A Quantitative Measure for Comparison between Topologies of Modern Telecommunications Networks

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Abstract

This paper proposes a new measure for network performance evaluation called topology lifetime. The measure provides insight into which one of a set of topologies is likely to last the longest before more capacity must be installed. The lifetime measure is not single valued, but considers growth as a function of a set of demand shifts. The lifetime measure is applied to several topologies: a dual ring, a chordal ring, a Manhattan Street network and an hierarchical network.

1 Introduction

According to current network design practices, communications networks are typically designed to (1) carry the load that is presented to the network, (2) provide a level of reliability, and (3) accommodate the expected growth in demand. During the design process, the following facts and issues are considered. The load on the network is not static. It is different at different times of the day and different days of the year, and hence the network must support all of the load distributions that occur. In order to guarantee a level of reliability, networks have alternate paths and sufficient additional capacity to carry the traffic even if a link or a node fails. Networks must also have sufficient spare capacity to support the future predicted traffic for at least the time it takes to install new facilities.

What is typically not considered in the design process is how well the network copes with **unpredicted** shifts in the traffic load. Currently, there are three factors that are making it more important to consider such shifts: (1) the way fibers are installed, (2) recent and expected advances in wavelength division multiplexing (WDM) and Dense WDM (DWDM), and (3) the new Internet services that are being introduced. Because of the first two factors, installed network topologies have nowadays much more capacity than in the past, and will be able to cope with greater growth and shifts in demand. The third factor is causing the shifts in demands to occur more frequently and more suddenly than they have in the past.

Until recently, the shift in network load has been a result of movements of population and businesses. This is a relatively slow and predictable process. The Internet is increasing the rate of this process which is becoming more and more unpredictable. Server farms may be located anywhere on the network. If a server farm provides a service that becomes popular and requires wide band channels, the demand to that part of the network increases. As the popularity and use of the Internet increases, so do the sudden shifts in network load.

We introduce the concept of *network lifetime* measure. It is a measure of the growth and shifts in the load (traffic demand perturbations) that a network can sustain. The longer the network can support growth and load changes, the longer it will be before we have to add new links. The measure is used to compare network topologies and not to estimate the actual time (weeks, months or years) before links must be added. The growth that a network can sustain is a function of the demand perturbations. One topology may have a longer lifetime than another when there is **linear growth** but not when the perturbations exceed a certain magnitude.

To calculate the linear growth that the network can sustain we consider traffic matrices of the load between each pair of cities for different times of the day and different days of the year (multihour traffic demand matrices). For each matrix we increase each element of the matrix by the same multiplier until we can no longer support the load on the network, regardless of how we reroute the traffic. The minimum of the multipliers of the matrices is the growth that the network can sustain before we have to add bandwidth. It should be clarified that this linear growth represents the traffic growth **relative** to the growth in capacity that can be achieved without installing new fiber and not the absolute traffic growth. Clearly, if the capacity per fiber increases further than the demand growth everywhere in the network forever, then we shall never need to install more fiber.

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Defining the growth associated with perturbations in the network load is more difficult because there are many ways that the load on a network can change. In a network with N cities, there are N ways that the load from one city can change relative to the others, $N(N-1)$ ways that the traffic between one pair of cities may change relative to the others, and many more ways that combinations of origin destination (OD) pairs and cities may change relative to the rest of the network. We start by defining the set of changes in load that we will consider - the set of feasible perturbations. In this paper the set of feasible perturbations includes change in the importance of any particular city. In a practical application we may augment the set of feasible perturbations to include changes in importance of combinations of cities or OD pairs that are likely to occur together.

In the remainder of the paper we rigorously define in Section 2 our measure of topology lifetime and in Section 3 we compare the lifetime of several topologies.

2 A Measure for Topology Lifetime

Consider a network with m nodes. Let the present traffic be represented by a finite set S of n traffic matrices:

$$S = \{[T_1(i,j)], [T_2(i,j)], ..., [T_n(i,j)]\}. \tag{1}$$

Each of the matrices in S is the traffic between all OD pairs at a certain time of day for different days of the year. For example, for some s, u and v, $[T_s(i,j)]$, can be the traffic matrix between 8 AM and 9 AM on a normal working day, $[T_u(i,j)]$ the traffic matrix between 8 PM and 9 PM on a normal Sunday, and $[T_v(i,j)]$ the traffic matrix between 10 AM and 11 AM on Mother's Day. Since we are concerned with a network that interconnects nodes, we shall assume that there is no traffic between a node and itself, i.e., $T_d(i,i) = 0$ for all i.

We define a *topology* in two ways (1) directed and (2) undirected. A *directed topology* is defined by:

- a Graph $G = \langle V, E\rangle$ where V is a set of nodes (vertices) and E is a set of directed links (edges),
- a set of capacities: $C = \{c_{ij}\}$ where c_{ij} is the capacity of link (i,j), and
- practical routing limitations.

The *undirected topology* is also defined by a graph $G = \langle V, E\rangle$ except that E is now a set of undirected links. Each of the elements c_{ij} in C represents the total capacity on the (i,j) link. In other words, c_{ij} must be greater than or equal to the traffic transmitted on link (i,j) between i and j plus the traffic transmitted on link (j,i) between j and i. Networks such as the token ring are directed topologies. The bidirectional networks that are currently used for telecommunications are also directed networks, where $c_{ij} = c_{ji}$ for every link (i,j).

Different routing techniques can be used to satisfy OD demands in a network. In practice, constraints are placed on the routes and these constraints limit the traffic on the network and the lifetime of the network. Typically, networks constrain the number of hops on a route to avoid overloading switching and transmission facilities and to avoid excessive delay. In this work, k is the hop limit, and when $k = \infty$ there is no hop limit.

Another typical routing constraint is caused by the switching capabilities of different nodes. Telecommunications networks are hierarchical. Local access switches cannot act as an intermediate node on a path. In order to send traffic to a destination that is not directly connected to a local switch, the traffic must be sent to a higher layer (tandem) switch. In Section 3 we investigate the lifetime of an hierarchical network.

For a given network $G = \langle V, E\rangle$ with link capacities c_{ij}, the traffic matrix $[T_d(i,j)]$ is said to be *feasible* for a given topology if the topology can support the load $[T_d(i,j)]$ without violating the routing contraints or exceeding the link capacities.

For any traffic matrix $[T_d(i,j)]$, we introduce the concept of *growth factor*, denoted by $\psi \in \mathcal{R}^+$. In particular, we are interested in the maximum value of ψ for which $\psi[T_d(i,j)]$ is feasible. Since we are interested in traffic growth, we shall focus on the range $\psi \geq 1$.

For a given topology and for a given traffic matrix $[T_d(i,j)]$, let ψ_d^* be the maximal value for ψ_d such that $\psi_d[T_d(i,j)]$ is feasible. Let

$$\psi^* = \min\{\psi_1^*, \psi_2^*, ..., \psi_n^*\}. \tag{2}$$

We use ψ^* as our measure for the life-time of a given topology with relation to the traffic load represented by a finite set of matrices S. The measure ψ^* signifies by how much current traffic load can grow until we need to add capacity to the current topology.

In the remainder of this paper we use capital letter T for a matrix and small letter t to denote an element in a matrix. For example, the $[i,j]$ element in the matrix $[T_d(i,j)]$ is denoted by $t_d(i,j)$.

So far the definition of ψ^* assumes that the network growth is uniform. However, we have argued that we must also consider changes in the traffic distribution. To allow for such unexpected changes in the traffic growth we introduce a parameter which we call the *unexpected traffic growth* (UTG) parameter, denoted U.

To incorporate the UTG in our original framework we construct new sets of matrices $S(U)$ as follows. When the traffic for a particular node j increases by U then, for every matrix T_d in S, we multiply all the elements $t_d(i,j)$ and $t_d(j,i)$ by $1+U$, for all $i \neq j$. All other elements in T_d are multiplied by $(1 - r_{d(j)})$, where

$$r_{d(j)} = \frac{U\left(\sum_{i=1}^{m} t_d(i,j) + \sum_{i=1}^{m} t_d(j,i)\right)}{\left(\sum_{i=1}^{m}\sum_{j=1}^{m} t_d(i,j)\right) - \left(\sum_{i=1}^{m} t_d(i,j) + \sum_{i=1}^{m} t_d(j,i)\right)}. \tag{3}$$

Notice that Eq. (3) must be modified if the condition $t_d(i,i) = 0$ does not hold.

We repeat this procedure for all the nodes and all the matrices in S, and obtain the set of matrices $S(U)$. The number of matrices in $S(U)$ is $n \times m$.

Again, for any matrix T_d and any node j, U must not exceed

$$U(d,(j))_{max} = \frac{NUM}{DEN}. \tag{4}$$

Where

$$NUM = \left(\sum_{i=1}^{m}\sum_{j=1}^{m} t_d(i,j)\right) - \left(\sum_{i=1}^{m} t_d(i,j) + \sum_{i=1}^{m} t_d(j,i)\right)$$

$$DEN = U\left(\sum_{i=1}^{m} t_d(i,j) + \sum_{i=1}^{m} t_d(j,i)\right)$$

since when $U = U(d,(j))_{max}$ all of the traffic in the network is at node j. Therefore, the value of U in $S(U)$ must not exceed

$$U_{max} = \min_{d,(i,j)} \{U(d,(j))_{max}\}. \tag{5}$$

Note that the set $S(U)$ can be augmented to include other shifts, such as one or more separate OD pairs or more than one node becoming more active simultaneously.

By analogy to the definition of ψ^* as our measure for the life-time of a given topology with relation to the set of matrices S, we define the function $\Psi^*(U)$ as our measure for the life-time of a given topology with relation to the set of matrices $S(U)$. Notice that $\psi^* = \Psi^*(0)$.

In general, when we shift traffic, it may not be possible to route the new traffic on the underlying network. In other words, the shifted traffic may not be feasible. In this case, in order to find a feasible solution we will have to consider $\psi < 1$. At present, we are mainly interested in networks with growth potential so it is unlikely that $\psi < 1$ will have to be considered.

For any given topology and a set of matrices S, we now have the curve $\Psi^*(U)$ which provides the network designer with a means for comparing different topologies. Notice that one topology could have a higher $\Psi^*(0)$ than another, but when U exceeds a certain value, the situation may be reversed. This provides the designer with insight into the possible effect of unexpected uneven traffic growth on topology life-time.

The way we normalized our traffic shifts, using U and r to keep the total traffic fixed, is very important and not at all arbitrary. By normalizing our traffic, the shift is independent of the growth (signified by the ψ variable). This way we cover both shift and growth and are able to have a meaningful single curve lifetime measure which includes the effect of different types of traffic shifts. In the remainder of the paper we shall derive the curves $\Psi^*(U)$ for several specific topologies.

To obtain the curve $\Psi^*(U)$, we first calculate ψ^* for the case $U = 0$, where ψ^* is the maximal ψ such that ψT is feasible. To find ψ^* we solve a series of feasibility problems, where we try values of ψ to determine if they are feasible. Each feasibility problem has two sets of constraints. The first set, the traffic requirement constraints, ensure that the traffic demand between any OD pair is fully satisfied. The second set (the capacity constraints) ensures that the total flow passing through any individual link of the network does not exceed the capacity of that link.

For any OD pair $[u,v]$, the traffic requirement constraint is:

$$\sum_{p \in P_{uv}} x_{uv}^p = t_{uv}$$

where t_{uv} is the traffic demand from node u to node v, P_{uv} is the set of working paths from node u to node v in the network and x_{uv}^p is the amount of flow sent from node u to node v through working path p.

The capacity constraint for a given link $(i,j) \in E$ is:

$$\sum_{\Gamma}\left(\sum_{p \in P_{uv}} \delta_{ij}^p x_{uv}^p\right) \le c_{ij}$$

where c_{ij} is the capacity of (i,j) link, Γ is the set of all OD pairs in G and δ_{ij}^p is an indicator function which is equal to one if path p uses link (i,j) and is zero otherwise.

The set of all constraints of type one and two, together with all nonnegativity constraints ($x_{uv}^p \ge 0$), form the feasibility problem **(FP)**.

$$\sum_{p \in P_{uv}} x_{uv}^p = t_{uv} \qquad \text{for all} \quad u,v \in V \tag{FP}$$

$$\sum_{\Gamma}\left(\sum_{p \in P_{uv}} \delta_{ij}^p x_{uv}^p\right) \le c_{ij} \qquad \text{for all} \quad (i,j) \in E$$

$$x_{uv}^p \ge 0 \qquad \text{for all} \quad u,v \in V \text{ and for all } p \in \Gamma$$

Note that t appears on the right hand side of the first set of constraints in our model.

To find ψ^* the values (t_{uv}) are substituted by (ψt_{uv}) for some $\psi \in \mathcal{R}^+$. The feasibility of **(FP)** can be checked in polynomial time using the Simplex algorithm. The only task is to find the maximum ψ value such that **(FP)** remains feasible.

3 Comparison of Several Topologies

In this section we derive the $\Psi^*(U)$ curves for a dual ring, chordal ring, Manhattan Street and hierarchical network.

Each of the networks have 8 nodes and bidirectional links. For each link $c_{ij} = 1000$, so that the capacity in each direction is 500. All of the nodes have switching capabilities, except in the hierarchical network where only the nodes in the upper layer can forward traffic between nodes.

Symmetrical traffic matrices are applied to all of the networks. The set S has two elements:

$$T_1(i,j) = \begin{bmatrix} 0 & 9 & 6 & 2 & 10 & 3 & 7 & 11 \\ 9 & 0 & 11 & 3 & 19 & 6 & 14 & 22 \\ 6 & 11 & 0 & 2 & 13 & 4 & 9 & 15 \\ 2 & 3 & 2 & 0 & 4 & 1 & 3 & 4 \\ 10 & 19 & 13 & 4 & 0 & 7 & 16 & 25 \\ 3 & 6 & 4 & 1 & 7 & 0 & 5 & 8 \\ 7 & 14 & 9 & 3 & 16 & 5 & 0 & 18 \\ 11 & 22 & 15 & 4 & 25 & 8 & 18 & 0 \end{bmatrix}$$

$$T_2(i,j) = \begin{bmatrix} 0 & 9 & 18 & 5 & 12 & 16 & 5 & 21 \\ 9 & 0 & 12 & 3 & 8 & 11 & 3 & 14 \\ 18 & 12 & 0 & 6 & 15 & 21 & 6 & 27 \\ 5 & 3 & 6 & 0 & 4 & 6 & 2 & 7 \\ 12 & 8 & 15 & 4 & 0 & 14 & 4 & 17 \\ 16 & 11 & 21 & 6 & 14 & 0 & 6 & 24 \\ 5 & 3 & 6 & 2 & 4 & 6 & 0 & 7 \\ 21 & 14 & 27 & 7 & 17 & 24 & 7 & 0 \end{bmatrix}$$

In the implementation process, the following routing strategies in terms of the maximum hop limits have been considered . For the dual ring example, seven-hop path limit is selected, which in our example it would provide all possible paths between any OD pair in the network. In the remaining examples, four-hop path limit was considered to be significantly sufficient to provide a large number of working paths between any OD pair in the network.

3.1 Dual ring

Dual rings were first used in data networks, such as FDDI [1], [2], to make the networks more reliable. They are now used to make SONET networks [3] and newly installed, long haul fiber networks more reliable. Dual rings are therefore entrenched in our telecommunications networks. Figure 1 is the topology of the dual ring in our example.

3.2 Chordal ring

It has long been recognized that dual rings can be improved. Chordal rings [4] were studied in the early 80's. They have the same number of links as the dual rings, but can be more reliable and have a smaller average distance between nodes [5]. Figure 2 is the topology of the chordal ring in our example.

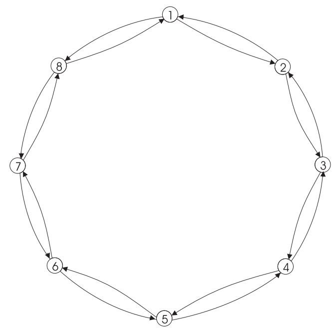

Figure 1: The eight node dual ring topology

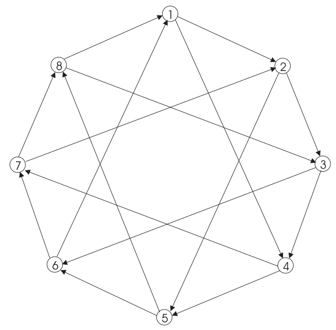

Figure 2: The eight node chordal ring topology

3.3 Manhattan Streets

At present, mesh networks, with the same number of links as dual rings, are being considered to replace these ring. One of the first regular, two-connected mesh networks was the Manhattan Street Network (MSN) [6]. This network is a grid of directed links that form rows and columns and is conceptually constructed on the surface of a torus. The adjacent rows and columns in this network have flows in opposite directions. These networks are much more reliable than the two-connected ring networks and have a shorter distance between nodes [7]. Unlike the chordal rings, there is a strategy for increasing the number of nodes in the MSN without rewiring the entire network [8]. Figure 3 is the topology of the MSN in our example.

3.4 Hierarchical undirected network

Our hierarchical network also has eight nodes. Six of the nodes are lower layer nodes (representing local access switches) and two are higher layer nodes (representing core network switches) as shown in Figure 9. The lower layer switches cannot switch traffic between two other nodes. A link between two lower layer nodes can only carry traffic that originates at one of the nodes and is destined for the

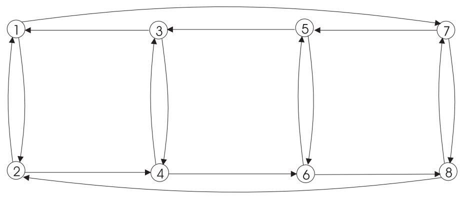

Figure 3: The eight node Manhattan Street Network Topology

other. This type of switching architecture is commonly used in the current telephone network. Figure 4, is the topology of the hierarchical network in our example.

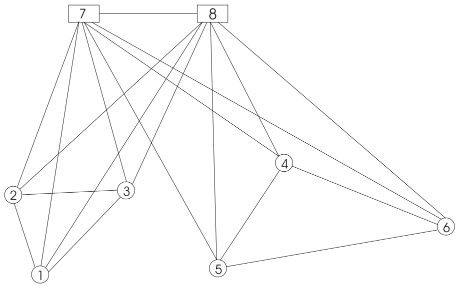

Figure 4: The eight node hierarchical network example

3.5 Lifetime comparison

Figure 7 is a plot of $\Psi(U)$ as a function of U for the four topologies. According to our metric, there is one network, the chordal ring, that can clearly support more growth than the others. There is also an example of two networks, the bidirectional loop and the Manhattan Street Network, that have different evaluations depending upon whether we want to plan for uniform growth or want to plan for perturbations of the traffic. There is also one network, the hierarchical network that is clearly inferior to the others, and will sustain the least growth.

4 Conclusions

We have presented a new measure for comparing networks. The measure determines which networks are likely to last longer if the traffic grows in a uniform manner, and which networks are likely to last longer under shifts in load.

Our measure is a composite of many components. We have shown how to use the composite measure to compare topologies. Such comparisons can be used to evaluate new networks or target topologies of an existing network.

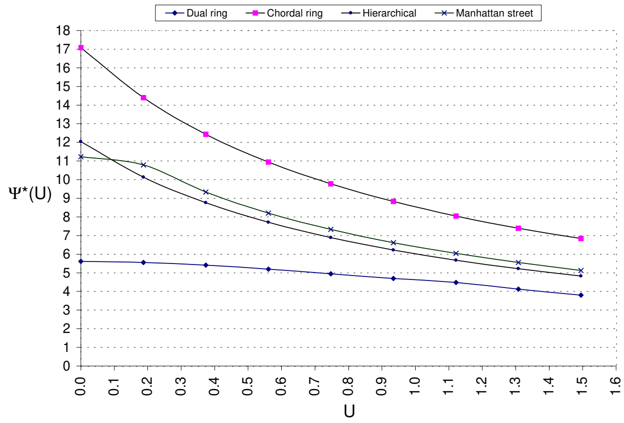

Figure 5: The lifetime function $\Psi^*(U)$ for the four topologies

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