, U_i)\}_{i\geq 1}$ with alternating periods when channel is available A_i and unavailable U_i, respectively; $(A, A_i)_{i\geq 1}$ and $(U, U_i)_{i\geq 1}$ are two independent sequences of i.i.d random variables. In each period of time that the channel becomes available, say A_i, we attempt to transmit the data unit of random size L_b. We focus on the situation when the data size has finite support on interval $[0, b]$. If $L_b < A_i$, we say that the transmission is successful; otherwise, we wait for the next period A_{i+1} when the channel is available and attempt to retransmit the data from the beginning. It was first recognized in [5] that this model results in power law distributions when the distributions of $L \equiv L_\infty$ and A have a matrix exponential representation, and this result was rigorously proved and further generalized in [7, 9, 2].

It was discovered in [7] that bounded data units result in truncated power law distributions for the number of retransmissions, see Example 3 in [7]. Such distributions are characterized by a power law main body and an exponentially bounded tail. However, the exponential behavior appears only for very small probabilities, often meaning that the number of retransmissions of interest may fall inside the region of the distribution that behaves as a power law. It was argued in Example 3 of [7] that the power law region will grow faster than exponential if the distributions of A and L_b are lighter than exponential. This phenomenon was further studied in [14], where partial approximations of the distribution of the number of retransmissions on the logarithmic and exact scales were provided in Theorems 1 and 3 of [14], respectively. In this paper, we present a uniform characterization of the entire body of such a distribution, both on the logarithmic as well as the exact scale.

Specifically, let N_b represent the number of retransmissions (until successful transmission) of a bounded random data unit of size $L_b \in [0, b]$ on the previously described channel. In order to study the uniform approximation in both n and b we construct a family of variables L_b, such that $\mathbb{P}[L_b \leq x] = \mathbb{P}[L \leq x]/\mathbb{P}[L \leq b]$, for $0 \leq x \leq b$ when $L = L_\infty$ is fixed. This scaling of L_b was also used in [14]. For the logarithmic scale, our result stated in Theorem 2, provides a uniform characterization of the entire body of $\log \mathbb{P}[N_b > n]$, i.e., informally

$$\log \mathbb{P}[N_b > n] \approx -\alpha \log n + n \log \mathbb{P}[A \leq b]$$

for all n and b sufficiently large. Note that the first term in the preceding approximation corresponds to the power law part $n^{-\alpha}$ of the distribution, while the second part describes the exponential (geometric $\mathbb{P}[A \leq b]^n$) tail. Hence, it may be natural to define the transition point n_b from the power law to the exponential tail as a solution to $n_b \log \mathbb{P}[A \leq b] \approx \alpha \log n_b$.

In addition, under more restrictive assumptions, we discover a new exact asymptotic formula for the retransmission distribution that works uniformly for all large n, b. Surprisingly, the approximation admits an explicit form (see Theorems 3 and 4)

$$\mathbb{P}[N_b > n] \approx \frac{\alpha}{n^\alpha \ell(n \wedge \mathbb{P}[A > b]^{-1})} \int_{-n \log \mathbb{P}[A \leq b]}^{\infty} e^{-z} z^{\alpha-1} dz, \tag{1.1}$$

where $x \wedge y = \min(x, y)$ and $\ell(\cdot)$ is a slowly varying function; note that the preceding integral is the incomplete Gamma function $\Gamma(x, \alpha)$.

Clearly, when $-n \log \mathbb{P}[A \leq b] \downarrow 0$, the preceding approximation converges to a true power law $\Gamma(\alpha + 1)/(\ell(n)n^\alpha)$. And, when $-n \log(\mathbb{P}[A < b]) \uparrow \infty$, approximation (1.1), by the property $\Gamma(x, \alpha) \approx e^{-x}x^{\alpha-1}$ as $x \to \infty$, has a geometric leading term $\mathbb{P}[A \leq b]^n$. Interestingly, when α is an integer, one can compute the exact expression for $\mathbb{P}[N_b > n]$ under more restrictive assumptions, see Proposition 2.2. Furthermore, our results show that the length of the power law region increases as the corresponding distributions of L and A assume lighter tails. All of the preceding results are validated via simulation experiments in Section 3. It is worth noting that our asymptotic approximations are in excellent agreement with the simulations.

This uniform approximation allows for a characterization of the entire body of the distribution $\mathbb{P}[N_b > n]$, so that one can explicitly estimate the region where the power law phenomenon arises. Introducing the relationship between n and $\mathbb{P}[A > b]$ also provides an assessment method of efficiency and is important for diminishing the power law effects in order to achieve high throughput. Basically, when the power law region is significant, it could lead to nearly zero throughput ($\alpha < 1$), implying that the system parameters should be more carefully adjusted in order to meet the new requirements. On the contrary, if the exponential tail dominates, the system performance is more desirable. Our analytical work could be applicable in network protocol design, possibly including packet fragmentation techniques [10, 11] and failure-recovery mechanisms. In particular, since we know the entire body of the distribution, we could approximate the mean value of N_b, and thus, via Wald's identity, the mean delay and throughput.

Also, from an engineering perspective, our results further suggest that careful re-examination and possible redesign of retransmission based protocols in communication networks might be necessary. Specifically, current engineering trends towards infrastructure-less, error-prone wireless technology encourage the study of highly variable systems with frequent failures. Hence, our results could be of potential use in improving the design of future complex and failure-prone systems.

The rest of the paper is organized as follows. After a detailed description of the channel model in the next Subsection 1.1, we present our main results in Section 2. Then, we conclude the paper with Section 3 that contains simulation examples to verify our theoretical work, which is followed by Section 4, where we include some of the technical proofs.

1.1 Description of the Channel

In this section, we formally describe our model and provide necessary definitions and notation. Consider transmitting a generic data unit of random size L_b over a channel with failures. Without loss of generality, we assume that the channel is of unit capacity. The channel dynamics is modeled as an on-off process $\{(A_i, U_i)\}_{i\geq 1}$ with alternating independent periods when channel is available A_i and unavailable U_i, respectively. In each period of time that the channel becomes available, say A_i, we attempt to transmit the data unit and, if $A_i > L_b$, we say that the transmission was successful; otherwise, we wait for the next period A_{i+1} when the channel is available and attempt to retransmit the data from the beginning. A sketch of the model depicting the system is drawn in Figure 1.

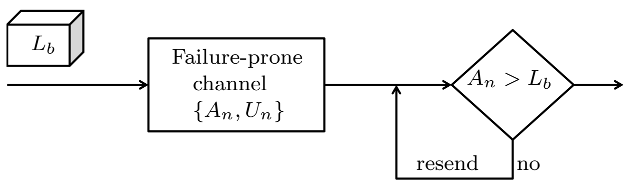

Figure 1: Packets sent over a channel with failures

Assume that $\{U, U_i\}_{i\geq 1}$ and $\{A, A_i\}_{i\geq 1}$ are two mutually independent sequences of i.i.d. random variables.

Definition 1.1 *The total number of (re)transmissions for a generic data unit of length L_b is defined as*

$$N_b \triangleq \inf\{n : A_n > L_b\}$$

We denote the complementary cumulative distribution functions for A and L, respectively, as

$$\bar{G}(x) \triangleq \mathbb{P}[A > x]$$

and

$$\bar{F}(x) \triangleq \mathbb{P}[L > x],$$

where L is a generic random variable that is used to define the distribution of L_b.

Throughout the paper we assume that $\bar{G}(x)$ and $\bar{F}(x)$ are absolutely continuous and have infinite support, i.e., $\bar{G}(x) > 0$ and $\bar{F}(x) > 0$ for all $x \geq 0$. Then, the distribution of L_b is defined as

$$\mathbb{P}[L_b \leq x] = \frac{\mathbb{P}[L \leq x]}{\mathbb{P}[L \leq b]}, \quad 0 \leq x \leq b. \tag{1.2}$$

To avoid trivialities, we always assume $\mathbb{P}[L \leq b] > 0$.

In this paper we use the following standard notations. For any two real functions $a(t)$ and $b(t)$ and fixed $t_0 \in \mathbb{R}\bigcup\{\infty\}$, we use $a(t) \sim b(t)$ as $t \to t_0$ to denote $\lim_{t\to t_0} a(t)/b(t) = 1$. Similarly, we say that $a(t) \gtrsim b(t)$ as $t \to t_0$ if $\liminf_{t\to t_0} [a(t)/b(t)] \geq 1$; $a(t) \lesssim b(t)$ has a complementary definition.

2. MAIN RESULTS

In this section, we present our main results. Under mild conditions, we first prove a general upper bound for the distribution of N_b in Proposition 2.1. Next, we show that $-\log \mathbb{P}[N_b > n]$ is actually a power law in the region where $n/\log n \leq o(1/\bar{G}(b))$, which represents the main body of the distribution, see Theorem 1. Then, in Theorem 2, we present our first main result, which under more stringent assumptions, characterizes the entire body of the distribution on the logarithmic scale uniformly for all large n and b, i.e., informally we show that

$$\log \mathbb{P}[N_b > n] \approx -\alpha \log n + n \log \mathbb{P}[A \leq b],$$

as previously mentioned in the introduction. Our results on the exact asymptotics are given in the next Subsection 2.1 in Theorems 3 and 4.

Recall that the distribution of L_b has finite support on $[0, b]$, given by (1.2). First, we prove the following general upper bound.

Proposition 2.1 *Assume that*

$$\log \mathbb{P}[L > x] \lesssim \alpha \log \mathbb{P}[A > x] \quad \text{as } x \to \infty$$

and let b_0 be such that $\mathbb{P}[L \leq b_0] > 0$, then for any $\epsilon > 0$, there exists n_0, such that, for all $n \geq n_0, b \geq b_0$,

$$\log \mathbb{P}[N_b > n] \leq (1-\epsilon)\left[n \log \mathbb{P}[A \leq b] - \alpha \log n\right].$$

PROOF. See Section 4. □

Next, we determine the region where the power law asymptotics holds on the logarithmic scale.

Theorem 1 *If*

$$\log \mathbb{P}[L > x] \sim \alpha \log \mathbb{P}[A > x] \quad \text{as } x \to \infty, \tag{2.1}$$

then, for any $\epsilon > 0$, there exist positive n_0, δ, such that for all $n \geq n_0$, $n\mathbb{P}[A > b] \leq \delta \log n$, we have

$$\left|\frac{-\log \mathbb{P}[N_b > n]}{\alpha \log n} - 1\right| \leq \epsilon. \tag{2.2}$$

PROOF. See Section 4. □

Remark 1 Note that this result extends the region of validity of Theorem 1(i) in [14] since $\delta \log n/\bar{G}(b) >> \bar{G}(b)^{-\eta}, 0 < \eta < 1$. In addition, $n/\log n \leq o(1/\bar{G}(b))$ can be shown to be the largest region where the power law asymptotics holds.

Next, one can easily characterize the logarithmic asymptotics of the very end of the exponential tail of $\mathbb{P}[N_b > n]$ for small b and large n. In particular, for fixed b, it can be shown that $\log \mathbb{P}[N_b > n] \sim n \log(1 - \bar{G}(b))$ as $n \to \infty$, see Theorem 1 in [14].

However, our main goal in this paper is to determine the entire body of the distribution of $\mathbb{P}[N_b > n]$ uniformly in n and b. To this end, in the region where $n\bar{G}(b) > \delta \log n$, we need more restrictive assumptions than in (2.1) in order to approximate $\log \mathbb{P}[N_b > n]$. The reason why this is the case is that $\mathbb{P}[N_b > n]$ behaves like a power law in the region where $n/\log n << 1/\bar{G}(b)$, while for $n/\log n >> 1/\bar{G}(b)$, it follows essentially a geometric distribution. Hence, more restrictive assumptions are required since the geometric distribution is much more sensitive to the changes in its parameters (informally, $((1+\epsilon)x)^{-\alpha} \approx x^{-\alpha}$ but $e^{-(1+\epsilon)x} \not\approx e^{-x}$).

Definition 2.1 *A function $\ell(x)$ is slowly varying if $\ell(x)/\ell(\lambda x) \to 1$ as $x \to \infty$ for any fixed $\lambda > 0$.*

We also assume that functions $\ell(x)$ are positive and bounded on finite intervals.

Theorem 2 *If $\mathbb{P}[A > x] = \ell(\mathbb{P}[L > x]^{-1})\mathbb{P}[L > x]^{1/\alpha}$, $\alpha > 0$, $x \geq 0$, $\ell(x)$ slowly varying, then for any $\epsilon > 0$, there exist n_0, b_0, such that for all $n \geq n_0, b \geq b_0$,*

$$\left|\frac{-\log \mathbb{P}[N_b > n]}{-n\log \mathbb{P}[A \leq b] + \alpha \log n} - 1\right| \leq \epsilon.$$

Remark 2 Note that the statement of this theorem can be formulated in an equivalent form

$$\left|\frac{-\log \mathbb{P}[N_b > n]}{n\mathbb{P}[A > b] + \alpha \log n} - 1\right| \leq \epsilon,$$

since $-n\log \mathbb{P}[A \leq b] \approx n\mathbb{P}[A > b]$ for large b.

Remark 3 This theorem unifies and extends Theorem 1 in [14]. In particular, it proves the result uniformly in n and b, while Theorem 1 in [14] characterized the initial power law part of the distribution ($n \leq \bar{G}(b)^{-\eta}, 0 < \eta < 1$) and the very end with exponential tail (fixed b, $n \to \infty$).

PROOF. The case where $n \leq \delta \log n/\bar{G}(b)$ is already covered by Theorem 1 since for δ sufficiently small, $\alpha \log n \leq -n\log(1-\bar{G}(b)) + \alpha \log n \leq (\alpha+\epsilon)\log n$. Hence, it remains to prove the result for $n \geq \delta \log n/\bar{G}(b)$. To this end, observe that

$$\begin{aligned}\mathbb{P}[N_b > n] &= \mathbb{E}[1-\bar{G}(L_b)]^n \\ &= \int_0^b \left(1 - \ell(\bar{F}(x)^{-1})\bar{F}(x)^{\frac{1}{\alpha}}\right)^n \frac{dF(x)}{F(b)},\end{aligned}$$

and by $F(b) \leq 1$ and the absolute continuity of $F(x)$,

$$\begin{aligned}\mathbb{P}[N_b > n] &\geq \int_{\bar{F}(b)}^1 \left(1 - \ell(z^{-1})z^{\frac{1}{\alpha}}\right)^n dz \\ &\geq \int_{\bar{F}(b)}^{\lambda\bar{F}(b)} \left(1 - \ell(z^{-1})z^{\frac{1}{\alpha}}\right)^n dz,\end{aligned}$$

where $\lambda > 1$ is chosen so that $\lambda\bar{F}(b) \leq 1$, since $\bar{F}(b) < 1$. Now, we can use the slowly varying property of $\ell(x)$ in the region $\{\bar{F}(b)^{-1}, \lambda^{-1}\bar{F}(b)^{-1}\}$ and thus, for b_0 large enough, $b \geq b_0$,

$$\mathbb{P}[N_b > n] \geq \int_{\bar{F}(b)}^{\lambda\bar{F}(b)} \left(1 - (1-\epsilon)\ell(\bar{F}(b)^{-1})z^{\frac{1}{\alpha}}\right)^n dz.$$

Then, by changing variables $u = (1-\epsilon)\ell\left(\bar{F}(b)^{-1}\right)z^{1/\alpha}$ and setting $C_{b,\epsilon}^{1/\alpha} = (1-\epsilon)\ell(\bar{F}(b)^{-1})$, we obtain

$$\begin{aligned}\mathbb{P}[N_b > n] &\geq \int_{(1-\epsilon)\ell(\bar{F}(b)^{-1})\bar{F}(b)^{\frac{1}{\alpha}}}^{\lambda^{1/\alpha}(1-\epsilon)\ell(\bar{F}(b)^{-1})\bar{F}(b)^{\frac{1}{\alpha}}} (1-u)^n \frac{\alpha u^{\alpha-1}}{C_{b,\epsilon}} du \\ &= \frac{\alpha}{C_{b,\epsilon}} \int_{(1-\epsilon)\bar{G}(b)}^{\lambda^{1/\alpha}(1-\epsilon)\bar{G}(b)} (1-u)^n u^{\alpha-1} du,\end{aligned}$$

where $\bar{G}(b) = \ell(\bar{F}(b)^{-1})\bar{F}(b)^{\frac{1}{\alpha}}$ from our main assumption.

Now, if $\alpha \geq 1$, $u^{\alpha-1}$ is monotonically increasing and thus

$$\begin{aligned}\mathbb{P}[N_b > n] &\geq \frac{\alpha(1-\epsilon)^{\alpha-1}\bar{G}(b)^{\alpha-1}}{C_{b,\epsilon}} \int_{(1-\epsilon)\bar{G}(b)}^{\lambda^{1/\alpha}(1-\epsilon)\bar{G}(b)} (1-u)^n du \\ &= \frac{\alpha(1-\epsilon)^{\alpha-1}\bar{G}(b)^{\alpha-1}}{C_{b,\epsilon}} \left.\frac{(1-u)^{n+1}}{n+1}\right|_{\lambda^{1/\alpha}(1-\epsilon)\bar{G}(b)}^{(1-\epsilon)\bar{G}(b)} \\ &= \frac{\alpha(1-\epsilon)^{\alpha-1}\bar{G}(b)^{\alpha-1}}{C_{b,\epsilon}(n+1)} \Big[\left(1-(1-\epsilon)\bar{G}(b)\right)^{n+1} \\ &\qquad - (1-\lambda^{1/\alpha}(1-\epsilon)\bar{G}(b))^{n+1}\Big] \\ &= \frac{\alpha(1-\epsilon)^{\alpha-1}\bar{G}(b)^{\alpha-1}\left(1-(1-\epsilon)\bar{G}(b)\right)^{n+1}}{C_{b,\epsilon}(n+1)} \\ &\quad \times \left(1 - \left[\frac{1-\lambda^{1/\alpha}(1-\epsilon)\bar{G}(b)}{1-(1-\epsilon)\bar{G}(b)}\right]^{n+1}\right) \\ &\geq \frac{\alpha(1-\epsilon)^{\alpha}\bar{G}(b)^{\alpha-1}}{C_{b,\epsilon}(n+1)} \left(1-(1-\epsilon)\bar{G}(b)\right)^{n+1}. \end{aligned} \tag{2.3}$$

The last inequality is implied by standard limit $(1-y/n)^n \to e^{-y}$ as $n \to \infty$, i.e.,

$$\begin{aligned}\lim_{n\to\infty}\left(\frac{1-\lambda^{1/\alpha}(1-\epsilon)\bar{G}(b)}{1-(1-\epsilon)\bar{G}(b)}\right)^{n+1} &= \lim_{n\to\infty}\frac{e^{-\lambda^{1/\alpha}(1-\epsilon)(n+1)\bar{G}(b)}}{e^{-(1-\epsilon)(n+1)\bar{G}(b)}} \\ &= \lim_{n\to\infty} e^{-(\lambda^{1/\alpha}-1)(1-\epsilon)(n+1)\bar{G}(b)} = 0,\end{aligned}$$

since $\lambda^{1/\alpha} > 1$ and the assumption $n\bar{G}(b) \geq \delta \log n$.

Then, using the standard property of slowly varying functions (see [4]) that $\ell(x) \leq Hx^{\eta}$, for any $\eta > 0$ and some sufficiently large constant H, we derive $C_{b,\epsilon}/(1-\epsilon)^{\alpha} = \ell(\bar{F}(b)^{-1})^{\alpha} \leq \sqrt{H}\bar{F}(b)^{-\epsilon(1-\alpha\epsilon)} \leq H\bar{G}(b)^{-\epsilon\alpha}$ and thus, by (2.3), obtain

$$\begin{aligned}\mathbb{P}[N_b > n] &\geq \frac{\alpha\bar{G}(b)^{\alpha-1+\epsilon\alpha}}{(n+1)H}\left(1-(1-\epsilon)\bar{G}(b)\right)^{n+1} \\ &= \frac{\alpha_H\bar{G}(b)^{\alpha-1+\epsilon\alpha}}{n+1}\left(1-(1-\epsilon)\bar{G}(b)\right)^{n+1},\end{aligned}$$

where we define $\alpha_H = \alpha/H$. Next, by taking the logarithm,

$$\begin{aligned}\log \mathbb{P}[N_b > n] \geq\ & \log \alpha_H + (\alpha(1+\epsilon)-1)\log \bar{G}(b) \\ & + (n+1)\log(1-(1-\epsilon)\bar{G}(b)) - \log(n+1) \\ \geq\ & \log \alpha_H - (\alpha(1+\epsilon)-1)\log n \\ & + (\alpha(1+\epsilon)-1)\log(\delta\log n) \\ & + (n+1)\log(1-(1-\epsilon)\bar{G}(b)) - \log(1+\epsilon)n,\end{aligned}$$

where in the last inequality we used $\log \bar{G}(b) \geq \log(\delta \log n) - \log n$. Then, we can pick n_0 such that, for all $n \geq n_0$, $\log(\delta\log n) \geq (\log(1+\epsilon) - \log\alpha_H)/(\alpha(1+\epsilon)-1)$. Hence,

$$\begin{aligned}\log \mathbb{P}[N_b > n] &\geq -\alpha(1+\epsilon)\log n + (n+1)\log(1-(1-\epsilon)\bar{G}(b)) \\ &\geq -\alpha(1+\epsilon)\log n + n(1+\epsilon)\log(1-\bar{G}(b)) \\ &= -(1+\epsilon)\left(\alpha\log n - n\log(1-\bar{G}(b))\right).\end{aligned}$$

Finally, dividing by $(\alpha\log n - n\log(1-\bar{G}(b))) > 0$ yields

$$\frac{\log \mathbb{P}[N_b > n]}{\alpha\log n - n\log(1-\bar{G}(b))} \geq -(1+\epsilon)$$

Symmetric arguments hold for the case where $\alpha < 1$. We omit the details. $\square$

2.1 Exact Asymptotics

In this section, under more restrictive assumptions, we derive the exact approximation for $\mathbb{P}[N_b > n]$ that works uniformly for all n, b sufficiently large (Theorems 3 and 4). Although some of the assumptions can be relaxed, we refrain from such generalizations here in order to gain analytical tractability. Interestingly enough, this characterization is explicit in that

$$\mathbb{P}[N_b > n] \approx \frac{\alpha}{n^\alpha \ell(n \wedge \mathbb{P}[A > b]^{-1})} \int_{-n \log \mathbb{P}[A \le b]}^{\infty} e^{-z} z^{\alpha-1} dz, \tag{2.4}$$

where $x \wedge y = \min(x, y)$ and $\ell(\cdot)$ is slowly varying. Implicitly, the argument of $\ell(x)$ is altered depending on whether $n\mathbb{P}[A > b] \le C$ or $n\mathbb{P}[A > b] > C$ for some constant C. Hence, we can choose $C = 1$ since $\ell(n \wedge 1/\mathbb{P}[A > b]) \approx \ell(n \wedge C/\mathbb{P}[A > b])$ for large n, b. Note that when $-n \log \mathbb{P}[A \le b] \downarrow 0$, the power law dominates, whereas when $-n \log \mathbb{P}[A \le b] \to \infty$, the integral determines the tail with the geometric (exponential) leading term.

We would like to point out that approximation (2.4) actually works well when $\mathbb{P}[A > b]^{-1}$ is large rather than simply b; this can be concluded by examining the proofs of the theorems in this section. Hence, formula (2.4) can be accurate for relatively small values of b provided that A is light-tailed. This may be the reason why we obtain such accurate results in our simulation examples in Section 3 for $b \le 10$.

First, in Theorem 3, we precisely describe the region where the distribution of N_b exhibits the power law behavior, $n\mathbb{P}[A > b] \le C$, for any fixed constant C. Then, Theorem 4 covers the remaining region, $n\mathbb{P}[A > b] > C$, where $\mathbb{P}[N_b > n]$ approaches the geometric tail. Proposition 2.3 gives an explicit exact asymptote for the exponential tail with b possibly small and $n \to \infty$.

Theorem 3 *Let $\mathbb{P}[L > x]^{-1} = \ell(\mathbb{P}[A > x]^{-1})\mathbb{P}[A > x]^{-\alpha}$, $\alpha > 0$, $x \ge 0$, and $C > 0$ be a fixed constant. Then, for any $\epsilon > 0$, there exists n_0 such that for all $n \ge n_0$, and $n\mathbb{P}[A > b] \le C$,*

$$\left| \frac{\mathbb{P}[N_b > n] n^\alpha \ell(n)}{\alpha \Gamma(-n \log \mathbb{P}[A \le b], \alpha)} - 1 \right| \le \epsilon, \tag{2.5}$$

where $\Gamma(x, \alpha)$ is the incomplete Gamma function defined as $\int_x^\infty e^{-z} z^{\alpha-1} dz$.

Proof. This result can be proved using similar techniques as in [9]. We omit the details. □

Remark 4 Related result was derived in Theorem 3 of [14] where it was required that $n \le \bar{G}(b)^{-\eta}, 0 < \eta < 1$. Note that here we broaden the region where the result holds by requiring $n \le C/\bar{G}(b)$, which is larger than $n \le \bar{G}(b)^{-\eta}$. Furthermore, this is the largest region where the exact power law asymptotics $O(n^{-\alpha}/\ell(n))$ holds since for $n\bar{G}(b) > C$, $\Gamma(n\bar{G}(b), \alpha) \le \Gamma(C, \alpha) \to 0$ as $C \to \infty$.

Remark 5 Note here that the incomplete Gamma function $\Gamma(\alpha, x) = \int_x^\infty z^{\alpha-1} e^{-z} dz$ can be easily computed using the well known asymptotic approximation (see Sections 6.5.32 in [1]), as $x \to \infty$,

$$\Gamma(\alpha, x) \sim x^{\alpha-1} e^{-x} \left[1 + \frac{\alpha - 1}{x} + \frac{(\alpha-1)(\alpha-2)}{x^2} + \dots \right].$$

Now, we characterize the remaining region where $n\mathbb{P}[A > b] > C$. Informally speaking, this is the region where $\mathbb{P}[N_b > n]$ has a lighter tail converging to the exponential when $n >> \bar{G}(b)^{-1}$. In the following theorem, we need more restrictive assumptions for $\ell(x)$; see the discussion before Theorem 2. In particular, we assume that $\ell(x)$ is slowly varying, eventually differentiable and monotonic.

Theorem 4 *Assume that $\mathbb{P}[L > x]^{-1} = \ell(\mathbb{P}[A > x]^{-1})\mathbb{P}[A > x]^{-\alpha}, \alpha > 0, x \ge 0$, where $\ell(x)$ is such that $\ell(x \log x)/\ell(x) \to 1$, as $x \to \infty$ and let $C > \alpha > 0$ be a fixed constant. Then, for any $\epsilon > 0$, there exist b_0, n_0, such that for all $n \ge n_0, b \ge b_0, n\mathbb{P}[A > b] > C$,*

$$\left| \frac{\mathbb{P}[N_b > n] n^\alpha \ell(\mathbb{P}[A > b]^{-1})}{\alpha \Gamma(-n \log \mathbb{P}[A \le b], \alpha)} - 1 \right| \le \epsilon. \tag{2.6}$$

Remark 6 Note that many slowly varying functions satisfy the condition $\ell(x \log x)/\ell(x) \to 1$, as $x \to \infty$. For example, it is easy to check that $(\log x)^\beta, (\log\log x)^\beta, \beta > 0$, and $e^{(\log x)^\gamma}, 0 < \gamma < 1$, satisfy the preceding condition.

Remark 7 Observe that Theorems 3 and 4 cover the entire distribution $\mathbb{P}[N_b > n]$ for all large n and b. Interestingly, the formula for the approximation is the same except for the argument of the slowly varying part, which equals to n and $\mathbb{P}[A > b]^{-1}$, respectively. Furthermore, when $n\mathbb{P}[A > b] = C$ the formulas are asymptotically identical as $\ell(n) \sim \ell(\mathbb{P}[A > b]^{-1})$ as $n \to \infty$ and $n\mathbb{P}[A > b] = C$.

Proof. Recall that

$$\begin{aligned} \mathbb{P}[N_b > n] &= \mathbb{E}[1 - \bar{G}(L_b)]^n \\ &= \int_0^b \left(1 - \bar{G}(x)\right)^n \frac{dF(x)}{F(b)} \\ &= \int_0^{x_0} \left(1 - \bar{G}(x)\right)^n \frac{dF(x)}{F(b)} \\ &\quad + \int_{x_0}^b \left(1 - \bar{G}(x)\right)^n \frac{dF(x)}{F(b)}. \end{aligned} \tag{2.7}$$

Next, without loss of generality, we assume that $\ell(x)$ is eventually non-decreasing. Now, from the fact that $\ell(x)$ is eventually differentiable ($x \ge x_0$) and slowly varying, it is easy to show that $\ell'(x)x/\ell(x) \to 0$ as $x \to \infty$. Hence, using the preceding observation, it follows that $d\bar{F}(x) = (1 + o(1))\alpha\bar{G}(x)^{\alpha-1}\ell^{-1}(1/\bar{G}(x))d\bar{G}(x)$ as $x \to \infty$. Thus, for the upper bound we have

$$\begin{aligned} &\mathbb{P}[N_b > n] \\ &\le \left(1 - \bar{G}(x_0)\right)^n - (1+\epsilon)^{1/2} \int_{x_0}^b (1 - \bar{G}(x))^n \frac{\alpha \bar{G}(x)^{\alpha-1} d\bar{G}(x)}{\ell(1/\bar{G}(x))F(b)} \\ &= \left(1 - \bar{G}(x_0)\right)^n + (1+\epsilon)^{1/2} \int_{\bar{G}(b)}^{\bar{G}(x_0)} (1 - z)^n \frac{\alpha z^{\alpha-1} dz}{\ell(1/z)F(b)} \\ &\le \left(1 - \bar{G}(x_0)\right)^n + \frac{\alpha(1+\epsilon)^{1/2}}{F(b)\ell(1/\bar{G}(x_0))} \int_{\bar{G}(b)}^1 (1 - z)^n z^{\alpha-1} dz, \end{aligned} \tag{2.8}$$

where the equality follows from the absolute continuity of $G(x)$ and change of variables, whereas the last inequality follows from the monotonicity of $\ell(x)$. Next, by changing the variables $z = 1 - e^{-u/n}$, we compute the integral in (2.8)

$$\int_{-n\log\left(1-\bar{G}(b)\right)}^{\infty} \frac{e^{-u(n+1)/n}(1-e^{-u/n})^{\alpha-1}}{n} du \leq \int_{-n\log\left(1-\bar{G}(b)\right)}^{\infty} \frac{e^{-u}(1-e^{-u/n})^{\alpha-1}}{n} du,$$

where for the inequality we use $e^{-u/n} \leq 1$. Thus, for $\alpha \geq 1$, from the preceding expression using the inequality $1-e^{-x} \leq x$, for $x \geq 0$, and since $\ell(x\log x)/\ell(x) \to 1$, we obtain the upper bound for (2.8)

$$\begin{aligned}\mathbb{P}[N_b > n] &\leq \left(1-\bar{G}(x_0)\right)^n + \frac{\alpha(1+\epsilon)}{F(b)n\ell(1/\bar{G}(b))} \int_{-n\log\left(1-\bar{G}(b)\right)}^{\infty} e^{-u}\left(\frac{u}{n}\right)^{\alpha-1} du \\ &= \left(1-\bar{G}(x_0)\right)^n + \frac{\alpha(1+\epsilon)}{F(b)n^{\alpha}\ell(1/\bar{G}(b))} \int_{-n\log\left(1-\bar{G}(b)\right)}^{\infty} e^{-u}u^{\alpha-1} du \\ &\leq e^{-n\bar{G}(x_0)} + \frac{\alpha(1+\epsilon)}{F(b)n^{\alpha}\ell(1/\bar{G}(b))}\Gamma(-n\log\left(1-\bar{G}(b)\right), \alpha).\end{aligned}$$

Now, from the continuity of $\bar{G}(x)$, we can pick x_0 such that $\bar{G}(x_0) = -\bar{G}(b)\log\bar{G}(b)$ and observe that

$$\frac{\mathbb{P}[N_b > n]n^{\alpha}\ell(1/\bar{G}(b))}{\alpha\Gamma(-n\log(1-\bar{G}(b)), \alpha)} \leq \frac{e^{-n\bar{G}(x_0)}n^{\alpha}\ell(1/\bar{G}(b))}{\alpha\Gamma(-n\log(1-\bar{G}(b)), \alpha)} + \frac{1}{F(b)}.$$

Next, we show that the first expression on the right is negligible. Observe that $\Gamma(-n\log(1-\bar{G}(b)), \alpha) \geq \Gamma((1-\delta)n\bar{G}(b), \alpha)$ for b large enough. Then, using the asymptotics $\Gamma(u, \alpha) \sim u^{\alpha-1}e^{-u}$ as $u \to \infty$, we have

$$\sup_{u>C} \frac{u^{\alpha-1}e^{-u(1-\delta)}}{\Gamma((1-\delta)u, \alpha)} < \infty.$$

From the preceding bounds and letting $u = n\bar{G}(b)$, we have

$$\frac{\mathbb{P}[N_b > n]n^{\alpha}\ell(1/\bar{G}(b))}{\alpha\Gamma(-n\log\left(1-\bar{G}(b)\right), \alpha)} \leq O\left(\frac{e^{-u(\log(1/\bar{G}(b))-1)}n^{\alpha}}{(n\bar{G}(b))^{\alpha-1}\bar{G}(b)^{\delta}}\right) + \frac{1}{F(b)}, \tag{2.9}$$

where we use the properties of the slowly varying functions [see Theorem 1.5.6 in [4]], $\ell(x) \geq x^{-\delta}$, for x large and $\delta > 0$, to upper bound $\ell(1/\bar{G}(b))$. Now, we pick b_0 large enough such that $\delta\log(\bar{G}(b)^{-1}) - 1 = \delta_{\epsilon,\beta} > 0$. Therefore, the preceding expression inside $O(\cdot)$ in (2.9) is upper bounded by

$$\begin{aligned}&(1+\epsilon)\frac{e^{-u\delta_{\epsilon,\beta}}\bar{G}(b)^{u(1-\delta)}n}{\bar{G}(b)^{\alpha-1+\delta}} \\ &= (1+\epsilon)ue^{-u\delta_{\epsilon,\beta}}\bar{G}(b)^{u(1-\delta)-\alpha-\delta} \\ &\leq \sup_{u\geq 0} ue^{-u\delta_{\epsilon,\beta}}(1+\epsilon)\bar{G}(b)^{C(1-\delta)-\alpha-\delta},\end{aligned}$$

where we are using our assumption that $u > C$. Also, since $C > \alpha$, we can pick $\delta = (C-\alpha)/\left(2(C+1)\right)$. And thus, for b_0 large enough, $O(\sup_{u\geq 0} ue^{-u\delta_{\epsilon,\beta}}(1+\epsilon)\bar{G}(b)^{(C-\alpha)/2}) \leq \epsilon$. Hence,

$$\frac{\mathbb{P}[N_b > n]n^{\alpha}\ell(1/\bar{G}(b))}{\alpha\Gamma(-n\log\left(1-\bar{G}(b)\right), \alpha)} \leq \epsilon + (1+\epsilon) = 1 + 2\epsilon,$$

which completes the proof after replacing ϵ with $\epsilon/2$.

Next, we prove the lower bound starting from (2.7). Proceeding with similar arguments as in the proof for the upper bound, we obtain

$$\begin{aligned}&\mathbb{P}[N_b > n] \\ &\geq -(1-\epsilon)^{1/2}\int_{x_0}^{b}(1-\bar{G}(x))^n \frac{\alpha\bar{G}(x)^{\alpha-1}d\bar{G}(x)}{\ell(1/\bar{G}(x))F(b)} \\ &= (1-\epsilon)^{1/2}\int_{\bar{G}(b)}^{\bar{G}(x_0)}(1-z)^n\frac{\alpha z^{\alpha-1}dz}{\ell(1/z)F(b)} \\ &\geq \frac{\alpha(1-\epsilon)^{1/2}}{F(b)\ell(1/\bar{G}(b))}\int_{-(n+1)\log\left(1-\bar{G}(b)\right)}^{-(n+1)\log G(x_0)} \frac{e^{-u}(1-e^{-\frac{u}{n+1}})^{\alpha-1}}{n+1}du,\end{aligned}$$

where we use the monotonicity of $\ell(x)$. Next, using the inequality $(1-e^{-x}) \geq (1-\delta)x$, for some $\delta > 0$ and all $x \geq 0$ small enough, we have

$$\begin{aligned}&\mathbb{P}[N_b > n-1] \\ &\geq \frac{\alpha(1-\epsilon)^{1/2}(1-\delta)^{\alpha-1}}{F(b)\ell(1/\bar{G}(b))n}\int_{-n\log(1-\bar{G}(b))}^{-n\log G(x_0)} e^{-u}\left(\frac{u}{n}\right)^{\alpha-1}du \\ &= \frac{\alpha(1-\epsilon)}{F(b)n^{\alpha}\ell(1/\bar{G}(b))}\int_{-n\log\left(1-\bar{G}(b)\right)}^{-n\log G(x_0)} e^{-u}u^{\alpha-1}du \\ &= \frac{\alpha(1-\epsilon)}{F(b)n^{\alpha}\ell(1/\bar{G}(b))}\left[\int_{-n\log\left(1-\bar{G}(b)\right)}^{\infty} e^{-u}u^{\alpha-1}du \right. \\ &\qquad \left. - \int_{-n\log G(x_0)}^{\infty} e^{-u}u^{\alpha-1}du\right]\end{aligned}$$

where we set $(1-\delta)^{\alpha-1} = (1-\epsilon)^{1/2}$. Then, by continuity, we pick x_0 such that $\Gamma(-n\log G(x_0), \alpha) \leq \epsilon\Gamma(-n\log G(b), \alpha)$ and thus

$$\frac{\mathbb{P}[N_b > n]n^{\alpha}\ell(1/\bar{G}(b))}{\alpha\Gamma(-n\log\left(1-\bar{G}(b)\right), \alpha)} \geq (1-\epsilon).$$

Now, if $\alpha < 1$, the lower bound follows similarly as the preceding upper bound while for the upper bound we use symmetric arguments. □

From the preceding two theorems we observe that $\mathbb{P}[N_b > n]$ behaves as a true power law when $n\mathbb{P}[A > b] \to 0$ ($n << \mathbb{P}[A > b]^{-1}$) and has an exponential tail (geometric) when $n\mathbb{P}[A > b] \to \infty$ ($n >> \mathbb{P}[A > b]^{-1}$). More specifically:

(i) If $n\mathbb{P}[A > b] \to 0$, then clearly $-n\log\mathbb{P}[A \leq b] \to 0$ and we have by Theorem 3, as $n \to \infty, n\mathbb{P}[A > b] \to 0$,

$$\mathbb{P}[N_b > n] \sim \frac{\alpha}{\ell(n)n^{\alpha}}\Gamma(\alpha) = \frac{\Gamma(\alpha+1)}{\ell(n)n^{\alpha}}.$$

(ii) If $n\mathbb{P}[A > b] \to \infty$, then $-n\log\mathbb{P}[A \leq b] \to \infty$ and thus, as $n \to \infty, b \to \infty, n\mathbb{P}[A > b] \to \infty$,

$$\mathbb{P}[N_b > n] \sim \frac{\alpha}{\ell(1/\bar{G}(b))n}\bar{G}(b)^{\alpha-1}(1-\bar{G}(b))^n,$$

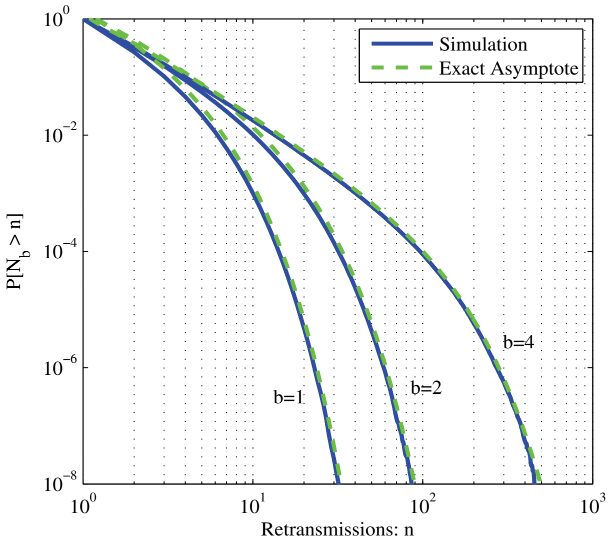

Figure 2: *Example 1(a). Exact asymptotics for $\alpha > 1$.*

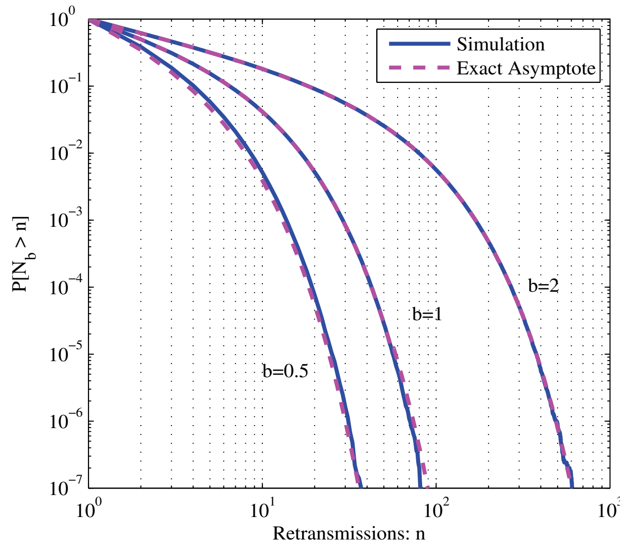

Figure 3: *Example 1(b). Exact asymptotics for $\alpha < 1$.*

which follows from Theorem 4 and the asymptotic expansion of the Gamma function (see Remark 5 of Theorem 3).

Interestingly, one can compute the distribution of $\mathbb{P}[N_b > n]$ exactly when the parameter α takes integer values.

Proposition 2.2 *If $\mathbb{P}[L > x] = \mathbb{P}[A > x]^{\alpha}$, for all $x \geq 0$ and α is a positive integer, then*

$$\mathbb{P}[N_b > n] = \frac{1}{\mathbb{P}[L \leq b]} \sum_{i=1}^{\alpha} \frac{\alpha!\, n!\, \mathbb{P}[A > b]^{\alpha - i}}{(\alpha - i)!(n+i)!} \mathbb{P}[A \leq b]^{n+i}.$$

PROOF. It follows directly from (2.7) using integration by parts. □

Finally, in the following proposition, we describe the tail of $\mathbb{P}[N_b > n]$ for fixed and possibly small b. Furthermore, the assumptions of this proposition could be weakened at the expense of additional technical complications, which we avoid here for reasons of simplicity.

Proposition 2.3 *Let b be fixed. If $\mathbb{P}[L > x] = \mathbb{P}[A > x]^{\alpha}$, $\alpha > 0$, $x \geq 0$, then*

$$\mathbb{P}[N_b > n] \sim \frac{\alpha}{\mathbb{P}[L \leq b]} \frac{\mathbb{P}[A > b]^{\alpha-1}\mathbb{P}[A \leq b]^{n+1}}{n+1} \quad \text{as } n \to \infty.$$

PROOF. See Section 4. □

3. SIMULATION EXPERIMENTS

In this section, we illustrate the validity of our theoretical results with simulation experiments. In all of the experiments, we observed that our exact asymptotics is literally indistinguishable from the simulation. In the following examples, we present the simulation experiments resulting from 10^8 (or more) independent samples of $N_{b,i}, 1 \leq i \leq 10^8$. This number of samples was needed to ensure at least 100 independent occurrences in the lightest end of the tail that is presented in the figures ($N_{b,i} \geq n_{\max}$), thus providing a good confidence interval.

Example 1. This example illustrates the exact asymptotics presented in Theorems 3 and 4, i.e., approximation (2.4), which combines the results from both theorems. We assume that L and A follow exponential distributions with parameters $\lambda = 2$ and $\mu = 1$, respectively. It is thus clear that $\bar{F}(x) = e^{-2x} = \bar{G}(x)^{\alpha}$, where $\alpha = 2$ and $\ell(x) \equiv 1$. Now, approximation (2.4) states that $\mathbb{P}[N_b > n]$ is given by $(1 - e^{-2b})^{-1} 2n^{-2}\Gamma(ne^{-b}, 2)$. Note that we added a factor $\mathbb{P}[L \leq b]^{-1} = (1 - e^{-2b})^{-1}$, as in Propositions 2.2 and 2.3, for increased precision when b is small; we add such a factor to approximation (2.4) in other examples as well. We simulate different scenarios when the data sizes L_b are upper bounded by b equal to 1, 2 and 4. The simulation results are plotted on log-log scale in Fig. 2.

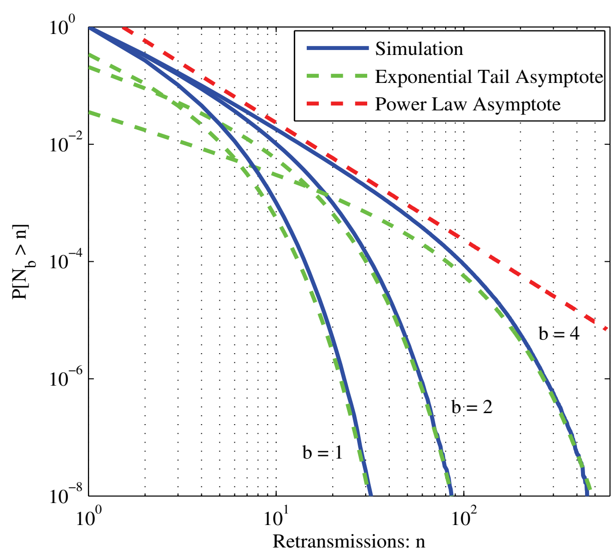

Figure 4: *Example 2. Exponential tail asymptotics.*

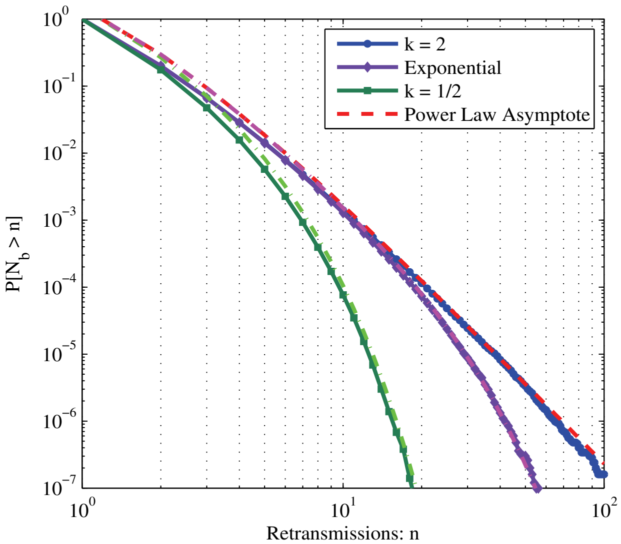

Figure 5: *Example 3. Power law region increases for lighter tails of L, A.*

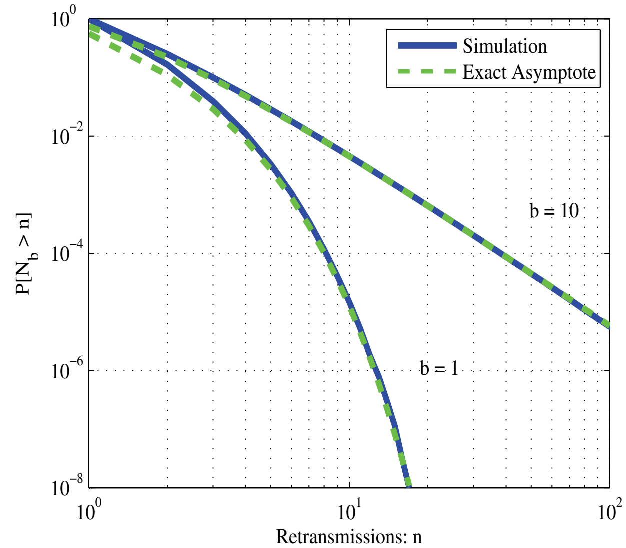

Figure 6: *Example 4. Exact asymptotics where $\mathbb{E}A > \mathbb{E}L_b$.*

From Fig. 2, we observe that the numerical asymptote approximates the simulation exactly for all different scenarios, even for very small values of n (large probabilities). We further validate our approximation by considering scenarios where L, A are exponentially distributed but $\alpha < 1$; in fact, this case tends to induce longer delays due to larger average data size compared to the channel availability periods. In this case, we obtain $\alpha = 0.5$ by assuming $\lambda = 1$ and $\mu = 2$. Again, the simulation results and the asymptotic formulas are basically indistinguishable for all n, as illustrated in Fig. 3.

For both cases, we deduce that for b small the power law asymptotics covers a smaller region of the distribution of N_b and, as n increases, the exponential tail becomes more evident and eventually dominates. As b becomes large - recall that $b \to \infty$ corresponds to the untruncated case where the power law phenomenon arises - the exponential tail becomes less distinguishable.

Example 2. This example demonstrates the exact asymptotics for the exponential tail as $n \to \infty$ and b is fixed, as in Proposition 2.3. Note that this proposition gives the exact asymptotic formula for the region $n \gg 1/\bar{G}(b)$ and lends merit to our Theorems 2 and 4. Informally, we could say that a point n_b such that $-n_b \log(1 - \bar{G}(b)) \approx n_b\bar{G}(b) = \alpha \log n_b$ represents the transition from power law to the exponential tail. We assume that L, A are exponentially distributed with $\lambda = 2$ and $\mu = 1$ (as in the first case of Example 1). Roughly speaking, we can see from Fig. 4 that the exponential asymptote appears to fit well starting from $n_b \approx \alpha e^b$, i.e., $n_b \approx 6, 15, 100$ for $b = 1, 2, 4$, respectively.

Example 3. This example highlights the importance of the distribution type of channel availability periods $\bar{G}(x) = \mathbb{P}[A > x]$. We consider some fixed b, namely $b = 8$ and assume that the matching between data sizes and channel availability, as defined in Theorems 3 and 4, is determined by the parameter $\alpha = 4$. We assume Weibull[1] distributions for L, A with the same index k and μ_L, μ_A respectively, such that $\alpha = (\mu_A/\mu_L)^k$. The simulations include three different cases for the aforementioned distributions: Weibull with index $k = 1$ (exponential) where $\mu_L = 1$ and $\mu_A = 4$, Weibull (normal-like) with index $k = 2$ ($\mu_L = 1, \mu_A = 2$) and Weibull with $k = 1/2$ ($\mu_L = 1, \mu_A = 16$). Fig. 5 illustrates the exact asymptotics from equation (2.4), shown with the lighter dashed lines; the main power law asymptote appears in the main body of all three distributions. We observe that heavier distributions (Weibull with $k = 1/2$) correspond to smaller regions for the power law main body of the distribution $\mathbb{P}[N_b > n]$. On the other hand, the case with the lighter Gaussian like distributions for $k = 2$ follows almost entirely the power law asymptotics in the region presented in Fig. 5. This increase in the power law region can be inferred from our theorems, which show that the transition from the power law main body to the exponential tail occurs roughly at $n_b \approx \bar{G}(b)^{-1}$. Hence, the lighter the tail of the distribution of A, the larger the size of the power law region.

Example 4. In this example, we deal with the case where the system availability periods are longer than the mean size of the data. This is true for a system that does not fail frequently and the packet sizes are small on average, where one would expect high throughput and lighter distributions for the retransmissions. However, our simulations demonstrate the dominant power law body when b is larger than the mean data size, i.e., we allow longer, albeit infrequent, packets to be transmitted over the channel. In this scenario, L, A are both exponentially distributed with $\lambda = 5$ and $\mu = 1$, yielding $\mathbb{E}L_b \approx 0.2 << \mathbb{E}A = 1$. Fig. 6 indicates the emergence of the power law in the main body of the distribution when the value of b is larger, namely $b = 10$. On the other hand, when $b = 1$, the exponential tail dominates and the power law basically disappears. From Fig. 6, we can

[1]In general, a Weibull distribution with index k has a complementary cumulative distribution function $\mathbb{P}[X > x] = e^{-(x/\mu)^k}$, where μ is the parameter that determines the mean.

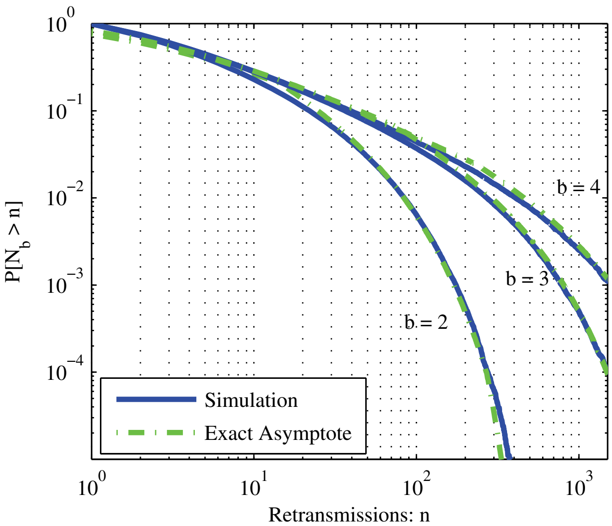

Figure 7: *Example 5(a). Uniform approximation from (2.4) for the case where L follows the Gamma distribution.*

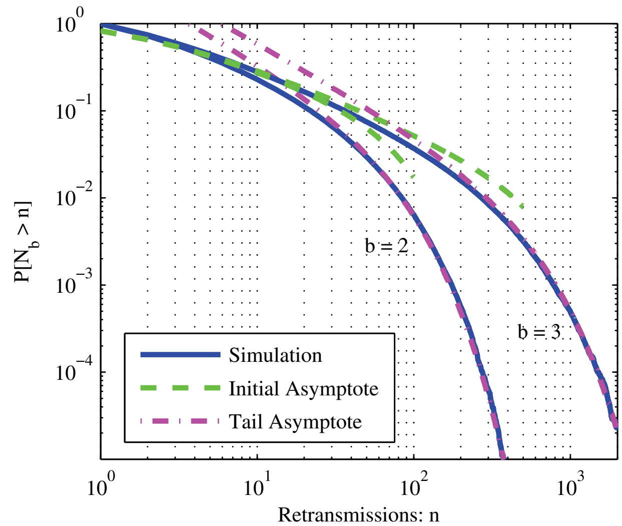

Figure 8: *Example 5(b). The asymptotes from Theorems 3 and 4 for the case where L follows the Gamma distribution.*

verify that, for even a small number of retransmissions, e.g., $n = 10$, $\mathbb{P}[N_b > 10]$ increases from 10^{-5} to roughly 10^{-2} - an almost 1000-fold increase - as b grows from 1 to 10. This dramatic change of the distribution of N_b for relatively modest increase of the maximum data size b emphasizes the importance of carefully adjusting the data characteristics (e.g., fragmentation) to the channel statistics.

Example 5. In this last example, we study the case where there is a more general functional relationship between the distributions of availability periods A and data sizes L, as Theorems 3 and 4 assume. In particular, we consider the case $\bar{F}(x) = \bar{G}(x)^\alpha/\ell(\bar{G}(x)^{-1})$, where $\ell(x)$ is slowly varying. We validate the approximation (2.4) in this more general setting.

In particular, the availability periods A are exponentially distributed with parameter μ while the data sizes L follow the Gamma distribution with parameters (λ, k); the tail of the Gamma distribution function is defined as $\lambda^k\Gamma(k)^{-1} \int_x^\infty e^{-\lambda x}x^{k-1}dx = \Gamma(\lambda x, k)/\Gamma(k)$ and, therefore, the tail distribution of L can be approximated by $\bar{F}(x) \sim (\lambda^{k-1}/\Gamma(k))\, x^{k-1}e^{-\lambda x}$ for large x. We can easily verify that $\bar{F}(x) = f(\mu^{-1}\log\bar{G}(x)^{-1})\bar{G}(x)^\alpha$, where $\alpha = \lambda/\mu$ and

$$f(x) = \lambda^{k-1}\Gamma(k)^{-1}\int_0^\infty e^{-z}(z/\lambda + x)^{k-1}dz.$$

Hence, the slowly varying function in Theorems 3 and 4 is $\ell(x) = 1/f(\mu^{-1}\log x)$. Also, from the preceding integral representation for $f(x)$, it can be easily shown that $\ell(x) \approx \Gamma(k)\alpha^{1-k}\log^{1-k}x$, which is indeed slowly varying, and $\bar{F}(x) \approx (\alpha^{k-1}/\Gamma(k))\log(\bar{G}(x)^{-1})^{k-1}\bar{G}(x)^\alpha$. We take $\lambda = 2, k = 2$ and $\mu = 2$ and run simulations for $b = \{2, 3, 4\}$. In Fig. 7, we demonstrate the results using the approximation (2.4). Surprisingly, our analytic approximation works nicely even for small values of n and b although the conditions in our theorems require n and b to be large.

In Fig. 8, we elaborate on the preceding example. To this end, we plot two asymptotes: (i) the 'Initial Asymptote' corresponding to the power law asymptote provided by Theorem 3 and (ii) the 'Tail Asymptote' from Theorem 4. Combining the two, we derive the approximation (2.4), as we have already shown in Fig. 7. Hereby, we see from Fig. 8 that both asymptotes are needed to approximate the entire distribution well, i.e., the 'Initial Asymptote' fits well the first part of the distribution, whereas the 'Tail Asymptote' is inaccurate in the beginning but works well for the tail. Recall that these two asymptotes differ only in the argument of the slowly varying function $\ell(\cdot)$, which is equal to n for the 'Initial Asymptote' and $\bar{G}(b)^{-1}$ for the tail.

4. PROOFS

In this section, we present the proofs of Propositions 2.1, 2.3 and Theorem 1.

PROOF. Proof of Proposition 2.1.
By assumption, there exists $0 < \epsilon < 1$ such that for all $x > x_\epsilon$

$$\bar{F}(x) \le \bar{G}(x)^{\alpha(1-\epsilon)}. \tag{4.1}$$

Next, since $\mathbb{P}[N_b > n|L_b] = (1 - \bar{G}(L_b))^n$,

$$\begin{aligned}
\mathbb{P}[N_b > n] &= \mathbb{E}[1 - \bar{G}(L_b)]^n \\
&= \mathbb{E}[1 - \bar{G}(L_b)]^{n(1-\epsilon+\epsilon)} \\
&= (1 - \bar{G}(b))^{n(1-\epsilon)}\Big[\mathbb{E}[1 - \bar{G}(L_b)]^{n\epsilon}1(L_b \le x_0) \\
&\qquad + \mathbb{E}[1 - \bar{G}(L_b)]^{n\epsilon}1(L_b > x_0)\Big] \\
&\le (1 - \bar{G}(b))^{n(1-\epsilon)}\Big[(1 - \bar{G}(x_0))^{n\epsilon} \\
&\qquad + \int_{x_0}^b \left(1 - \bar{G}(x)\right)^{n\epsilon}\frac{dF(x)}{F(b)}\Big] \\
&\le (1 - \bar{G}(b))^{n(1-\epsilon)} \\
&\qquad \times \left[\eta_{x_0}^{n\epsilon} + \int_0^b \left(1 - \bar{F}(x)^{\frac{1}{\alpha(1-\epsilon)}}\right)^{n\epsilon}\frac{dF(x)}{F(b)}\right],
\end{aligned}$$

where $\eta_{x_0} = 1 - \bar{G}(x_0)$ and the last inequality follows from (4.1).

Now, by extending the preceding integral to ∞, we obtain

$$\begin{aligned}\mathbb{P}[N_b > n] &\le \frac{1}{F(b)}(1-\bar{G}(b))^{n(1-\epsilon)} \Bigg[\eta_{x_0}^{n\epsilon} F(b) \\ &\qquad + \int_0^\infty \left(1 - \bar{F}(x)^{\frac{1}{\alpha(1-\epsilon)}}\right)^{n\epsilon} dF(x)\Bigg] \\ &= \frac{1}{F(b)}(1-\bar{G}(b))^{n(1-\epsilon)} \\ &\quad \times \left[\eta_{x_0}^{n\epsilon} F(b) + \mathbb{E}\left(1 - \bar{F}(L)^{\frac{1}{\alpha(1-\epsilon)}}\right)^{n\epsilon}\right] \\ &\le \frac{1}{F(b)}(1-\bar{G}(b))^{n(1-\epsilon)} \\ &\quad \times \left[\eta_{x_0}^{n\epsilon} F(b) + \mathbb{E}\, e^{-n\epsilon \bar{F}(L)^{\frac{1}{\alpha(1-\epsilon)}}}\right],\end{aligned}$$

where we use the elementary inequality $1 - x \le e^{-x}$, $x \ge 0$, and thus

$$\mathbb{P}[N_b > n] \le \frac{(1-\bar{G}(b))^{n(1-\epsilon)}}{F(b)} \left[\eta_{x_0}^{n\epsilon} F(b) + \mathbb{E}\, e^{-n\epsilon U^{\frac{1}{\alpha(1-\epsilon)}}}\right], \tag{4.2}$$

where $\bar{F}(L) = U$ is uniformly distributed on $[0,1]$ by Proposition 2.1 in Chapter 10 of [12].

Next, we upper bound the expectation in the preceding expression by

$$\begin{aligned}\mathbb{E}\, e^{-n\epsilon U^{\frac{1}{\alpha(1-\epsilon)}}} &= \int_0^1 e^{-x^{\frac{1}{\alpha(1-\epsilon)}} n\epsilon} dx \\ &= \int_0^{n\epsilon} \frac{\alpha(1-\epsilon)}{(n\epsilon)^{\alpha(1-\epsilon)}} e^{-z} z^{\alpha(1-\epsilon)-1} dz \\ &\le \frac{\alpha(1-\epsilon)}{(n\epsilon)^{\alpha(1-\epsilon)}} \int_0^\infty e^{-z} z^{\alpha(1-\epsilon)-1} dz \\ &= \frac{\alpha(1-\epsilon)}{(n\epsilon)^{\alpha(1-\epsilon)}} \Gamma\left(\alpha(1-\epsilon)\right),\end{aligned}$$

which follows from the definition of the Gamma function $\Gamma(a) = \int_0^\infty e^{-t} t^{a-1} dt$. Therefore, by replacing the preceding bound in (4.2), we obtain

$$\mathbb{P}[N_b > n] \le (1-\bar{G}(b))^{n(1-\epsilon)} \left[\eta_{x_0}^{n\epsilon} + \frac{H_\epsilon}{n^{\alpha(1-\epsilon)}}\right],$$

where $H_\epsilon = \alpha(1-\epsilon)\Gamma(\alpha(1-\epsilon))\epsilon^{-\alpha(1-\epsilon)}/F(b_0)$. Now, for any $\epsilon > 0$, we can choose n_0, such that for all $n \ge n_0$, $\eta_{x_0}^{n\epsilon} \le \epsilon H_\epsilon n^{-\alpha(1-\epsilon)}$, so that

$$\begin{aligned}\mathbb{P}[N_b > n] &\le (1-\bar{G}(b))^{n(1-\epsilon)} \left[\epsilon \frac{H_\epsilon}{n^{\alpha(1-\epsilon)}} + \frac{H_\epsilon}{n^{\alpha(1-\epsilon)}}\right] \\ &= (1-\bar{G}(b))^{n(1-\epsilon)} \frac{H_\epsilon}{n^{\alpha(1-\epsilon)}}(1+\epsilon).\end{aligned}$$

And by taking the logarithm in the preceding expression, we obtain

$$\begin{aligned}\log \mathbb{P}[N_b > n] &\le \log\left(H_\epsilon(1+\epsilon)\right) + n(1-\epsilon)\log(1-\bar{G}(b)) \\ &\quad - \alpha(1-\epsilon)\log n \\ &= \log\left(H_\epsilon(1+\epsilon)\right) \\ &\quad + (1-\epsilon)\left[n\log(1-\bar{G}(b)) - \alpha\log n\right].\end{aligned}$$

Next, since $-n\log(1-\bar{G}(b)) > 0$ and $\alpha \log n \ge 0$, $n \ge 1$,

$$\begin{aligned}\frac{\log \mathbb{P}[N_b > n]}{-n\log(1-\bar{G}(b)) + \alpha\log n} &\le \frac{\log H_\epsilon}{-n\log(1-\bar{G}(b)) + \alpha\log n} \\ &\quad - (1-\epsilon) \\ &\le \frac{\log H_\epsilon}{\alpha \log n} - (1-\epsilon).\end{aligned}$$

Since $\alpha \log n$ is increasing in n, we can choose n_0 such that for any $n \ge n_0$, $\log H_\epsilon / \alpha \log n \le \epsilon$, and thus,

$$\frac{\log \mathbb{P}[N_b > n]}{-n\log(1-\bar{G}(b)) + \alpha\log n} \le -(1-2\epsilon),$$

which completes the proof by replacing ϵ with $\epsilon/2$. □

Proof. Proof of Theorem 1.
Note that the upper bound follows from Proposition 2.1. For the lower bound, we have, for $x \ge x_0$,

$$\begin{aligned}\mathbb{P}[N_b > n] &= \mathbb{E}[1 - \bar{G}(L_b)]^n \\ &\ge \mathbb{E}[1 - \bar{F}(L_b)^{\frac{1}{\alpha(1+\epsilon)}}]^n \\ &\ge \int_{x_0}^b \left(1 - \bar{F}(x)^{\frac{1}{\alpha(1+\epsilon)}}\right)^n \frac{dF(x)}{F(b)} \\ &= \int_{\bar{F}(b)^{\frac{1}{\alpha(1+\epsilon)}}}^{\bar{F}(x_0)^{\frac{1}{\alpha(1+\epsilon)}}} (1-z)^n z^{\alpha(1+\epsilon)-1} \frac{\alpha(1+\epsilon)dz}{F(b)},\end{aligned}$$

where the last equality follows from the absolute continuity of $F(x)$ and change of variables. Next, by setting $z = 1 - e^{-u/n}$, and $\alpha_\epsilon = \alpha(1+\epsilon)$ we obtain

$$\mathbb{P}[N_b > n] \ge \frac{\alpha_\epsilon}{F(b)n} \int_{-n\log\left(1-\bar{F}(b)^{\frac{1}{\alpha_\epsilon}}\right)}^{-n\log\left(1-\bar{F}(x_0)^{\frac{1}{\alpha_\epsilon}}\right)} e^{-u(n+1)/n}(1 - e^{-\frac{u}{n}})^{\alpha_\epsilon - 1} du.$$

Now, by continuity, for any $\delta > 0$, we can choose x_0 such that $-\log(1 - \bar{F}(x_0)^{1/\alpha_\epsilon}) = \delta$, implying

$$\mathbb{P}[N_b > n] \ge \frac{\alpha_\epsilon(1-\delta)}{F(b)n} \int_{\delta \log n}^{\delta n} e^{-u}(1 - e^{-\frac{u}{n}})^{\alpha_\epsilon - 1} du,$$

where we use our assumption $n\bar{G}(b) \le \delta \log n$ to upper bound the lower limit of the integral; for the last inequality, note that $e^{-u/n} \ge 1 - u/n \ge (1-\delta)$. Next, by $F(b) < 1$ and using $1 - e^{-y} \ge (1-\delta)y$, for all y small enough, it follows that

$$\begin{aligned}\mathbb{P}[N_b > n] &\ge \frac{\alpha_\epsilon(1-\delta)^{\alpha_\epsilon}}{n^{\alpha_\epsilon}} \int_{\delta\log n}^{\delta n} e^{-u} u^{\alpha_\epsilon - 1} du \\ &\ge \frac{\alpha_\epsilon(1-\delta)^{\alpha_\epsilon} h_\epsilon}{n^{\alpha_\epsilon}} (\log n)^{\alpha_\epsilon - 1} \int_{\delta\log n}^{(1+\epsilon)\delta\log n} e^{-u} du \\ &= \frac{\alpha_\epsilon(1-\delta)^{\alpha_\epsilon} h_\epsilon}{n^{\alpha_\epsilon}} (\log n)^{\alpha_\epsilon - 1} n^{-\delta}\left(1 - n^{-\delta\epsilon}\right),\end{aligned}$$

where in the second inequality we use $n \ge (1+\epsilon)\log n$ for n large and the monotonicity of $u^{\alpha_\epsilon - 1}$; also, we set $h_\epsilon =$

$\max(1,(1+\epsilon)^{\alpha_\epsilon-1})$. Therefore, for n large such that $n^{-\delta\epsilon}<\delta$, we obtain the lower bound

$$\mathbb{P}[N_b>n]\geq\frac{\alpha_\epsilon(1-\delta)^{\alpha_\epsilon+1}h_\epsilon}{n^{\alpha_\epsilon}}\frac{(\log n)^{\alpha_\epsilon-1}}{n^\delta}.$$

Now, $\log n\geq\log n_0$ and we can choose $\delta<\epsilon\alpha$, such that $1/n^\delta\geq 1/n^{\epsilon\alpha}$, implying

$$\mathbb{P}[N_b>n]\geq\frac{h_{\epsilon,\delta}}{n^{\alpha(1+2\epsilon)}},$$

where we set $h_{\epsilon,\delta}=\alpha_\epsilon(1-\delta)^{\alpha_\epsilon+1}h_\epsilon(\log n_0)^{\alpha_\epsilon-1}$. And by taking the logarithm, we obtain

$$\log\mathbb{P}[N_b>n]\geq\log h_{\epsilon,\delta}-\alpha(1+2\epsilon)\log n.$$

Finally, since $\log n$ is increasing in n, we can choose n_0 such that for all $n\geq n_0$, $\alpha\epsilon\log n\geq-\log h_{\epsilon,\delta}$, i.e.,

$$\log\mathbb{P}[N_b>n]\geq-\alpha(1+3\epsilon)\log n,$$

which completes the proof by replacing ϵ with $\epsilon/3$. $\square$

Proof. Proof of Proposition 2.3.
Similarly as before, by the assumption,

$$\begin{aligned}\mathbb{P}[N_b>n]&=\mathbb{E}[1-\bar{G}(L_b)]^n\\&=\mathbb{E}[1-\bar{F}(L_b)^{\frac{1}{\alpha}}]^n\\&=\int_0^b\left(1-\bar{F}(x)^{\frac{1}{\alpha}}\right)^n\frac{dF(x)}{F(b)}.\end{aligned}$$

And by setting $\bar{F}(x)^{\frac{1}{\alpha}}=z$, we obtain

$$\begin{aligned}\mathbb{P}[N_b>n]&=\int_{\bar{F}(b)^{\frac{1}{\alpha}}}^1(1-z)^nz^{\alpha-1}\frac{\alpha dz}{F(b)}\\&=\frac{\alpha}{F(b)}\int_{\bar{G}(b)}^1(1-z)^nz^{\alpha-1}dz,\end{aligned}$$

following the assumption $\bar{G}(x)^\alpha=\bar{F}(x)$. Next, we break the preceding integral into two parts

$$\begin{aligned}\mathbb{P}[N_b>n]=\frac{\alpha}{F(b)}\Bigg[&\int_{\bar{G}(b)}^{\bar{G}(b)(1+\epsilon)}(1-z)^nz^{\alpha-1}dz\\&+\int_{\bar{G}(b)(1+\epsilon)}^1(1-z)^nz^{\alpha-1}dz\Bigg].\end{aligned}$$

To obtain the lower and upper bounds, we consider two different cases for α.
(i) If $\alpha\geq 1$,

$$\mathbb{P}[N_b>n]\geq\frac{\alpha}{F(b)}\bar{G}(b)^{\alpha-1}\int_{\bar{G}(b)}^{\bar{G}(b)(1+\epsilon)}(1-z)^ndz.$$

since $z^{\alpha-1}$ is monotonically increasing. Then,

$$\begin{aligned}&\mathbb{P}[N_b>n]\\&\geq\frac{\alpha}{F(b)}\bar{G}(b)^{\alpha-1}\left|-\frac{(1-z)^{n+1}}{n+1}\right|_{\bar{G}(b)}^{\bar{G}(b)(1+\epsilon)}\\&=\frac{\alpha\bar{G}(b)^{\alpha-1}}{F(b)(n+1)}\left[(1-\bar{G}(b))^{n+1}-(1-\bar{G}(b)(1+\epsilon))^{n+1}\right]\\&=\frac{\alpha\bar{G}(b)^{\alpha-1}(1-\bar{G}(b))^{n+1}}{F(b)(n+1)}\left[1-\left(\frac{1-\bar{G}(b)(1+\epsilon)}{1-\bar{G}(b)}\right)^{n+1}\right].\end{aligned}$$

Next, by recalling that $(1-\bar{G}(b)(1+\epsilon))/(1-\bar{G}(b))<1$ and b is fixed, we have as $n\to\infty$,

$$\mathbb{P}[N_b>n]\gtrsim\frac{\alpha}{F(b)}\frac{\bar{G}(b)^{\alpha-1}(1-\bar{G}(b))^{n+1}}{n+1}.\tag{4.3}$$

Also, for the upper bound,

$$\begin{aligned}\mathbb{P}[N_b>n]\leq\frac{\alpha}{F(b)}\Bigg[&(\bar{G}(b)(1+\epsilon))^{\alpha-1}\int_{\bar{G}(b)}^{\bar{G}(b)(1+\epsilon)}(1-z)^ndz\\&+\int_{\bar{G}(b)(1+\epsilon)}^1(1-z)^ndz\Bigg],\end{aligned}\tag{4.4}$$

since $z^{\alpha-1}\leq 1$. Next, we evaluate the expression in the brackets,

$$\begin{aligned}&(\bar{G}(b)(1+\epsilon))^{\alpha-1}\left.\frac{(1-z)^{n+1}}{n+1}\right|_{\bar{G}(b)(1+\epsilon)}^{\bar{G}(b)}+\left.\frac{(1-z)^{n+1}}{n+1}\right|_1^{\bar{G}(b)(1+\epsilon)}\\&=\frac{\bar{G}(b)^{\alpha-1}(1+\epsilon)^{\alpha-1}}{n+1}\left[(1-\bar{G}(b))^{n+1}-(1-\bar{G}(b)(1+\epsilon))^{n+1}\right]\\&\quad+\frac{(1-\bar{G}(b)(1+\epsilon))^{n+1}}{n+1}\\&=\frac{(1-\bar{G}(b))^{n+1}}{n+1}\\&\quad\times\left[\bar{G}(b)^{\alpha-1}(1+\epsilon)^{\alpha-1}\left(1-\left(\frac{1-\bar{G}(b)(1+\epsilon)}{1-\bar{G}(b)}\right)^{n+1}\right)\right.\\&\quad\left.+\left(\frac{1-\bar{G}(b)(1+\epsilon)}{1-\bar{G}(b)}\right)^{n+1}\right].\end{aligned}$$

Similarly as before, $(1-\bar{G}(b)(1+\epsilon))/(1-\bar{G}(b))<1$ and by taking the limit as $n\to\infty$ in (4.4), we have

$$\mathbb{P}[N_b>n]\lesssim\frac{\alpha}{F(b)}\frac{\bar{G}(b)^{\alpha-1}(1-\bar{G}(b))^{n+1}(1+\epsilon)^{\alpha-1}}{n+1}.$$

Therefore, by letting $\epsilon\to 0$,

$$\mathbb{P}[N_b>n]\lesssim\frac{\alpha}{F(b)}\frac{\bar{G}(b)^{\alpha-1}(1-\bar{G}(b))^{n+1}}{n+1}.\tag{4.5}$$

Finally, by equations (4.3) and (4.5), we have

$$\mathbb{P}[N_b>n]\sim\frac{\alpha}{F(b)}\frac{\bar{G}(b)^{\alpha-1}(1-\bar{G}(b))^{n+1}}{n+1}.$$

(ii) If $\alpha\leq 1$, similar arguments work, we omit the details. $\square$

5. ACKNOWLEDGMENTS

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