

# The Fair Allocation of Power to Electric Vehicles on a Smart Grid

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***Abstract***-Our objective is to allocate power to electric vehicles fairly given limited power. This is an alternative of pricing strategy. We assume the charging system for electric vehicles is built on a smart grid network where the power companies can receive customers' requirements on desired traveling distance and departure time and control the power distribution. We propose and compare several fairness mechanisms to allocate power. We also develop a new metric based on customers' demands and apply it to the fairness mechanism. Strategies for preventing customers from 'gaming' the system are also presented.

***Index terms***-Power allocation control, Max-min Fairness, Electric Vehicles, Charging

## I . Introduction

With increasing concentration on clean energy, there's a trend of number of electric vehicles (EV) is growing. However, it's unlikely for power companies to construct power generation facilities in advance to meet the growing demand for charging energy, and therefore there are times when the power demand exceeds the power generation.

At present, there are several proposals to reduce power consumption during these periods by adjusting the price of power [1]. However, this strategy depends largely on customers' financial capability instead of customers' requirements, thereby damping customers' average level of satisfaction and reducing incentive to purchase electric vehicles.

In a smart grid, we are able to collect information in real time and control the distribution of power accordingly [2]. In this paper, we propose several fairness metrics and apply them to fairness mechanisms that are used in communication networks to allocate flows, to allocate

power.

We investigate two fairness definitions that control 1) the largest amount of power required among all customers and 2) the greatest delay time that any customer will suffer. We will refer to these mechanisms as min-max Energy Requirement (MMER) and min-max Delay Time (MMDT) respectively.

In this work, we only control the power allocated for charging electric vehicles at home, rather than for charging electric vehicles at any other locations. We assume that, for each residence in a smart grid, the power company can monitor the battery level of electric vehicles and will receive customer's requirements for traveling distance for the next day and their expected departure time every time when the electric vehicles are plugged in. We also assume that there's no inveracity in customer's requirements. The baseline system is Round Robin Charging System (RR). Our objective is to compare 1) the fraction of delayed EVs; 2) average delayed time for all EVs and 3) average delayed time for delayed EVs under different rules for controlling the distribution of power available for charging electric vehicles. Two important conclusions are: 1) To ensure 94% electric vehicles departing without delay, MMDT only need 5% more than the power demand, while RR need at least twice as much as power demand; 2) DDER performs worse than RR, especially when supply and demand ratio is less than 1.6.

The rest of this paper is organized as follows.

Since we assume that the power is controlled by switches which will be chosen to be turned on or off for every 5 minute, we will discuss the reasons for this controlling method in section II. Our model and baseline system will be presented in section III. In section IV, we present two fairness mechanisms, which are MMER and MMDT. In section V we apply different fairness

mechanisms to our model with real-world data, present and intuitively analyze the simulation results under series of power supply and demand ratio. In section VI, we discuss about mechanisms to prevent customers from “gaming” the system when they are asked for their expected driving distances and expected departure time. In section VII, we conclude this paper.

## II. Power Control

In this section, we are going to discuss the methods of controlling power supply, and show the advantages of the method using switches.

There are many ways to control power supply in order to meet customers’ requirements [3] [4]. For example, a possible method is to adjust charging current according to the demands. However, it’s difficult to implement the hardware system, and will cause great damage to battery [5].

However, turning a switch on/off to control power supply requires simple implementation without extra technology and causes minor battery damage.

## III. System Model and Baseline System

In this section, we: 1) Describe the model we use and data source; 2) Introduce several evaluation metrics and variables we use; 3) Present the mechanism of the baseline system.

The model we implement assumes: 1) the arrivals of vehicles for charging follows Poisson distribution, where the arrival rate changes as a function of the time of day. Most vehicles arrive during the early evening hours. In 2009 American Community Survey Reports [6], we collected the distribution of number of persons leaving for work as a function of time, shifted the time axis by 10 hours, i.e., 8 a.m. corresponds to 6 p.m., and then calculated the arrival rate; 2) the time periods available for charging (plug-in time) satisfy normal distribution, with average of time 14 hours and standard deviation of 4 hours. Any generated plug-in time which is less than 6 hours or more than 22 hours will be truncated; 3) thus the desired departure time for each vehicle will be the arrival time plus the plug-in time; 4) vehicles arrive with battery levels uniformly distributed from 0% to 30%; 5) the distance that a vehicle must travel during the commute is distributed

according to Omnibus Household Survey [7], and we fitted it to exponential distribution with mean of 1 mile. We truncated the generated data which is less than 0 mile or more than 70 miles. Considering the power needed for other purposes except for working, we added extra 20 miles to each generated random number; 6) the distribution of power available for charging during a day is the amount of power supplied minus the power consumption distribution based on available statistics [8]. It’s assumed that the power supplied by power companies remains constant for each day, but will be set to different value according to the supply and demand (the demand only refers to the total power demand for charging) ratio we pick; 7) the charging mode is a typical one with 120V, 15A, 1.8kw. 100 mile = 28 kwh. 1 kwh = 6.67 units of 5-minute interval; 8) full battery level is 100 miles, which requires 186.7 units of 5-minute interval.

We compare different fairness mechanisms in the following three respects:

1) fraction of delayed vehicles:

$$\frac{\text{number of delayed vehicles}}{\text{number of arrived vehicles}},$$

2) average delay for delayed vehicles:

$$= \frac{\text{total delay time of delayed vehicles}}{\text{number of delayed vehicles}},$$

3) average delay for all vehicles:

$$= \frac{\text{total delay time of delayed vehicles}}{\text{number of all vehicles}}.$$

In each of the respect, we also evaluate the mechanisms in series of supply and demand ratio

$$R := \frac{\text{total power supply}}{\text{total power demand}}.$$

The baseline system we use is round robin charging system (RR), since the mechanism fairly assigns power in circular order with no priorities. The RR operates as follows.

Initially, assume  $n$  cars queue to be charged, but the power company can only charge  $m$  cars ( $m < n$ ). In the  $1^{st}$  round of charging, the  $m$  cars which arrive first will be charged, at the same time, new arrivals will be added to the end of the queue. At the beginning of the  $2^{nd}$  round, the  $m$  cars which are charged will be moved to the very end of the queue, so the rest of  $n-m$  cars will be served in the  $2^{nd}$  round. Thus, at the end of  $i^{th}$  round, the queue is always

arranged as the following order: cars which are not charged in this round are followed by new arrivals, and cars which are charged in  $i^{th}$  round.

#### IV. Fairness Mechanisms

In this section, we: 1) define Min-Max Energy Requirement fairness, and present the iterations, section A; 2) define Min-Max Delay Time fairness, elaborately explain the meaning of the new metric ST, section B.

##### A. Min-Max Energy Requirement

Analogous to the max-min fairness in communication networks, our definition of min-max fairness is to minimize the maximum energy requirement that a customer demand.

We define the charging system as being min-max fair with respect to energy requirement if we cannot increase the power amount distributed to any vehicle without decreasing the power amount assigned to any vehicles with lower power demand. In other words, when assigning power to a vehicle, we will not decrease the amount assigned to other vehicles with higher power demand. The objective is to distribute energy as much as possible without exceeding customer's expected power demand or total power supply amount.

The energy requirement for every round of charging can be calculated as

$$ER = (\text{desired\_distance} / \text{full\_range} * \text{full\_battery\_energy} - \text{current\_energy}) / 5\_min\_energy * \text{charging\_times} \quad (1),$$

where *full\_range* refers to the maximum distance a car can travel if the battery is full, *5\_min\_energy* refers to the power amount achieved with 5-minute charging.

Suppose  $n$  cars need to be charged.

- i. Calculate  $ER_i$ ,  $i=1,2,\dots,n$ ;
- ii. Sort the  $n$  cars with energy requirements in descending order;
- iii. Given the power available for charging, determine the number of cars can be served  $m$ ;
- iv. Charge the first  $n$  cars in the queue;
- v. Set  $\text{charging\_times}_i = \text{charging\_times}_{i+1}$ ,  $i=1,2,\dots,m$ ;
- vi. Calculate  $ER_i$ ,  $i=1,2,\dots,n$ ,  
if  $ER_i=0$ , remove the car from the queue  
else Go to next iteration.

##### B. Min-Max Delay Time

Delay time is defined as the difference between the completion time of charging minus the expected departure time. In MMDT, we try to minimize the maximum delay time. A system is MMDT fair if we cannot shorten the delay time of a vehicle without increasing the delay time of other vehicles with shorter delay time.

In order to realize the fairness, we want delayed vehicles to be charged first. However, by simply putting high priorities to delayed vehicles can't decrease the fraction of cars that are delayed, so we propose a new metric ST to minimize both the delay time and fraction of delayed vehicles by considering both desired energy requirement and departure time.

The new metric is measured as

$$ST = (\text{Desired departure time} - \text{current time}) / 5\_minute - \text{Units of energy required} \quad (2),$$

where the term  $(\text{desired departure time} - \text{current time}) / 5\_minute$  is to convert time to number of 5-minute intervals, while the energy required is also converted to number of 5-minute intervals.

Similar as the iterations of MMER, the car will be queued in ascending order with respect to ST.

When a vehicle is not delayed, ST is positive and can be regarded as the spare time that is not used for charging. The mechanism is to minimize the maximum spare time in order to prevent the vehicle from finishing charging many hours before the desired departure time, that is, more time/energy can be saved for other vehicles. While if a vehicle is delayed, ST will become negative. According to formula (2), the more a vehicle is delayed, the smaller ST will be, and the higher priority will be put on the vehicle.

To conclude, the new metric ST guarantees: 1) for the vehicles that are not delayed, it saves spare time as much as possible for other vehicles without exceeding its own expected departure time; 2) for delayed vehicles, it put highest priorities to the vehicles that are delayed most.

#### V. Simulation

The smart grid based electric vehicles (EVs) charging management system is simulated in Matlab R2011a with three changeable parameters: supply and demand ratios (R), number of days (n), and total arrival EVs per day (N). The essential purpose of the simulation is to compare our proposed two types of fairness schemes with the baseline

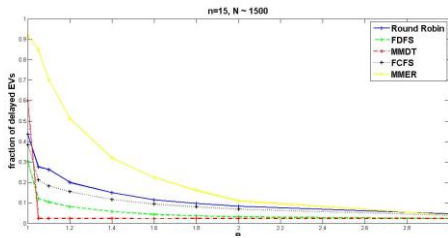


Fig.1 Fraction of delay Vs. R

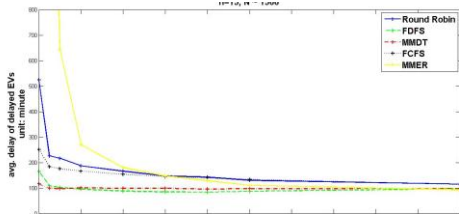


Fig.2 Avg. delay of delayed EVs Vs. R

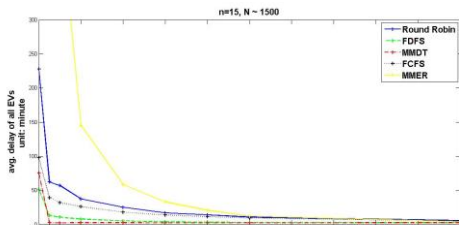


Fig.3 Avg. delay of all EVs Vs. R

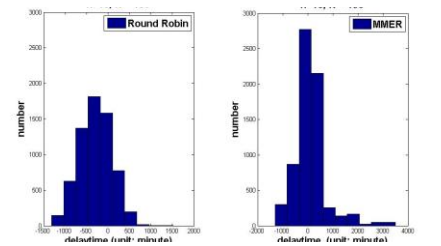


Fig.4 Delay dist. of RR and MMER at R=1.2

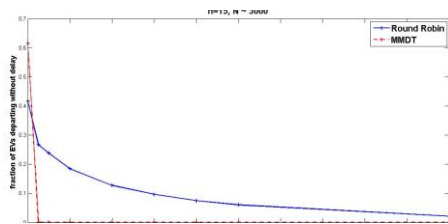


Fig.5 Fraction of EVs without delay Vs. R

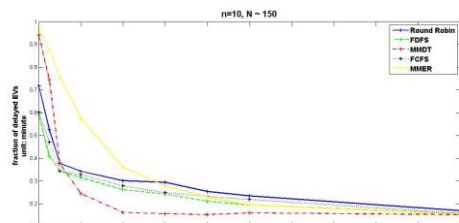


Fig.6 Fraction of delayed EVs Vs. R for Uniform desired departure time dist.

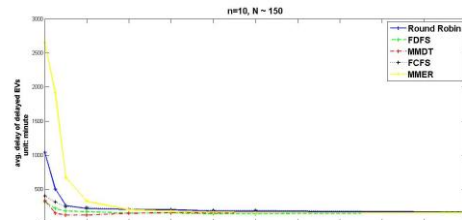


Fig.7 Avg. delay of delayed EVs Vs. R for Uniform desired departure time dist.

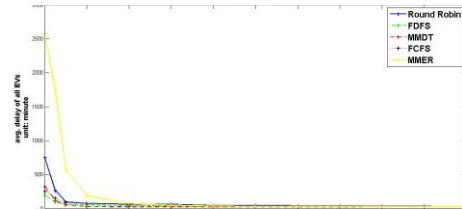


Fig.8 Avg. delay of delayed EVs Vs. R for Uniform desired departure time dist.

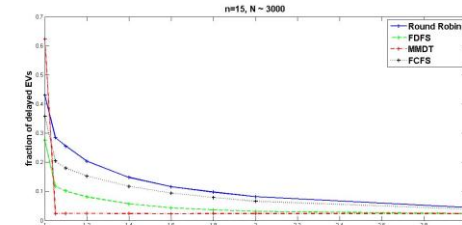


Fig.9 Fraction of delayed EVs Vs. R with N ~ 3000

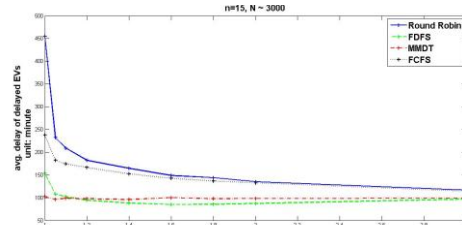


Fig.10 Avg. delay of delayed EVs Vs. R with N ~ 3000

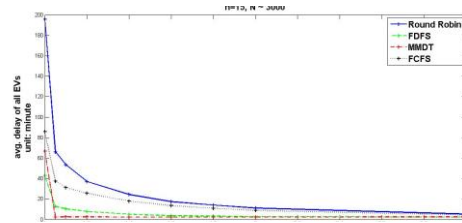


Fig.11 Avg. delay of all EVs Vs. R with N ~ 3000

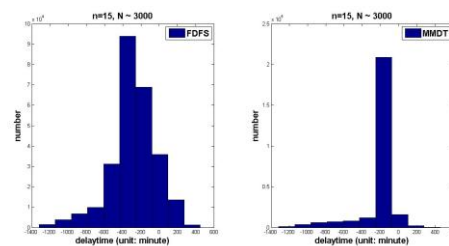


Fig.12 Delay dist. Of FDFS and MMDT at R=1.2

system and to find out the most effective charging scheme in terms of three different metrics: the fraction of delayed EVs, the average delay of delayed cars and the average delay of all cars. For the purpose of comparison and explanation of our proposed fairness schemes, two other basic charging schemes, First Come First Serve (FCFS) and First Depart First Serve (FDFS), are also included in the real simulation. FCFS gives higher priority to the cars that arrive earlier; FDFS gives higher priority to the cars that expect to depart earlier.

#### A. Simulation Setup

For each applied metric,  $R$  is ranging from 1 to 3 (increasing in steps of 0.05 from 1 to 1.1; increasing in steps of 0.2 from 1.2 to 2; increasing in steps of 1 from 2 to 3). By setting the arrival process of EVs appropriately,  $N$  is around 3000 in the simulation, though not very realistic since Obama Administration's goal is one million electric vehicles on the road by 2015, it can still help us to learn the performance of various charging scheme. To obtain accurate simulation results,  $n$  is set to be 10 and then increased to 15. However, the measurements are not taken until the 5<sup>th</sup> day so that the initializations are accurate enough and insensitive to variations. In addition, the measurements end up on the  $n - 2^{th}$  day to ensure all EVs have departed by the  $n^{th}$  day. The simulation results with  $n=10$  and  $n=15$  are quite the same. Therefore our results (Fig.1 to Fig. 12) obtained from  $n=15$  are both reasonable and reliable.

Specifically speaking, Fig.1, Fig.2, and Fig.3 are three different metrics for five charging schemes (Round Robin, FCFS, FDFS, MMER and MMDT) with respect to a series of values  $R$  using real world data, where total arrivals per day is assumed to be 1500 or so. Fig.4 is the delay distribution of MMER and the baseline schemes at  $R=1.2$  using real world data. Fig.5 is the statistical result of applying MMDT fairness scheme to real world data in comparison with the baseline system. Fig.6, Fig.7, Fig.8 are the same as Fig.1, Fig.2, Fig.3 except that the desired departure time is a uniform distribution. Fig.9, Fig.10, Fig.11 are three different metrics for four charging schemes (Round Robin, FCFS, FDFS and MMDT) with respect to a series of values  $R$  using real world data, where total arrivals per day is doubled to 3000 or so. Fig.12 is the

delay distribution of MMDT and FDFS schemes at  $R=1.2$  using real world data. Fig.13 is a toy case of MMDT fairness scheme.

#### B. Results Analysis

We will show six important conclusions and their corresponding illustrations in what follows.

1. Whatever charging scheme applies, there is always a small fraction of EVs that cannot depart on time.

It is shown in Fig.1 that all the charging schemes will bottom out to a delay fraction floor and even the best charging scheme converges to a 2.5% of delayed EVs, which means there are always 2.5% of cars cannot depart without delay and we can do nothing about it. This is because the cars request lot of energy on their arrivals while their expected departure time is too early to complete charging process, even if they were always given the highest charging priority and the available energy is more than sufficient. Therefore, this part of cars has been removed when we compare and evaluate our proposed charging schemes.

2. MMER charging scheme shows the worst performance in comparison with all other charging schemes.

It is demonstrated in Fig.1, Fig.2 and Fig.3 that, when  $R \leq 1.6$  (the available energy is 1.6 times less than the required energy), MMER fairness scheme (the yellow line) has the largest fraction of delayed EVs (nearly 50% cars are delayed at  $R < 1.2$ ), the largest amount of average delay for delayed cars (over 4.5 hours at  $R < 1.2$ ) and the largest amount of average delay for all cars (nearly 2.5 hours at  $R < 1.2$ ), all of which are totally unacceptable in practice. The reason is MMER charges the cars that require most energy first. So after a period of time, these particular cars' requested energy become fewer and their charging priority become lower accordingly. Instead, the continuously arrived cars (most of which request more energy than those cars that have been in the charging list for a long time) obtain high priority for charging and prevent the earlier arrived cars from completing their charging process. Fig.4 proves the above explanation to be reasonable by showing a large fraction of cars that can depart in advance in the baseline system are delayed, though not much, using MMER fairness scheme. What's more, the total delay time of applying MMER increases a lot compared with the

baseline system.

3. Using additional information of departure time greatly improves the performance of charging system.

For FDFS and MMDT charging schemes which take into account the information of departing time, we observe a general improvement on the fraction of delayed EVs and the average delay of all cars/ delayed cars. In particular, FDFS and MMDT ensure more than 90% cars departing without delay as long as  $R > 1.1$  (the green line and red line in Fig.1), compared with the baseline system (the blue line in Fig.1) of the same performance requiring  $R \geq 1.8$ . The goal of FDFS and MMDT can account for the drop in the fraction of delayed EVs. An interesting exception happens at  $R < 1.05$ , where the fraction of delayed EVs using MMDT fairness scheme (the red line in Fig.1) is much larger than that of the baseline system (the blue line in Fig.1). The problem is similar to MMER which we have explained before. Because MMDT minimizes the maximum delay time, when the available energy is quite insufficient, the cars with small amount of delay time have less chance to finish charging on time and the number of such cars exceeds that of the baseline system.

Moreover, the average delay of delayed cars and the average delay of all cars by employing the additional information of departure time drop down to a great extent of the baseline system in the whole range of values  $R$ . Typically, an average of 100 minutes delay of the delayed cars is observed for FDFS and MMDT (the green line and red line in Fig.2) at  $R = 1.1$ , in comparison with 217 minutes delay for the baseline system (the blue line in Fig.2) with the same supply and demand. This indicates an over 50% decrease by employing the additional information of departure time. Again at  $R = 1.1$ , the average delay of all cars for the baseline system is 57 minutes (the blue line in Fig.3), while it decreases to 7 minutes in average, 12.3% of the baseline system, for FDFS and MMDT (the green line and red line in Fig.3). We suppose these results are closely related to the system model, where plug-in time is a Gaussian distribution and desired departure time is a truncated exponential distribution. This means the majority of arrived cars require relatively fewer energy (compared to full battery energy). Hence the total delay time of FDFS and MMDT is smaller than that of the baseline system.

4. MMDT fairness scheme achieves the best performance when the available energy is a little more than the required energy.

Here we would like to compare FDFS and MMDT, both of which take advantage of the information of departure time. It is shown in Fig.1 and Fig.3 that MMDT fairness scheme (the red line) always performs better than FDFS charging scheme (the green line) in terms of the fraction of delayed EVs and the average delay of all cars when  $R \geq 1.05$  (the available energy is 5% more than the required energy). This is because MMDT compromises between the advanced departure time and the delayed departure time. Put it in another way, for the cars that can depart many hours earlier than their expected using FDFS, MMDT fairness scheme deliberately defers their departure to a late date. This can be seen clearly from Fig.12. However, considering the average delay of delayed cars, as shown in Fig.2, MMDT fairness scheme might not be the best one because it aims at minimizing the maximum delay time rather than the average delay time.

It is also noted that FDFS has a smaller fraction of delayed EVs (the green line in Fig.1) and smaller amount of average delay of all cars (the green line in Fig.3) when  $R$  is very close to 1. Let us take a simple example to illustrate why the number of delayed EVs using FDFS is less than that of MMDT when the available energy is very insufficient. Two cars A and B shown in Fig.13 each with 3.5 unit energy and 2 unit energy respectively, but only one car can be charged at each time unit (5 minute interval) due to the limited energy resources. Car A expects to depart in 15 minutes and Car B expects to depart in 10 minutes. According to MMDT fairness scheme, the car with smaller value of spare time is chosen to be charged at current time unit. So Car A is charged for the first time unit and Car B is charged for the next time unit. 10 minutes later we find that both cars must have been delayed by using MMDT. With FDFS, however, Car A would have been charged continuously on the first and second time unit (as Car A expects to depart earlier than Car B) so that it can depart on time. 10 minutes later only one car (Car B) would be delayed. Small number of delayed EVs may lead to small amount of total delay time and therefore fewer average delay of all cars when  $R < 1.05$ .

Car A (unit of 5 minute)		Car B (unit of 5 minute)		Car A	Car B
ER	Desire Depart Time – Current Time	ER	Desire Depart Time – Current Time	MMDT Metric	
3.5	3	2	2	-0.5	0
2.5	2	2	1	-1.5	-1
2.5	1	1	0	-1.5	-1

Fig.13. Example of Applying MMDT Fairness Scheme

5.MMDT also works very well as we generalize the system model to uniformly distributed desired departure time.

Since desired departure time distribution plays an important role in our project, we try the uniform distribution other than the real world distribution. The results shown in Fig.6, Fig.7 and Fig.8 demonstrate that our proposed MMDT fairness scheme still obtains the best performance when the available energy is 10% more than the required energy.

6.Increasing the number of total arrivals per day will not affect the performance of our proposed MMDT fairness scheme.

Fig. 9, Fig.10 and Fig.11 compare our proposed MMDT fairness scheme to other three charging schemes, Round Robin, FCFS and FDFS by doubling total arrival EVs per day. It is found that all conclusions obtained previously could be directly applied here.

To conclude, 1) MMER fairness scheme works poorly in terms of what metrics we applied: the fraction of delayed EVs, the average delay of delayed cars, and the average delay of all cars, because it takes no advantage of the information of departure time. It performs even worse than the baseline Round Robin system when the available energy is twice less than the required energy. 2) MMDT fairness scheme generally achieves best performance in terms of two of the three metrics: the fraction of delayed EVs and the average delay of all cars. Based upon the statistical results shown in Fig.5, MMDT fairness scheme with  $R=1.05$  makes sure 94% EVs (excluding the fraction of EVs that must be delayed whatever charging scheme is applied) departing without delay, compared with the baseline Round Robin system of the same performance

requiring  $R \geq 2$ .

## VI.Discussion on lies

Although using the additional information of desired departure time could improve the performance of electric vehicles charging management system to a great extent, we must have the additional information of desired departure time accurate and reliable. Otherwise customers could have gambled on this system.

Assuming punishment of lies is known to every customer, we came up with two strategies to impose the punishment of lies about desired departure time and desired traveling distance. 1) *Punish by fine*. We can fine customers for telling a lie appropriately. 2) *Punish by extra time*, as shown in Fig.14. The punishment is computed as the “actual departure time–claimed departure time + time of lying“. For customers who lie about their traveling distance, we first convert distance to departure time and then follow the same mechanism *punish by extra time*.

Day	Claimed Depart Time	Actual Depart Time	Punishment	Real Depart Time
1	8am	10am	$10-8+1=3$	8am
2	7am	10am	$10-7+2=5$	10am
3	5am	10am	$10-5+3=8$	10am
4	2am	10am	$10-2+4=12$	10am
5	.....			

Fig. 14. Example of Punish by Extra Time

## VII.Conclusion

In this paper, we mainly develop two kinds of fairness to distribute power for electric vehicles based on the definitions in communication networks. We also propose a new metric describing the status of electric vehicles in terms of delay time and energy requirement. The simulation results show that the mechanism using the new fairness metric performs much better than the baseline system.

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