

The Fair Allocation of Power to Air Conditioners on a Smart Grid

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Abstract—Our objective is to allocate power to air conditioners fairly when demand exceeds supply or when the load in regions of the network exceeds the capacity of the links. This is an alternative to the current practice of brown outs or black outs and the proposed pricing mechanisms. We hypothesize a smart grid in which the power company can communicate with the customers, measure the temperature and power consumption of each residence, and remotely set each thermostat. We develop simple techniques to predict changes in power consumption for modest changes in the setting of a thermostat, algorithms to apply two fairness metrics used in communications networks to power allocation, and an algorithm to assign power to different parts of a network on a capacity constrained local distribution network. We also propose a practical procedure to apply these algorithms.

Index Terms—power distribution control, proportional fairness, max-min fairness, smart grid.

I. INTRODUCTION

IN a smart grid we will measure and control the distribution of power. In this paper we are concerned with the local distribution of power during peak loads, when the demand exceeds the supply [1] [2]. At present, power companies are forced to have brown-outs or black-outs to shed parts of the load. There are several proposals to reduce consumption during these periods by adjusting the price of power [3] [4]. This strategy does not reflect an increase in the cost of generating power, thereby creating wind fall profits for power companies and reducing their incentive to increase supplies. Pricing also places the greatest burdens on those who can least afford to pay. Instead of pricing we propose using fairness mechanisms that are used in communications networks to allocate flows, to allocate the available power.

In this work, we only control the power allocated to air conditioners, rather than the total power consumed by a customer. This is reasonable because most power shortages occur during heat spells and are caused by an increased use of air conditioning. In addition, air conditioning is the largest user of residential electricity, consuming 22.3% over the year 2010 [5]. By controlling air conditioning we can eliminate the need for most brown-outs and targeted black-outs.

We assume that, for each residence in a smart grid, the power company can monitor the thermostat setting, the inside and outside temperature, and the power being consumed by air conditioners and other appliances. We also assume that the power company controls the setting of the thermostat. Our objective is to describe and compare rules for controlling the thermostat settings in a fair manner.

There are many ways to define fairness. We will investigate two fairness definitions that control 1) the lowest temperature

that any residence can obtain and 2) the fraction of the requested cooling that each residence obtains. We will refer to these mechanisms as min-max fairness and proportional fairness, and describe them in section II and section III.

Our model of the local distribution network is described in section II. We assume that the network is a tree network, with the sources at the trunk of the tree and the sinks, appliances, at the leaves. The network supplies controlled appliances, the air conditioners, and uncontrolled appliances, everything else that consumes power.

The power that we can distribute to air conditioners is the power from the sources minus the power consumed by uncontrolled appliances. And, the capacity available for air conditioning on any branch of the tree is the capacity of the branch minus the power flowing through the branch for uncontrolled appliances. The power and the link capacities available for air conditioning change continuously. Our objective is to distribute as much of the power as possible, while meeting all of the capacity constraints, and, treating all of the customers fairly. The algorithms that we use are described in section III.

In section IV we apply the algorithms to several examples and show the difference in the temperature allocation when using the two fairness definitions. An important difference between min-max fairness and proportional fairness is that with proportional fairness customers can lower the temperature that they receive by requesting a lower temperature, while with min-max fairness they cannot. If we use proportional fairness, we must prevent game playing. For instance, the power company may not allow a user to request a lower temperature when there is a power shortage than the user requests when there isn't a shortage.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we: 1) Define the fairness for power assignment to specific appliances based upon two concepts of fairness in communications networks, section A; 2) Develop the relationship between the power consumed and the thermostat setting in each air conditioners, section B; 3) Specify the information that the power company maintains, and the general procedure that it follows to control air conditioners, section C; And, 4) Describe our model of the distribution network, section D.

A. Fairness

Our definition of min-max fairness is analogous to max-min fairness that is used in communication networks [6] [7] [8]. In communication networks the objective is to maximize the

minimum capacity that a customer obtains, and in the power network the objective is to minimize the maximum thermostat setting.

We define a system as being min-max fair if we cannot lower any residence's temperature without raising the temperature of a residence with a higher temperature. The objective is to distribute as much of the available power as possible without cooling any residence to a temperature that is lower than the customer's thermostat setting or exceeding the capacity on any of the branches of the distribution network leading to the residence. The goal is to make each customer as comfortable as possible without making another customer less comfortable.

The second fairness definition that we investigate is proportional fairness. In proportional fairness we try to make all of the customers equally content by giving each customer the same fraction of their requested temperature reduction. A system is proportional fair if we cannot provide any residence with a larger fraction of their requested cooling without reducing the proportion of the cooling given to those that are receiving a smaller fraction of their requested cooling. The objective of proportional fairness is also to distribute as much of the available power as possible without exceeding the capacity constraints of the distribution network. Proportional fairness has also been applied to capacity distribution in communications networks [8] [9].

B. Predicting the relationship between power and temperature

In communications networks we fairly allocate a scarce resource to the customer. In the power network, power is the scarce resource, but we are interested in fairly allocating cooling. Each residence will require a different amount of power to achieve the same cooling. In order to fairly allocate cooling, we must know the relationship between power and cooling for each residence.

The relationship between power and cooling is a function of the size and physical construction of the residence, the number and activities of the people occupying the space, the external temperature and sunlight, whether or not windows or doors are closed, the efficiency of the air conditioner, and possibly other factors. Rather than trying to determine these factors for all of the residences, we will predict the power required for each residence to reach a temperature based upon the amount of power required to reach a nearby temperature in the recent past.

We make the estimates using two models, the first assumes a linear relationship between power and cooling, and the second predicts the slope of the change in temperature with power. As the power company adjusts the thermostats, for each customer it uses the model that gave the most accurate results in its last adjustment.

The first model predicts the power consumption for the i th customer as:

$$P_{1,i} = K_{1,i} \times (T_{O,i} - T_i) \quad (1)$$

Where $T_{O,i}$ is the outside temperature at i th customer, T_i is the inside temperature, and $K_{1,i}$ is a constant that is determined from the last stable operating point of the customers residence.

The second model predicts the power consumption as:

$$P_{2,i} = K_{2,i} \times (T_{-1,i} - T_i) + P_{-1,i} \quad (2)$$

Where $T_{-1,i}$ and $P_{-1,i}$ are the temperature and power consumed at the previous operating point, and $K_{2,i}$ is the slope of the power consumption, is calculated from the previous two operating points.

The predictive rule:

When a customer first turns on his air conditioner, the power company sets his thermostat using the same rule that is applied to the other air conditioners, but does not predict the amount of power that the air conditioner requires. The power consumed by this air conditioner is assigned to the power consumption of the uncontrolled appliances until the temperature at the customer's location reaches an equilibrium point. The equilibrium point occurs when the inside temperature reaches the thermostat setting, or when the inside temperature reaches a higher temperature and the air conditioner is unable to lower the temperature further. The power company measures the average power consumed at the equilibrium point and estimates $K_{1,i}$ for this air conditioner. From this point on the air conditioner is a controlled appliance, and initially $K_{1,i}$ is used to estimate the power consumed by this air conditioner.

When the thermostat setting for the air conditioner is changed, and the inside temperature reaches a new equilibrium point, the average power is measured at the new equilibrium point. The new operating point is used to recalculate $K_{1,i}$, and the two stable operating points are used to calculate $K_{2,i}$. We continue to use $K_{1,i}$ to estimate the power that will be consumed if the thermostat setting is changed.

From this time forward, each time the inside temperature reaches a new stable operating point, we determine which of the two prediction techniques would have been more accurate, and use that prediction technique to determine the power consumed when the thermostat setting is changed. We also use the new operating point to calculate new values of $K_{1,i}$ and $K_{2,i}$.

C. The General Procedure Followed by the Power Company

To implement the power assignment algorithm, the power company must maintain information about each of the air conditioners that it is controlling.

For each air conditioner, A_i , the information is:

$$A_i = [T_{O,i}, T_{U,i}, T_{a,i}, T_i, P_i, T_{-1,i}, P_{-1,i}, K_{1,i}, K_{2,i}, S_i]$$

Where:

- $T_{O,i}$ is the outdoor temperature of customer i
- $T_{U,i}$ is the thermostat temperature set by customer i
- $T_{a,i}$ is the thermostat temperature set in the power assignment algorithm.
- T_i, P_i is the actual achieved temperature and power consumed of customer i .
- $T_{-1,i}, P_{-1,i}$ are the temperature and power consumed in the previous round.
- $K_{1,i}, K_{2,i}$ are the coefficients used in the two predictive models.
- S_i is an indicator that which predictive model is chosen by the air conditioner.

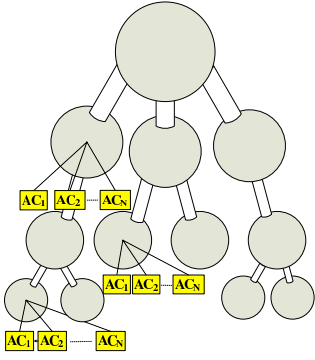


Fig. 1. Tree Distribution Network

This database is operated on by two programs that run asynchronously with respect to one another. The first program operates whenever a customer location reaches a new stable operating point. It updates the information used to predict the power that will be required at achieve a temperature, as described in section B. The second program runs continuously, and determines the thermostat settings that will fairly allocate the available power. The algorithms to determine the thermostat settings are described in section III.

D. Network Model

The power distribution network is a tree network, as shown in Fig.1. The capacity of the branches of the tree may differ, with the higher capacity links typically closer to the root of the tree. The sources deliver power to the root of the tree and the sinks are located at step down transformers at the nodes. The sinks consist of controlled appliances, the air conditioners, and uncontrolled appliances, everything else. In a more general model, there may also be smaller sources, alternative energy sources distributed on the tree, but we do not consider these sources in the current paper.

The amount of power that the power company can assign to air conditioners is the total power delivered from the sources minus the power that is delivered to the uncontrolled appliances. Each transmission link must be able to carry the power delivered to all of the controlled and uncontrolled appliances at the nodes below the link. The capacity available for air conditioning on a link is the capacity of the link minus the power that is delivered to the uncontrolled sources. The power that is delivered from sources may remain the same for extended periods, but the power consumed by the uncontrolled appliances changes continuously, which is why the power company must continuously redistribute the power to air conditioners.

III. POWER ASSIGNMENT ALGORITHM

We present the power assignment algorithm in two parts. In the first part, section A, a specific amount of power is assigned to air conditioners using both min-max and proportional fairness. In the second part, section B, the capacity available for air conditioners on each of the links of the tree network is used to determine how much power is assigned to air conditioners in different regions of the network. The same power assignment

algorithm with capacity constrain is used for both fairness mechanisms.

A. Fairness

In this section, we describe the algorithms that the power company uses to set the thermostats to achieve min-max and proportional fairness. The algorithms are applied to a group of customers when power P is available for cooling. The capacity of the distribution network does not impose any additional constraints on this group of customers.

When the power company sets the thermostats for the i th customer to $T_{a,i}$, it predicts that the customers will require power:

$$P_D = \sum_{S_i=1} K_{1,i} \times (T_{O,i} - T_{a,i}) + \sum_{S_i=2} K_{2,i} \times (T_{-1,i} - T_{a,i}) + P_{-1,i} \quad (3)$$

1) *Min-Max Fairness*: The procedure to implement min-max fairness is an iterative procedure. The power company determines the common temperature (T_P) to which it can lower the temperature for a group of customers. Those customers that request a temperature higher than this temperature are removed from the group, and the power required to achieve the temperature that they request is subtracted from the available power. The power company recalculates the value of T_P to which it can lower the remaining customers. The new T_P is lower than the previous value because the customers that were removed from the set receive less power than they would have. Therefore the remaining customers receive more power and a lower temperature. We repeat the procedure until none of the remaining customers would receive a lower temperature than they requested or all of the customers have received their requested temperature.

$A_P = A_i$ is a set of air conditioners whose thermostats will be set to T_P . Initially A_P includes all of the air conditioners. The amount of power required to reach this temperature is calculated from Eq(3), with $T_{a,i}$ set to T_P for all i . The total power available to reach T_P is P . Initially, P is all of the power that is available for air conditioning.

- i. T_P is calculated as
$$T_P = \frac{\sum_{S_i=1} (K_{1,i} \times T_{O,i}) + \sum_{S_i=2} (K_{2,i} \times T_{-1,i} + P_{-1,i}) - P}{\sum K_i}$$
- ii. Form a set $\mathbb{N}_{\mathbb{R}}$ of air conditioners with $T_{U,i} > T_P$.

If $\mathbb{N}_{\mathbb{R}}$ is empty,

set the thermostats $T_{a,i}$ of all of the air conditioners in A_P to T_P ,

stop.

Otherwise,

set the thermostats for the customers in $\mathbb{N}_{\mathbb{R}}$ to $T_{U,i}$, remove the air conditioners in $\mathbb{N}_{\mathbb{R}}$ from A_P .

If A_P is empty,

stop.

Otherwise,

reduce P by the power required for the air conditioners in $\mathbb{N}_{\mathbb{R}}$ to reach $T_{U,i}$, as calculated by Eq(3),

go to step i.

In each iteration of the algorithm, the power assigned to the customers in $\mathbb{N}_R$ is less than the power that is allocated by setting their air conditioners to T_P . The power surplus, as calculated in Eq(4), can be provided to each of the remaining customers in A_P , and T_P in the next round will be lower.

$$\begin{aligned}
 P_{surp} = & \sum_{i \in N_R} \sum_{S_i=1} K_{1,i} \times (T_{O,i} - T_P) \\
 & - \sum_{i \in N_R} \sum_{S_i=1} K_{1,i} \times (T_{O,i} - T_{U,i}) \\
 & + \sum_{i \in N_R} \sum_{S_i=2} [K_{2,i} \times (T_{-1,i} - T_P) + P_{-1,i}] \\
 & - \sum_{i \in N_R} \sum_{S_i=2} [K_{2,i} \times (T_{-1,i} - T_{U,i}) + P_{-1,i}] \quad (4)
 \end{aligned}$$

The system remains fair because the customers in customers in $\mathbb{N}_R$ who have higher thermostat settings than the customers in A_P , cannot receive a lower temperature.

2) *Proportional Fairness*: A system is defined as being proportionally fair when we cannot give any customers a larger fraction of their requested temperature reduction without taking power from customers who are receiving a smaller fraction of their requested temperature reduction. The procedure of implementing proportional fairness is straight forward. Customer i requests temperature $T_{U,i}$, and receives temperature $T_{a,i}$, where,

$$T_{O,i} - T_{a,i} = (T_{O,i} - T_{U,i}) \times \alpha_P \quad (5)$$

$\alpha_P \leq 1$ is the same for every customer. It is determined from Eq(3) and Eq(5) as:

$$\alpha_P = \min \left\{ \frac{P - \sum_{S_i=2} [K_{2,i} \times (T_{-1,i} - T_{O,i}) + P_{-1,i}]}{\sum K_i \times (T_{O,i} - T_{U,i})}, 1 \right\}$$

B. Capacity Constrains

The power available for air conditioning, P_A , is the power delivered to the system minus the power consumed by the uncontrolled appliances. If the capacity of link L_i proceeding node N_i is C_i , the power flowing to all the nodes below L_i in a tree network must flow over this link. This set of nodes below L_i is $\mathbb{N}_{C,i}$. The capacity available for air conditioning in $\mathbb{N}_{C,i}$ is $C_{N,i} = C_i - P_{U,i}$, where $P_{U,i}$ is the power being distributed to the uncontrolled devices in $\mathbb{N}_{C,i}$.

Our objective is to fairly distribute the power available for air conditioning without exceeding the capacity constraints on any links. The algorithm that we use is a discrete implementation of a water-filling algorithm. A water-filling algorithm would incrementally increases the power supplied to air conditioners from zero toward the maximum power available for air conditioning. Initially all of the air conditioners in the system would be in the set of air conditioners that are not constrained by the capacity of their links, and one of the fairness rules defined in the previous section would be used to assign power to the air conditioners. When a link reaches its capacity, we cannot assign any more power to the air conditioners below the link, and we remove them from the set of air conditioners that are receiving more power. We continue assigning more

power to air conditioners in the set until we have assigned all of the power available for air conditioning, we have reached the capacity of all of the links, or all of the air conditioners remaining in the set have reached the thermostat settings of the customers. We are treating the customers who receive less of a temperature reduction fairly because we have not provided more cooling to other customers by denying cooling to them. We had to limit their cooling because of the capacity constraints of the distribution network.

The problem with the water filling algorithm is that we may have to apply the fairness rules a large number of times between each point at which we reach the capacity constraints of a link. The discrete version of the algorithm determines the next power assignment that would saturate a link, and jumps directly to that step.

$N = N_i$ is the set of tree nodes in the Tree network. For each N_i :

$$N_i = [C_{N,i}, A_{N,i}, T_{P,i}, \alpha_{P,i}]$$

- $C_{N,i}$ is the capacity of the link leading to that node that is available for all air conditioners at or below the node.
- $\mathbb{A}_{N,i}$ is the set of air conditioners at or below node N_i .
- $T_{P,i}$ is the minimum thermostat setting for air conditioners in $\mathbb{N}_{C,i}$ if the min-max-fairness algorithm is used.
- $\alpha_{P,i}$ is the proportion temperature differential that determines the thermostat setting for the air conditioners in $\mathbb{N}_{C,i}$ when the proportional fairness algorithm is used.

Although we use both metrics in the description of the power assignment algorithm with capacity constrain, only one will be calculated depending on the fairness mechanism we select.

The discrete version of the water-filling algorithm calculates the fairness metric for the entire tree using the power available for air conditioning, and for each node using the capacity of the link leading to the node. When $T_{P,i}$ decreases, or $\alpha_{P,i}$ increases, the power assigned to air conditioners increases. Therefore, the power flows are incremented by lowering $T_{P,i}$ or increasing $\alpha_{P,i}$. If any node has a larger $T_{P,i}$, or a smaller $\alpha_{P,i}$ than the entire tree, the link leading to the branch cannot accept all of the power that would be assigned to it. If we incrementally increased the power in a water-filling algorithm, the node with the highest $T_{P,i}$ or lowest $\alpha_{P,i}$ would have saturated first. We freeze the fairness metrics for the air conditioners below this node, remove the power that they require from the available power for air conditioning, remove the flow to this node from the available capacity of its parent nodes, and repeat the procedure.

Initially, $P = P_A$, $N = N_i$, and for all i , $C_{N,i} = C_i - P_{U,i}$, and $A_{N,i}$ are all of the air conditioners at or below node N_i .

- i. Calculate T_P or α_P for all air conditioners in N using power P .
- ii. For each N_i , calculate $T_{P,i}$ or $\alpha_{P,i}$ for all $A_{N,i}$, using power $C_{N,i}$
- iii. If $T_{P,i} \leq T_P$ or $\alpha_{P,i} \geq \alpha_P$ for all i ,

stop.

- iv. Else:

- a) find the N_i with the highest $T_{P,i}$ or smallest $\alpha_{P,i}$,
- b) set the thermostats for $A_{N,i}$,

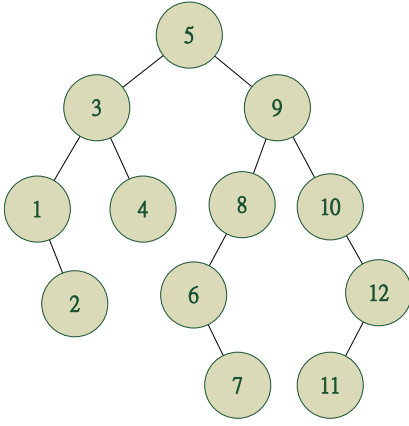


Fig. 2. Network Topology for the simulation

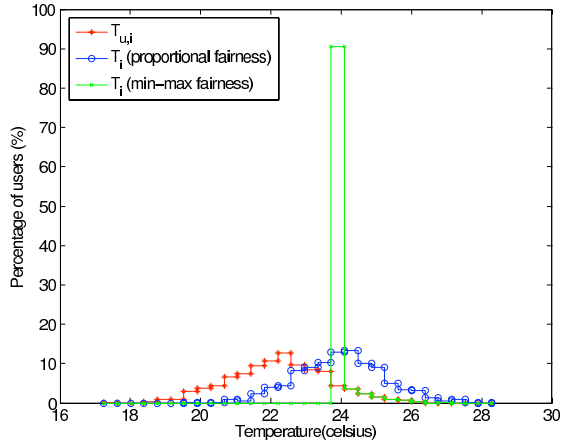


Fig. 3. Network Topology for the simulation

- c) remove N_i from N , and $P = P - C_{N,i}$.
- d) If N is empty
stop.
- e) For each N_j above N_i , $C_{N,j} = \min(C_{N,j} - C_{N,i}, 0)$, and $A_{N,j} = A_{N,j} - A_{N,i}$.
- f) **Go to step i.**

IV. SIMULATION RESULT

The network topology in our simulation is shown in Fig.2, with 840 air conditioners. The customers' thermostat settings $\{T_{U,i}\}$, are normally distributed with a mean of 22°C, and a standard deviation of 2. The outside temperatures at customer locations are also distributed normally, with a mean of 37°C and a standard deviation of 2.

The thermostat settings when fairness mechanisms are implemented in a network without capacity constraints is shown in Fig.3. The red line (asterisks) is the distribution of the customer's thermostat settings $T_{U,i}$. The green line (crosses) is the thermostat setting of the power company, T_i , with min-max fairness, and the blue line (circles) is the T_i with proportional fairness. In this example, the power required to provide all of the air conditioners with the cooling that the customers request is 6225 units, and available power for cooling is 5500 units. The power short fall for air conditioning is approximately

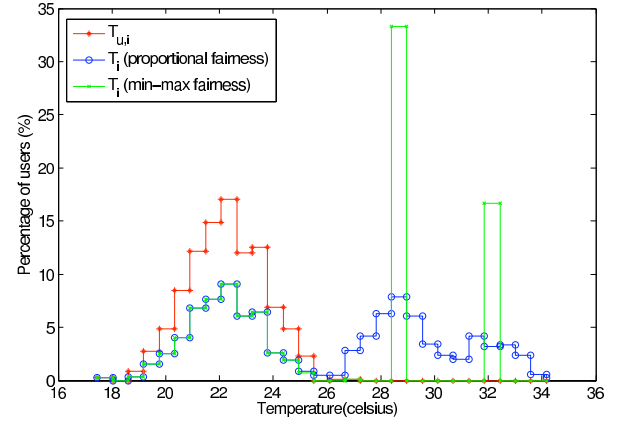


Fig. 4. Network Topology for the simulation

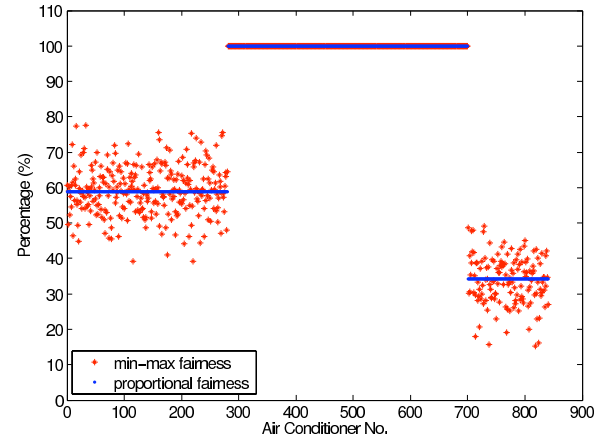


Fig. 5. Network Topology for the simulation

12%. If we assume that the energy being used for air conditioning is half of the energy being consumed, the alternative to fairly distributing power to the customers is to black out 6% of the customers. In this simulation, when we use min-max fairness, customers who request temperatures below 23.75°C have their thermostats set to 23.75°C while those who set their thermostats above that temperature have their thermostats set to their requested setting. With proportional fairness, the power company sets the thermostats for all customers at 88.35% of their requested cooling. On the average, the thermostat settings are 1.74°C higher than the customer's request. The mean thermostat settings in both min-max and proportional fairness are around 23.80°C.

To show the impact of link capacity constraints, the power allowed over L_3 for air conditioning is limited to 1200 units (58.94% of the power demand over L_3) and that allowed over L_{12} is 350 units (34.20% of the power demand over L_{12}). The thermostat settings are simulated again with both fairness mechanisms and are shown in Fig.4 and 5.

In this example, the customers that are not on a capacity constrained link receive the thermostat settings that they request. When we use min-max fairness, the customers constrained by L_3 have a minimum temperature of 28.24°C, and those constrained by L_{12} have a minimum temperature of 31.85°C. With proportional fairness, there are two shifted

versions of temperature settings distribution for the customers constrained by L_3 and L_{12} . The average thermostat settings in both fairness mechanisms are around 25.66 °C.

Fig.5 shows the percentage of the requested cooling that a customer receives. The percentage of cooling that a customer receives is a measure of how satisfied he is with the service. Approximately 280 customers under node 3, and 140 customers under node 12 are constrained by the capacity of the links. The other customers receive 100% of their requested cooling. In the area below a constrained link, proportional fairness provides customers with the same percentage of their requested cooling, thus everyone approaches the same level of satisfaction. With min-max fairness, thermostats tend to be set at the same common temperature, resulting in various levels of satisfactions.

V. CONCLUSION

This paper, we define two kinds of fairness for power distribution to air conditioners based upon the concepts of fairness from communication networks. We also specify the procedures to control the power consumed by this appliance, develop the relationship between power and temperature, and develop an algorithm to allocate power on a grid with capacity constraints. The proposed algorithms provide alternatives to black-outs or brown-outs that occasionally occur during heat waves.

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