A Quantitative Measure for Telecommunications Networks Topology Design

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abstract:

This paper proposes a new measure for network performance evaluation called topology lifetime. The measure provides insight into which one of a set of topologies is likely to last the longest before more capacity must be installed. The lifetime measure is not single valued, but considers growth as a function of a set of demand shifts (perturbation). One network may be better able to support a uniform growth in the traffic, while another may support more growth when unexpected shifts in the load occur. The ability of a network to support unexpected changes in load is becoming more important because of (1) current practices for installing fiber optics cables, (2) recent advances in dense wavelength division multiplexing, and (3) the increasing popularity of the Internet. The lifetime measure is applied to several topologies; a dual ring, a chordal ring, a Manhattan Street network and an hierarchical network. We also apply the measure to a realistic US IP Backbone network. In this paper, our objective is to show how to apply the measure to different networks, and to explain certain implications for comparisons between networks. We expect this measure to be useful both in the construction of new networks and in selecting between new links that may be added to an existing network.

Key Words: Network Topology, Telecommunications, DWDM, Linear Programming.

1 Introduction

According to current network design practices, communications networks are typically designed to (1) carry the load that is presented to the network [14], [15],[19], [20], [21], (2) provide a level of reliability [18], [21], and (3) accommodate the expected growth in demand [15], [16], [17]. During the design process, the following facts and issues are considered. The load on the network is not static. It is different at different times of the day and different days of the year, and hence the network must support all of the load distributions that occur [15], [17]. In order to guarantee a level of reliability, networks have alternate paths and sufficient additional capacity to carry the traffic even if a link or a node fails. Networks must also have sufficient spare capacity to support the future predicted traffic for at least the time it takes to install new facilities.

What is typically not considered in the design process is how well the network copes with **unpredicted** shifts in the traffic load. We are concerned that highly optimised networks, based on the above criteria, may not cope with such traffic shifts. Currently, there are three factors that are making it more important to consider such shifts: (1) the way fibers are installed, (2) recent and expected advances in wavelength division multiplexing (WDM) and Dense WDM (DWDM), and (3) the new Internet services that are being introduced. Because of the first two factors, installed network topologies have nowadays adequate capacity for a longer time than in the past, and will be able to cope with greater shifts in demand. The third factor is causing the shifts in demands to occur more frequently and more suddenly than they have in the past.

The total cost of installing a long haul fiber, including the material and labor, is usually much higher than the cost of the fiber itself. The current practice is to install conduits that can house 36 fibers, and there is a move to install several hundred fibers at a time. If a link will not require a large number of fibers in the foreseeable future, empty conduits are installed and fibers are added as the demand increases. Installing empty, rather than full, conduits allows the network owner to take advantage of advances in fiber technology that occur in the interim. As a result of the way fibers are installed, the physical topology of the network is fixed for a longer time into the future than if smaller units of capacity were installed as needed.

Up until 1995 fiber on the long haul network carried one wavelength that operated at 1.7 Gb/s. By 1997 WDM systems carried eight wavelengths per fiber, each operating at 2.5 Gb/s - a ten fold increase in capacity in two years. In 1998 WDM systems were available that had 16 wavelengths per fiber, and in 1999 systems were announced that have 40 wavelengths per fiber, with a commitment to increase the bits per wavelength to 10 Gb/s by mid-2000 - another twenty fold increase. Nowadays, WDM systems with 40 wavelengths or more are called Dense WDM or DWDM in short. The current trend is expected to reach 160 wavelengths per fiber and 40 Gb/s per wavelength by 2007 - 4,000 times the capacity per fiber that was possible 12 years earlier. As a result of DWDM, the fiber links that are installed have adequate bandwidth for a longer time and hence postpone new fiber installations and modifications of the network topology.

Until recently, the shift in network load has been a result of movements of population and businesses. This is a relatively slow and predictable process. The Internet is increasing the rate of this process which is becoming more and more unpredictable. Server farms may be located anywhere on the network. If a server farm provides a service that becomes popular, the demand to that part of the network increases. As the popularity and use of the Internet increases, so do the sudden shifts in network load.

We introduce the concept of *network lifetime* measure. It is a measure of the growth and shifts in the load (traffic demand perturbations) that a network can sustain. The longer the network can support growth and load changes, the longer it will be before we have to add new links. The measure is used to compare network topologies and not to estimate the actual time (weeks, months or years) before links must be added. The growth that a network can sustain is a function of the demand perturbations. One topology may have a longer lifetime than another when there is **linear growth** but not when the perturbations exceed a certain magnitude.

To calculate the linear growth that the network can sustain we consider traffic matrices of the load between each pair of cities for different times of the day and different days of the year (multihour traffic demand matrices). For each matrix, we increase each element of the matrix by the same multiplier until we can no longer support the load on the network, regardless of how we reroute the traffic. The minimum of the multipliers of the matrices is the growth that the network can sustain before we have to add bandwidth. It should be clarified that this linear growth represents the traffic growth **relative** to the growth in capacity that can be achieved without installing new fiber and not the absolute traffic growth. Clearly, if the capacity per fiber increases further than the demand growth everywhere in the network forever, then we shall never need to install more fiber.

Defining the growth associated with perturbations in the network load is more difficult because there are many ways that the load on a network can change. In a network with N cities, there are N ways that the load from one city can change relative to the others, $N(N-1)$ ways that the traffic between one pair of cities may change relative to the others, and many more ways that combinations of origin destination (OD) pairs and cities may change relative to the rest of the network. We start by defining the set of changes in load that we will consider - the set of feasible perturbations. In this paper the set of feasible perturbations includes all changes in the importance of one city or one OD pair. In a practical application we may augment the set to include changes in importance of combinations of cities or OD pairs that are likely to occur together.

For each perturbation in our feasible set we select a number of different magnitudes of the perturbation and follow the same procedure that we did for linear growth. First we modify each of the traffic matrices by the same magnitude of one perturbation, then we calculate the load multipliers to saturation for each matrix, and finally we determine the minimum multiplier. This results in a graph of the minimum multiplier as a function of the magnitude of the perturbation. There is a graph for each perturbation in the feasible set.

In order to reason about the relative merits of a topology, we must distill the large number of graphs. We cannot simply combine the results and plot the minimum multiplier for the same magnitude of all perturbations in the feasible set because some perturbations will subsume others and hide their effects. For instance, the minimum multiplier for a 10 percent increase in the traffic from a city will always be less than the minimum multiplier for a 10 percent increase in the traffic on one link from that city. Therefore, the result of a city becoming more important will hide the effect of a link between cities becoming more important. In order to look at different types of perturbations at the same time, we normalize the modified matrices. Our normalization procedure is formally specified in Section 2. Basically, we decrease some components as we increase others, so that the network load is the same on all networks before we calculate the growth.

The result is a single graph of the growth that can be sustained as a function of the magnitude of all perturbations. The value of the graph at zero perturbation is the linear growth that the network can sustain. In Section 4, we apply our measure to a three node network. In Section 5, we compare a number of topologies using this procedure, and finally in Section 6 we apply our measure to a network of realistic size and comment on critical links.

2 Definition of a Measure for Topology Lifetime

Consider a network with m nodes. Let the present traffic be represented by a finite set S of n traffic matrices:

$$S = \{[T_1(i,j)], [T_2(i,j)], ..., [T_n(i,j)]\}. \tag{1}$$

Each of the matrices in S is the traffic between all OD pairs at a certain time of day for different days of the year. For example, for some s, u and v, $[T_s(i,j)]$, can be the traffic matrix between 8 AM and 9 AM on a normal working day, $[T_u(i,j)]$ the traffic matrix between 8 PM and 9 PM on a normal Sunday, and $[T_v(i,j)]$ the traffic matrix between 10 AM and 11 AM on Mother's Day. Since we are concerned with a network that interconnects nodes, we shall assume that there is no traffic between a node and itself, i.e., $T_d(i,i) = 0$ for all i.

We define a *topology* in two ways (1) directed and (2) undirected. A *directed topology* is defined by:

- a Graph $G = \langle V, E \rangle$ where V is a set of nodes (vertices) and E is a set of directed links (edges),
- a set of capacities: $C = \{c_{ij}\}$ where c_{ij} is the capacity of link (i,j), and
- practical routing limitations.

The *undirected topology* is also defined by a graph $G = \langle V, E \rangle$ except that E is now a set of undirected links. Each of the elements c_{ij} in C represents the total capacity on the (i,j) link. In other words, c_{ij} must be greater than or equal to the traffic transmitted on link (i,j) between i and j plus the traffic transmitted on link (j,i) between j and i. Networks such as the token ring are directed topologies. The bidirectional networks that are currently used for telecommunications are also directed networks, where $c_{ij} = c_{ji}$ for every link (i,j). Undirected networks are more flexible than directed networks because the capacity on link (i,j) can be used for transmission in either direction. On an undirected channel, the capacity on a link can be divided so that different fractions of the bandwidth are used for transmission in each direction. Therefore, undirected networks have greater routing flexibility and longer lifetimes. In the past, most telecommunications applications, like voice telephone calls, required an equal amount of bandwidth in each direction. Therefore, it was reasonable to design networks with equal bandwidth in both directions on each of the channels. At present, more applications, like distributing entertainment television and the world wide web, require more bandwidth in one direction than the other. In such applications it is more economical for the network to be asymmetric and to have different bandwidths in opposite directions on a channel. Since we can change the transmission direction on a fiber, or even on a subset of the wavelengths on the fiber, to optimize network utilization, undirected networks may be used in the future. Our measure applies to both directed and undirected networks.

Different routing techniques can be used to satisfy OD demands in a network. In practice, constraints are placed on the routes and these constraints limit the traffic on the network and the lifetime of the network. Typically, networks constrain the number of hops on a route to avoid overloading switching and transmission facilities and to avoid excessive delay. In this work, k is the hop limit, and when $k = \infty$ there is no hop limit.

Another typical routing constraint is caused by the switching capabilities of different nodes. Telecommunications networks are hierarchical. Local access switches cannot act as an intermediate node on a path. In order to send traffic to a destination that is not directly connected to a local switch, the traffic must be sent to a higher layer (tandem) switch. In section 5.4 we investigate the lifetime of an hierarchical network.

For a given network $G = \langle V, E \rangle$ with link capacities c_{ij}, the traffic matrix $[T_d(i,j)]$ is said to be *feasible* for a given topology if the topology can support the load $[T_d(i,j)]$ without violating the routing contraints or exceeding the link capacities.

For any traffic matrix $[T_d(i,j)]$, we introduce the concept of *growth factor*, denoted by $\psi \in \mathbb{R}^+$. In particular, we are interested in the maximum value of ψ for which $\psi[T_d(i,j)]$ is feasible. Since we are interested in traffic growth, we shall focus on the range $\psi \geq 1$.

For a given topology and for a given traffic matrix $[T_d(i,j)]$, let ψ_d^* be the maximal value for ψ_d such that $\psi_d[T_d(i,j)]$ is feasible. Let

$$\psi^* = \min\{\psi_1^*, \psi_2^*, ..., \psi_n^*\}. \tag{2}$$

We use ψ^* as our measure for the life-time of a given topology with a given set of link capacities, routing constraints and a set of traffic load matrices (S). The measure ψ^* signifies by how much current traffic load can grow until we need to add capacity to the current topology.

In the remainder of this paper we use capital letter T for a matrix and small letter t to denote an element in a matrix. For example, the *[i,j]* element in the matrix $[T_d(i,j)]$ is denoted by $t_d(i,j)$.

So far the definition of ψ^* assumes that the network growth is uniform. However, we have argued that we must also consider changes in the traffic distribution. To allow for such unexpected changes in the traffic growth we introduce a parameter which we call the *unexpected traffic growth* (UTG) parameter, denoted U.

To incorporate the UTG in our original framework we construct new sets of matrices $S(U)$ as follows. When the traffic between the OD pair $[i,j]$ increases by U we multiply the elements $t_d(i,j)$ and $t_d(j,i)$ by $(1+U)$, for every matrix T_d in S, $(d = 1, 2, ..., n)$. All of the other elements in each matrix T_d are multiplied by $(1 - r_{1,d(i,j)})$, where,

$$r_{1,d(i,j)} = \frac{U(t_d(i,j) + t_d(j,i))}{\left(\sum_{i=1}^{m}\sum_{j=1}^{m} t_d(i,j)\right) - (t_d(i,j) + t_d(j,i))}. \tag{3}$$

This value of $r_{1,d(i,j)}$ maintains the total traffic (the sum of the elements in matrix T_d) constant, so that there is a shift in load without growth. Multiplying the load on both $[i,j]$ and $[j,i]$ OD pairs means that we assume that both increase at the same rate. This is a reasonable assumption even when the traffic matrix is asymmetric.

We repeat this procedure for all OD pairs to obtain a set of matrices $S_1(U)$. The number of matrices in $S_1(U)$ is $n \times m(m-1)/2$ where n is the number of matrices in S and m is the number of nodes in the network.

The value of U is constrained. For any matrix T_d and any OD pair $[i,j]$, U cannot exceed

$$U_1(d(i,j))_{max} = \frac{\left(\sum_{i=1}^{m}\sum_{j=1}^{m} t_d(i,j)\right) - (t_d(i,j) + t_d(j,i))}{t_d(i,j) + t_d(j,i)}. \tag{4}$$

When $U = U_1(d(i,j))_{max}$ all of the traffic has been moved to OD pair $[i,j]$ and no more traffic can be moved. Therefore, in $S_1(U)$ the value of U must not exceed

$$U(1)_{max} = \min_{d(i,j)}\{U_1(d(i,j))_{max}\}. \tag{5}$$

We follow a similar procedure when single nodes become more active. For a particular node j, for every matrix T_d in S, we multiply all the elements $t_d(i,j)$ and $t_d(j,i)$ by $1+U$, for all $i \neq j$. All other elements in T_d are multiplied by $(1 - r_{2,d(j)})$, where

$$r_{2,d(j)} = \frac{U\left(\sum_{i=1}^{m} t_d(i,j) + \sum_{i=1}^{m} t_d(j,i)\right)}{\left(\sum_{i=1}^{m}\sum_{j=1}^{m} t_d(i,j)\right) - \left(\sum_{i=1}^{m} t_d(i,j) + \sum_{i=1}^{m} t_d(j,i)\right)}. \tag{6}$$

Notice that Eq. (6) must be modified if the condition $t_d(i,i) = 0$ does not hold.

We repeat this procedure for all the nodes and all the matrices in S, and obtain the set of matrices $S_2(U)$. The number of matrices in $S_2(U)$ is $n \times m$.

Again, for any matrix T_d and any node j, U must not exceed

$$U_2(d(j))_{max} = \frac{\left(\sum_{i=1}^{m} \sum_{j=1}^{m} t_d(i,j)\right) - \left(\sum_{i=1}^{m} t_d(i,j) + \sum_{i=1}^{m} t_d(j,i)\right)}{U\left(\sum_{i=1}^{m} t_d(i,j) + \sum_{i=1}^{m} t_d(j,i)\right)}, \tag{7}$$

since when $U = U_2(d(j))_{max}$ all of the traffic in the network is at node j. Therefore, the value of U in $S_2(U)$ must not exceed

$$U(2)_{max} = \min_{d(i,j)} \{U_2(d(j))_{max}\}. \tag{8}$$

Considering Eqs. (5) and (8), the value of U must not exceed any of the bounds $U(1)_{max}$ and $U(2)_{max}$ and hence it cannot exceed the bound U_{max} defined by

$$U_{max} = \min\{U(1)_{max}, U(2)_{max}\}. \tag{9}$$

Define $S(U)$ as the union of $S_1(U)$ and $S_2(U)$. The number of $m \times m$ traffic matrices in $S(U)$ is $n \times m(m+1)/2$. Note that the set $S(U)$ can be augmented to include other shifts, such as two or more separate OD pairs or nodes becoming more active.

By analogy to the definition of ψ^* as our measure for the life-time of a given topology with given link capacities, routing constraints and the set of matrices S, we define the function $\Psi^*(U)$ as our measure for the life-time of a given topology with given link capacities, routing constraints and, in this case, the set of matrices $S(U)$. Notice that $\psi^* = \Psi^*(0)$.

In general, when we shift traffic, it may not be possible to route the new traffic on the underlying network. In other words, the shifted traffic may not be feasible. In this case, in order to find a feasible solution we will have to consider $\psi < 1$. At present, we are mainly interested in networks with growth potential so it is unlikely that $\psi < 1$ will have to be considered.

For any given topology, capacities, routing constraints and a set of matrices S, we now have the curve $\Psi^*(U)$ which provides the network designer with a means for comparing different topologies. Notice that one topology could have a higher $\Psi^*(0)$ than another, but when U exceeds a certain value, the situation may be reversed. This provides the designer with insight into the possible effect of unexpected uneven traffic growth on topology life-time.

The way we normalized our traffic shifts, using U and r to keep the total traffic fixed, is very important and not at all arbitrary. By normalizing our traffic, the shift is independent of the growth (signified by the ψ variable). This way we cover both shift and growth and are able to have a meaningful single curve lifetime measure which includes the effect of different types of traffic shifts. If, for example, we grow the OD pair $[i, j]$ by D and grow node i by D, without normalizing, then the increased traffic caused by growing the traffic from the node always includes the increased traffic caused by growing the traffic on the OD pair. In such a case, the optimal ψ obtained as a result of increasing node traffic activity will always be less than the optimal ψ related to the OD pair being more active and the $\Psi^*(D)$ curve will not consider link growth at all. In order to consider both link and node growth, we would need two separate curves. If we also consider growing pairs of cities, or other more complex patterns, we would need additional curves. When we normalize the traffic, so that the total load remains the same, we can include different shift patterns on the same curve. Thus, achieving a simple and useful lifetime measure. In the remainder of the paper we shall derive the curves $\Psi^*(U)$ for several specific topologies.

3 Discussion on Use and Extensions

In this paper we implicitly assume that the network load grows uniformly across all of the links and nodes in the networks, and that equal percentage deviations from the predicted growth are equally likely. This is a first approximation and it can be refined.

First, we do not expect the load to grow uniformly across the network. For example, in the United States there has been a population shift from the mid-west to the south. If the communications load increases by 10%, we may expect a 15% increase in the south and only a 5% increase in the mid-west. In a real application, growth should be our best estimate of how the network is growing, rather than uniform growth.

Second, equal percentage deviations from the expected growth are not always equally likely, particularly if the cities or links that we consider have vastly different sizes. If someone installs a server farm in a town in North Dakota, it may cause a 1000% percent increase in the traffic. This increase is much more likely than a 1000% increase in the traffic in New York City.

We can deal with the different likelihoods by weighting the percentage change according to the relative size of the entities. However, this is not a desirable solution because it will require the designer to optimise these weights which is a significant burden. Users of tools do not like to make hard decisions on many parameters. A more natural approach for networks is to analyze a hierarchy of approximately equal sized members. In Chapter 6, we consider a possible USA IP backbone network. All of the links are 2.4 Gbps in each direction. There are also lower rate access networks that we can analyze separately. In the network of equal size components it is reasonable to assume that all deviations from the expected growth are equally likely. An added advantage to the hierachical approach is that we can more easily perform our computations.

For large networks, it may not be possible (due to computing limitations) to consider both the traffic shifts from any OD pair to another as well as the traffic shifts on the nodes (e.g., one node becomes more important in terms of traffic demand than others). In this case, we recommend using just the traffic shifts on nodes. First, it significantly shortens the solution time, and second, we found empirically that in many cases, node traffic shifts subsume OD pair traffic shifts; namely, we obtain the same lifetime curve whether or not we consider traffic shift on OD pairs.

4 Topology Lifetime Measure for a Three Node Network

In this section we consider a small example of a three node network (see Figure 1) to show how to calculate the topology lifetime, namely the $\Psi^*(U)$ curve. Our three node network has a symmetrical traffic matrix, all its links are bidirectional and all its nodes have switching capabilities. When we mention an OD pair $[i, j]$, we mean both $[i, j]$ and $[j, i]$; accordingly, this three node example has three OD pairs and not six.

The traffic is represented by a single symmetric 3×3 matrix, T. That is, the set S, has only one element T. Let:

$$T = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$$

Also assume that $c_{12} = c_{13} = c_{23} = 60$ (recall that c_{ij} is the total capacity between node i and node j). This means, since the topology is bidirectional, that for every link we have 30 units of capacity in each direction.

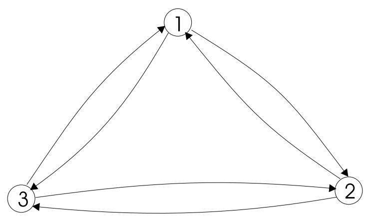

Figure 1: The three node topology

The set $S_1(U)$ has three matrices, $T_{1(1,2)}$, $T_{1(1,3)}$, and $T_{1(2,3)}$, related to the OD pairs [1,2], [1,3] and [2,3] respectively. The matrices are:

$$T_{1(1,2)} = \begin{bmatrix} 0 & 1(1+U) & 2(1-r_{1(1,2)}) \\ 1(1+U) & 0 & 3(1-r_{1(1,2)}) \\ 2(1-r_{1(1,2)}) & 3(1-r_{1(1,2)}) & 0 \end{bmatrix}$$

where $r_{1(1,2)} = U/5$.

$$T_{1(1,3)} = \begin{bmatrix} 0 & 1(1-r_{1(1,3)}) & 2(1+U) \\ 1(1-r_{1(1,3)}) & 0 & 3(1-r_{1(1,3)}) \\ 2(1+U) & 3(1-r_{1(1,3)}) & 0 \end{bmatrix}$$

where $r_{1(1,3)} = U/2$

$$T_{1(2,3)} = \begin{bmatrix} 0 & 1(1-r_{1(2,3)}) & 2(1-r_{1(2,3)}) \\ 1(1-r_{1(2,3)}) & 0 & 3(1+U) \\ 2(1-r_{1(2,3)}) & 3(1+U) & 0 \end{bmatrix}$$

where $r_{1(2,3)} = U$.

The set $S_2(U)$ also has three matrices, $T_{2,1}$, $T_{2,2}$, and $T_{2,3}$, related to the nodes 1, 2, and 3 respectively. The matrices are:

$$T_{2,1} = \begin{bmatrix} 0 & 1(1+U) & 2(1+U) \\ 1(1+U) & 0 & 3(1-r_{2(1)}) \\ 2(1+U) & 3(1-r_{2(1)}) & 0 \end{bmatrix}$$

where $r_{2(1)} = U$.

$$T_{2,2} = \begin{bmatrix} 0 & 1(1+U) & 2(1-r_{2(2)}) \\ 1(1+U) & 0 & 3(1+U) \\ 2(1-r_{2(2)}) & 3(1+U) & 0 \end{bmatrix}$$

where $r_{2(2)} = 2U$.

$$T_{2,3} = \begin{bmatrix} 0 & 1(1-r_{2(3)}) & 2(1+U) \\ 1(1-r_{2(3)}) & 0 & 3(1+U) \\ 2(1+U) & 3(1+U) & 0 \end{bmatrix}$$

where $r_{2(3)} = 5U$.

The six matrices in $S_1(U)$ and $S_2(U)$ form the set $S(U)$. To obtain the curve $\Psi^*(U)$, we first calculate ψ^* for the case $U = 0$, where ψ^* is the maximal ψ such that ψT is feasible. To find ψ^* we solve a series of feasibility problems, where we try values of ψ to determine if they are feasible. Each feasibility problem has two sets of constraints. The first set, the traffic requirement constraints, ensure that the traffic demand between any OD pair is fully satisfied. The second set (the capacity constraints) ensures that the total flow passing through any individual link of the network does not exceed the capacity of that link.

For any OD pair $[u, v]$, the traffic requirement constraint is:

$$\sum_{p \in P_{uv}} x_{uv}^p = t_{uv}$$

where t_{uv} is the traffic demand from node u to node v, P_{uv} is the set of working paths from node u to node v in the network and x_{uv}^p is the amount of flow sent from node u to node v through working path p.

The capacity constraint for a given link $(i, j) \in E$ is:

$$\sum_{\Gamma} (\sum_{p \in P_{uv}} \delta_{ij}^p x_{uv}^p) \le c_{ij}$$

where c_{ij} is the capacity of (i, j) link, Γ is the set of all OD pairs in G and δ_{ij}^p is an indicator function which is equal to one if path p uses link (i, j) and is zero otherwise.

The set of all constraints of type one and two, together with all nonnegativity constraints $(x_{uv}^p \ge 0)$, form the feasibility problem **(FP)**.

$$\begin{aligned} &\sum_{p \in P_{uv}} x_{uv}^p = t_{uv} && \text{for all} \quad u, v \in V && \textbf{(FP)} \\ &\sum_{\Gamma} (\sum_{p \in P_{uv}} \delta_{ij}^p x_{uv}^p) \le c_{ij} && \text{for all} \quad (i, j) \in E \\ &x_{uv}^p \ge 0 && \text{for all} \quad u, v \in V \text{ and for all } p \in \Gamma \end{aligned}$$

Note that t appears on the right hand side of the first set of constraints in our model.

To find ψ^* the values (t_{uv}) are substituted by (ψt_{uv}) for some $\psi \in \mathbb{R}^+$. The feasibility of **(FP)** can be checked in polynomial time using the Simplex algorithm. The only task is to find the maximum ψ value such that **(FP)** remains feasible.

Returning to our example, the bound on U, from the previous section, is $U_{max} = 0.2$. We consider the points $U = \frac{1}{8}U_{max}$, $\frac{2}{8}U_{max}$, $\frac{3}{8}U_{max}$, $\frac{4}{8}U_{max}$, $\frac{5}{8}U_{max}$, $\frac{6}{8}U_{max}$, $\frac{7}{8}U_{max}$, $\frac{8}{8}U_{max}$ and for each of these points, we derive $\Psi^*(U)$. Let $\Psi^*_{1(1,2)}(U)$, $\Psi^*_{1(1,3)}(U)$ and $\Psi^*_{1(2,3)}(U)$ be the maximal values for ψ such that $\psi T_1(1, 2)$, $\psi T_1(1, 3)$ and $\psi T_1(2, 3)$ are feasible, respectively. Then let $\Psi^*_{2(1)}(U)$, $\Psi^*_{2(2)}(U)$ and $\Psi^*_{2(3)}(U)$ be the maximal values for ψ such that $\psi t_2(1)$, $\psi t_2(2)$ and $\psi t_2(3)$ are feasible, respectively. These six optimization problems are identical to the first one performed to obtain ψ^*. After performing these optimization problems:

$$\Psi^*(U) = \min\{\Psi^*_{1(1,2)}(U), \Psi^*_{1(1,3)}(U), \Psi^*_{1(2,3)}(U), \Psi^*_{2(1)}(U), \Psi^*_{2(2)}(U), \Psi^*_{2(3)}(U)\}.$$

In Table 1 we present these results. We note, by comparing $\Psi^*_{1(1,2)}(U)$ and $\Psi^*_{2(1)}(U)$, that the normalization we have used does prevent an increase in traffic at a node from masking

U	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200
$\Psi^*_{1(1,2)}(U)$	11.721	11.721	11.721	12.208	12.208	12.208	12.208	12.208
$\Psi^*_{1(1,3)}(U)$	11.721	11.721	11.721	11.721	11.721	11.721	11.721	11.721
$\Psi^*_{1(2,3)}(U)$	11.721	11.721	11.721	11.721	11.233	11.233	11.233	11.233
$\Psi^*_{2(1)}(U)$	12.055	12.117	12.178	12.238	12.299	12.360	12.421	12.497
$\Psi^*_{2(2)}(U)$	12.055	12.117	12.178	12.238	12.299	12.360	12.421	12.497
$\Psi^*_{2(3)}(U)$	11.705	11.416	11.157	10.898	10.655	10.426	10.198	10.000

Table 1: The ψ^* values for the three node example

the effect of OD pair traffic shifts. We can also begin to see how we might use this measure as a network desgin tool. In the table, $\Psi^*(U) = \Psi^*_{2(3)}(U)$ for all values of U, implying that node 3 is the bottleneck. If we were to add capacity, we should add it at this node. This is important because it tells the designer on which links capacity should be added to improve network lifetime.

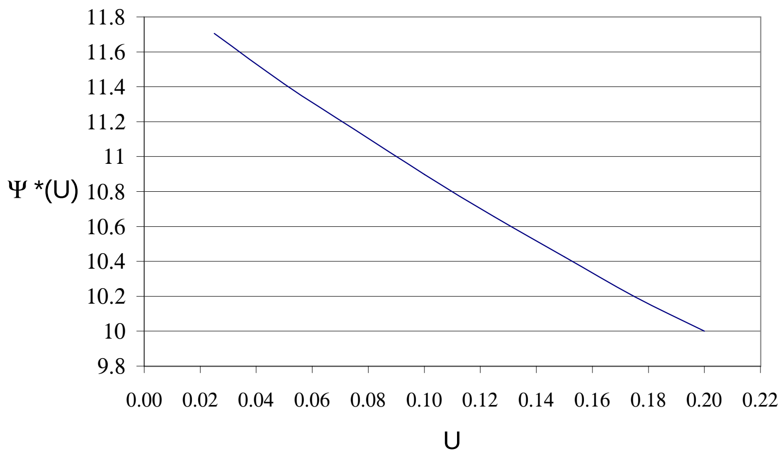

Figure 2: The lifetime function $\Psi^*(U)$ for the three node example

5 Lifetime Comparison between Several Topologies

In this section we derive the $\Psi^*(U)$ curves for a dual ring, chordal ring, Manhattan Street and hierarchical network. The first three networks are regular topologies. The hierarchical network has six nodes in the lower layer, and two in the upper layer.

All of the networks have 8 nodes and bidirectional links. For each link $c_{ij} = 1000$, so that the capacity in each direction is 500. All of the nodes have switching capabilities, except in the hierarchical network where only the nodes in the upper layer can forward traffic between nodes.

Symmetrical traffic matrices are applied to all of the networks. The set S has two elements:

$$T_1(i,j) = \begin{bmatrix} 0 & 9 & 6 & 2 & 10 & 3 & 7 & 11 \\ 9 & 0 & 11 & 3 & 19 & 6 & 14 & 22 \\ 6 & 11 & 0 & 2 & 13 & 4 & 9 & 15 \\ 2 & 3 & 2 & 0 & 4 & 1 & 3 & 4 \\ 10 & 19 & 13 & 4 & 0 & 7 & 16 & 25 \\ 3 & 6 & 4 & 1 & 7 & 0 & 5 & 8 \\ 7 & 14 & 9 & 3 & 16 & 5 & 0 & 18 \\ 11 & 22 & 15 & 4 & 25 & 8 & 18 & 0 \end{bmatrix} \quad T_2(i,j) = \begin{bmatrix} 0 & 9 & 18 & 5 & 12 & 16 & 5 & 21 \\ 9 & 0 & 12 & 3 & 8 & 11 & 3 & 14 \\ 18 & 12 & 0 & 6 & 15 & 21 & 6 & 27 \\ 5 & 3 & 6 & 0 & 4 & 6 & 2 & 7 \\ 12 & 8 & 15 & 4 & 0 & 14 & 4 & 17 \\ 16 & 11 & 21 & 6 & 14 & 0 & 6 & 24 \\ 5 & 3 & 6 & 2 & 4 & 6 & 0 & 7 \\ 21 & 14 & 27 & 7 & 17 & 24 & 7 & 0 \end{bmatrix}$$

In the implementation process, the following routing strategies in terms of the maximum hop limits have been considered . For dual ring example, seven-hop path limit is selected, which in our example it would provide all possible paths between any OD pair in the network. In the remaining examples, four-hop path limit was considered to be significantly sufficient to provide a large number of working paths between any OD pair in the network.

5.1 Dual ring

Dual rings were first used in data networks, such as FDDI [13], [7], to make the networks more reliable. They are now used to make SONET networks [11] and newly installed, long haul fiber networks more reliable. Dual rings are therefore entrenched in our telecommunications networks.

Figure 3 is the topology of the dual ring in our example.

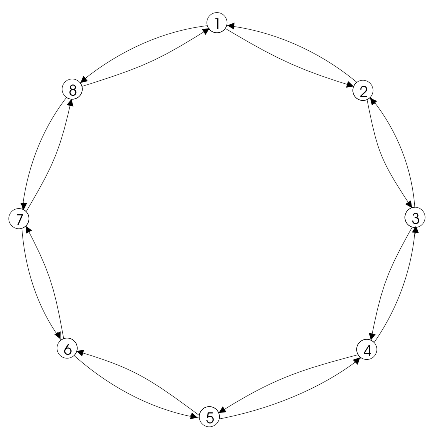

Figure 3: The eight node dual ring topology

5.2 Chordal ring

It has long been recognized that dual rings can be improved. Chordal rings [12] were studied in the early 80's. They have the same number of links as the dual rings, but can be more reliable and have a smaller average distance between nodes [6].

Figure 4 is the topology of the chordal ring in our example.

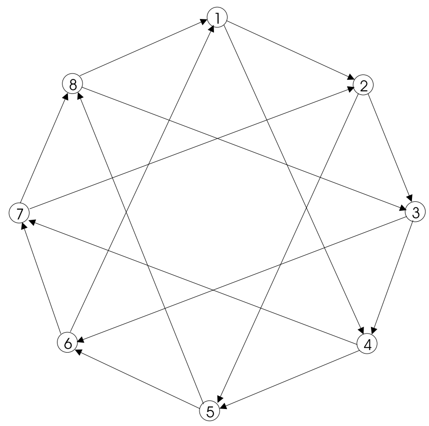

Figure 4: The eight node chordal ring topology

5.3 Manhattan Streets

At present, mesh networks, with the same number of links as dual rings, are being considered to replace these ring. One of the first regular, two-connected mesh networks was the Manhattan Street Network (MSN) [8]. This network is a grid of directed links that form rows and columns and is conceptually constructed on the surface of a torus. The adjacent rows and columns in this network have flows in opposite directions. These networks are much more reliable than the two-connected ring networks and have a shorter distance between nodes [3]. Unlike the chordal rings, there is a strategy for increasing the number of nodes in the MSN without rewiring the entire network [9].

MSN's with bidirectional links have also been studied [2], [1]. The bidirectional MSN is the basis for our undirected network.

Figure 5 is the topology of the MSN in our example.

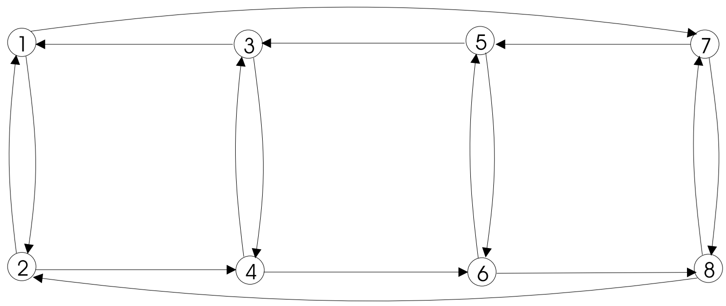

Figure 5: The eight node Manhattan Street Network Topology

5.4 Hierarchical undirected network

Our hierarchical network also has eight nodes. Six of the nodes are lower layer nodes (representing local access switches) and two are higher layer nodes (representing core network switches) as shown in Figure 6. The lower layer switches cannot switch traffic between two other nodes. A link between two lower layer nodes can only carry traffic that originates at one of the nodes and is destined for the other. This type of switching architecture is commonly used in the current telephone network.

Figure 6, is the topology of the hierarchical network in our example.

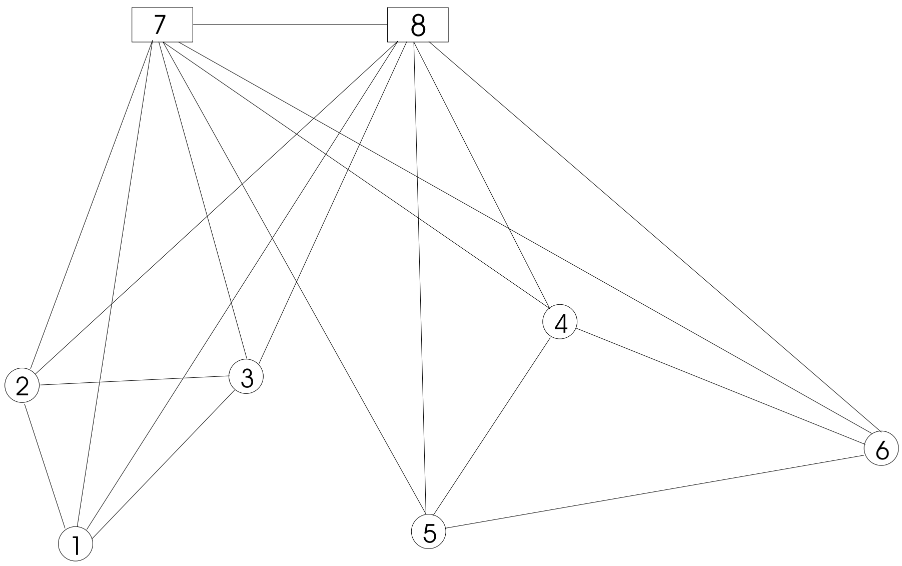

Figure 6: The eight node hierarchical network example

5.5 Lifetime comparison

Figure 7 is a plot of $\Psi(U)$ as a function of U for the four topologies. According to our metric, there is one network, the chordal ring, that can clearly support more growth than the others. There is also an example of two networks, the bidirectional loop and the Manhattan Street Network, that have different evaluations depending upon whether we want to plan for uniform growth or want to plan for perturbations of the traffic. There is also one network, the hierarchical network that is clearly inferior to the others, and will sustain the least growth.

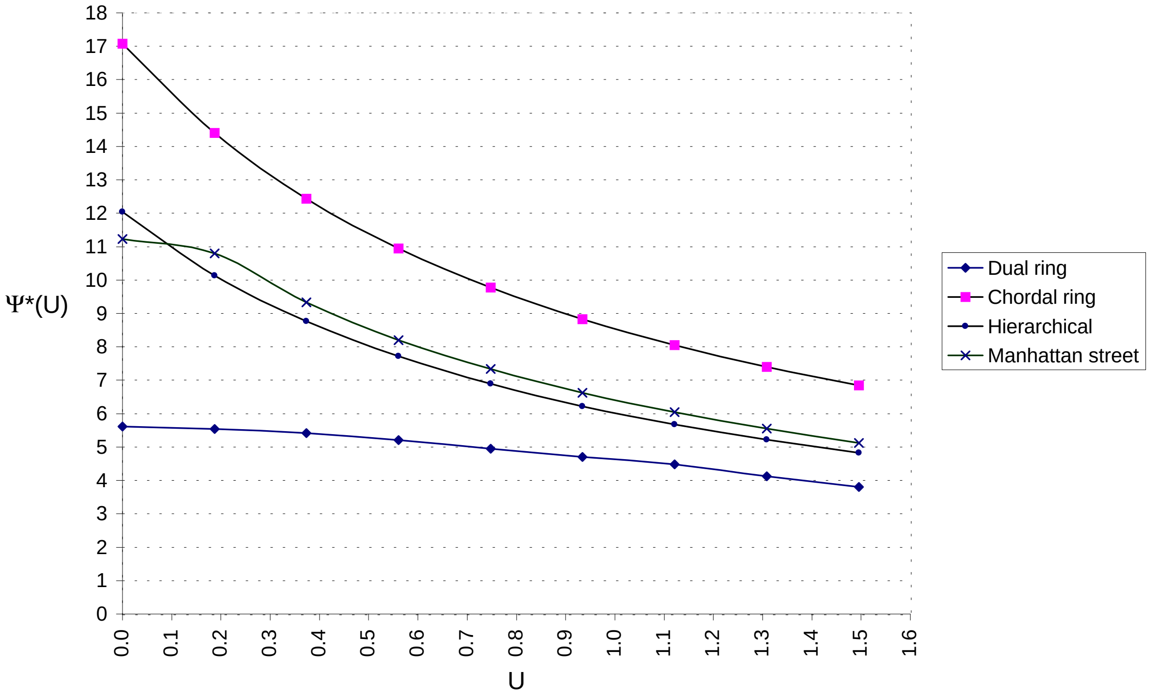

Figure 7: The lifetime function $\Psi^*(U)$ for the four topologies

It is important to understand that lifetime is not only a function of the topology. It depends very much also on the initial traffic, i.e., on the set of initial traffic matrices $S = \{[T_1(i,j)], [T_2(i,j)], ..., [T_n(i,j)]\}$. The implication of this fact is that the choice of optimal topology from the point of view of lifetime depends on the initial traffic load.

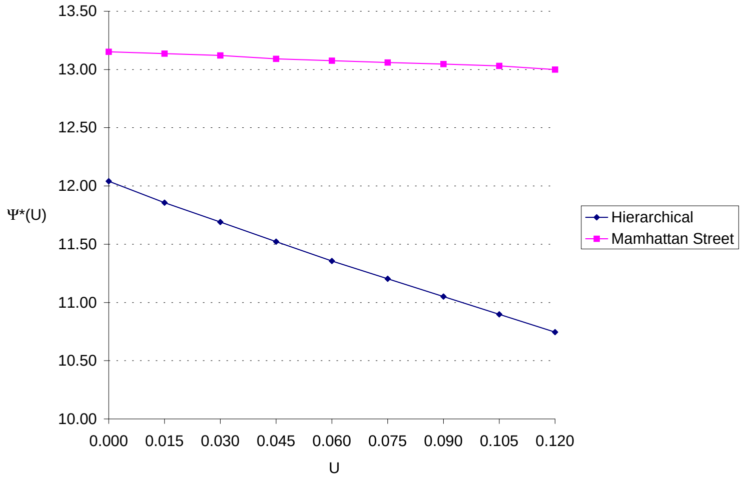

Figure 8: The lifetime function $\Psi^*(U)$ for the hierarchical and Manhattan Street topologies based on only $T_1(i, j)$

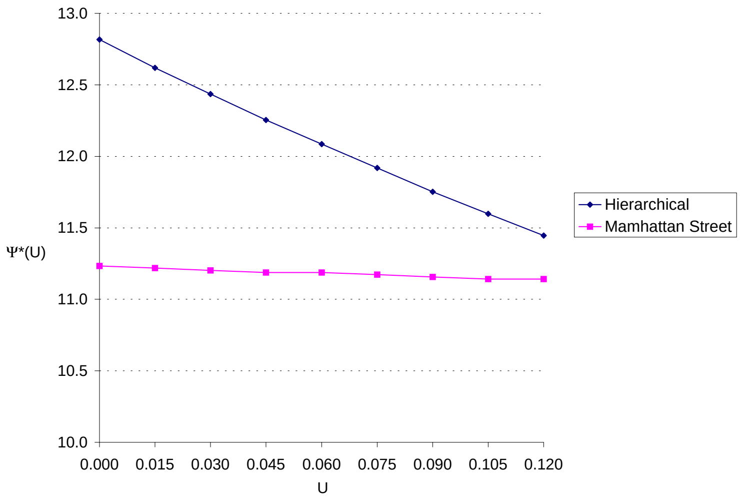

Figure 9: The lifetime function $\Psi^*(U)$ for the hierarchical and Manhattan Street topologies based on only $T_2(i, j)$

In figures 8 and 9 which demonstrate that the lifetime of a network is dependent on the initial traffic load distribution. In Figure 8 we compare between the lifetime function of the hierarchical and the Manhattan Street topologies where the set of initial traffic matrices (S) has only one element - the matrix $T_1(i,j)$, while in Figure 9, we compare between the lifetime of these two topologies under the case $S = \{[T_2(i,j)]\}$. Clearly, the results in Figure 8 demonstrate better lifetime for the Manhattan Street topology, while the results of Figure 9 show the opposite.

6 Application to an IP Backbone Network

We now consider the realistic IP backbone network presented in Figure 10. This network is similar to the part of the AT&T IP backbone network of 2Q2000 (provided in [4]) which is connected by OC48 links (all of the links are 2.4 Gbps in each direction).

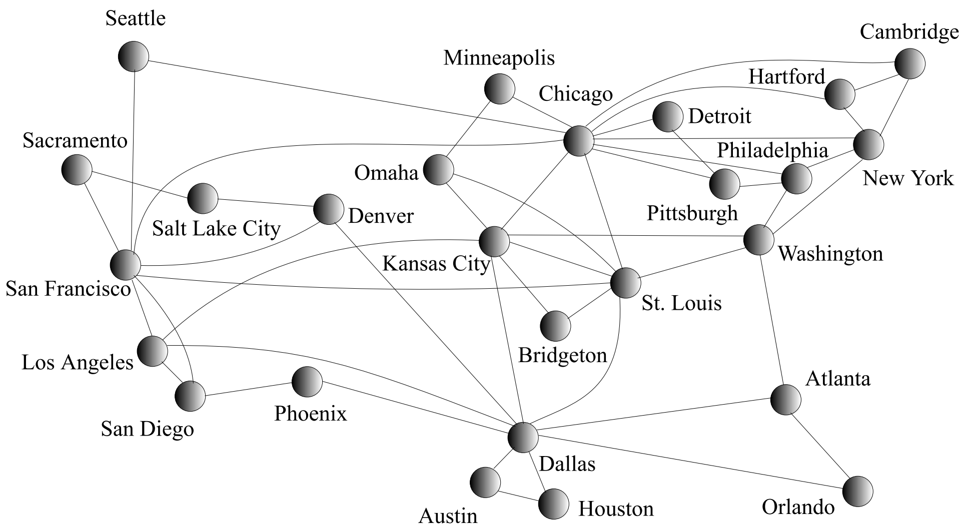

Figure 10: The IP backbone network

As discussed in Section 3, a proper way to apply the measure is in an hierarchical way, and at each layer to apply the measure to nodes and links of the same type. This way, we can expect the uniform shift principle to apply. Then, in Figure 11, we assign a number to each of the nodes of Figure 10. These numbers are then used in Figure 12 to present the OD traffic matrix we have used. The numbers in Figure 12 were randomly chosen and they do not represent any real measured traffic. The choice of symmetric traffic is also arbitrary and our model is also applicable to asymmetric traffic. We applied our procedure for this IP backbone network and we obtained the lifetime curve of Figure 13. The following assumptions were made: (1) the hop limit $k = 4$, (2) the number of working paths between any OD pair was unlimited, and (3) only node traffic shifts was considered.

From observing the lifetime curve, we conclude the following: The topology can tolerate some shifts, but for $U > 2$, the lifetime is getting close to one. We provided lifetime values of less then one to see how fast the curve decays. Of course, $\Psi^*(U)$ values of less than 1 means that the current topology cannot support such traffic shifts.

Node Number	Node Name	Node Number	Node Name
1	Cambridge	14	Houstin
2	Hartford	15	San Diego
3	New York	16	Los Angeles
4	Philadelphia	17	San Francisco
5	Pittsburgh	18	Seattle
6	Detroit	19	Minneapolis
7	Chicago	20	Omaha
8	Washington	21	Kansas City
9	St. Louis	22	Bridgeton
10	Atlanta	23	Denver
11	Orlando	24	Salt Lake City
12	Dallas	25	Sacramento
13	Austin	26	Pheonix

Figure 11: Node numbers

We also note that for the case $U = 0$, when lifetime ends 12 links become saturated together. The links are: $(8, 10)$, $(10, 12)$, $(7, 17)$, $(17, 18)$, $(7, 9)$, $(7, 21)$, $(17, 25)$, $(9, 20)$, $(12, 21)$, $(17, 23)$, $(9, 17)$, $(9, 12)$. This means that when the lifetime (assuming no shifts) reaches its end, significant investment will be required as long-range link such as Chicago - San Francisco will require upgrade. On the other hand, for the case $U = 1.3$, only five links become saturated $(7, 17)$, $(7, 9)$, $(17, 25)$, $(12, 23)$, $(9, 12)$, when lifetime ends, again, the link $(7, 17)$ (San Fransisco-Chicago)is still one of the links that reaches saturation and requires upgrade.

7 Conclusions

We have presented a new measure for comparing networks. The measure determines which networks are likely to last longer if the traffic grows in a uniform manner, and which networks are likely to last longer under shifts in load.

We are not aware of any previous measures that try to predict the ability of a network to

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	0	44	37	39	27	28	46	16	46	48	44	17	32	50	47	30	54	50	18	53	30	36	46	18	39	34
2	44	0	27	40	41	26	27	49	48	39	55	52	25	43	55	25	37	20	55	43	16	39	20	20	47	27
3	37	27	0	17	27	31	52	53	55	32	27	22	22	41	32	56	68	29	41	24	23	39	19	34	52	26
4	39	40	17	0	47	31	27	52	41	41	33	19	38	43	52	49	16	37	52	33	43	36	36	34	30	32
5	27	41	27	47	0	26	18	25	55	18	31	30	35	22	34	26	41	37	22	53	42	36	31	20	47	34
6	28	26	31	31	26	0	46	39	49	16	24	18	20	29	21	16	37	42	37	49	19	23	43	34	30	21
7	46	27	52	27	18	46	0	44	53	37	19	46	32	34	35	48	53	19	39	22	53	19	33	26	50	46
8	16	49	53	52	25	39	44	0	26	42	26	19	17	28	47	27	25	35	26	29	17	35	24	50	39	46
9	46	48	55	41	55	49	53	26	0	53	29	37	19	41	32	54	20	52	40	29	21	35	24	55	21	17
10	48	39	32	41	18	16	37	42	53	0	29	37	52	37	32	49	49	42	44	55	29	35	32	43	23	32
11	44	55	27	33	31	24	19	26	29	29	0	37	48	37	33	36	25	40	35	43	51	30	28	27	22	37
12	17	52	22	19	30	18	46	19	37	37	37	0	24	39	30	51	35	23	43	45	40	47	22	48	24	54
13	32	25	22	38	35	20	32	17	19	52	48	24	0	18	18	47	31	34	20	20	22	17	44	37	38	24
14	50	43	41	43	22	29	34	28	41	37	37	39	18	0	34	45	46	31	52	45	19	41	44	16	33	32
15	47	55	32	52	34	21	35	47	32	32	33	30	18	34	0	27	55	48	43	32	45	27	30	33	53	20
16	30	25	56	49	26	16	48	27	54	49	36	51	47	45	27	0	65	29	20	23	19	33	54	37	35	54
17	54	37	68	16	41	37	53	25	20	49	25	35	31	46	55	65	0	24	31	31	27	36	21	36	54	38
18	50	20	29	37	37	42	19	35	52	42	40	23	34	31	48	29	24	0	52	42	33	43	18	46	44	35
19	18	55	41	52	22	37	39	26	40	44	35	43	20	52	43	20	31	52	0	22	24	29	47	18	36	46
20	53	43	24	33	53	49	22	29	29	55	43	45	20	45	32	23	31	42	22	0	48	29	54	48	42	52
21	30	16	23	43	42	19	53	17	21	29	51	40	22	19	45	19	27	33	24	48	0	51	32	20	54	47
22	36	39	39	36	36	23	19	35	35	35	30	47	17	41	27	33	36	43	29	29	51	0	43	32	16	22
23	46	20	19	36	31	43	33	24	24	32	28	22	44	44	30	54	21	18	47	54	32	43	0	22	36	32
24	18	20	34	34	20	34	26	50	55	43	27	48	37	16	33	37	36	46	18	48	20	32	22	0	20	27
25	39	47	52	30	47	30	50	39	21	23	22	24	38	33	53	35	54	44	36	42	54	16	36	20	0	41
26	34	27	26	32	34	21	46	46	17	32	37	54	24	32	20	54	38	35	46	52	47	22	32	27	41	0

Figure 12: The OD traffic matrix

survive change. However, we believe that this criterion is about to become very important because of current fiber and Internet technology.

Our measure is a composite of many components. In Section 5 we show how to use the composite measure to compare topologies. These comparisons can be used to evaluate new networks or target topologies of an existing network. Earlier, in section 4 we showed how to calculate the value of the measure and how to view its components to isolate the bottlenecks in a network. This shows how to make improvements on a new topology before it is installed or to determine where links should be added in an existing network. This has been achieved for a network of realistic size in section 6. The main reason for defining and comparing the networks in Section 5 is to show how our measure is used, not to recommend one topology over another. Clearly different traffic distributions or different size networks would change the comparisons. However, it should be noted that the two networks that are currently being used in our telecommunications networks, the hierarchical network and the bidirectional ring, had the poorest performance.

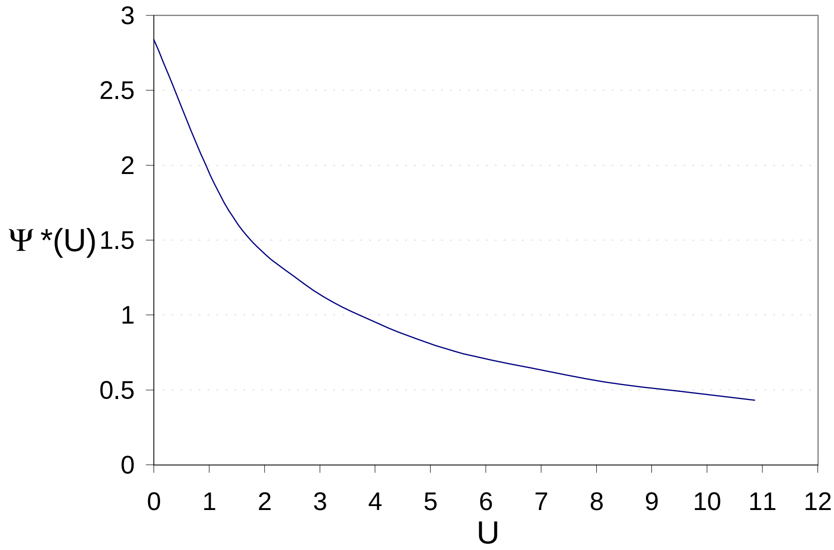

Figure 13: The lifetime curve of the IP backbone network

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