"Principles and methods of testing finite state machines-a survey" by D. Lee and M. Yannakakis

Pass 3 on "Adaptive Distinguishing Sequence" (ADS)

1. Definition and existence

- Definition: ADS is a rooted tree T where internal nodes are input symbols and edges are output symbols. Tree T has n leaves, and each of them is a state of FSM. By applying inputs based on previously observed outputs from root of T, we can identify the initial state when we reach certain leave of T.

- Length of sequence = depth (*T*), best possible of length $(\pi^2/12)n^2$ (proved by Sokolovskii)

- A FSM can have no preset distinguish sequence (PDS) but still have ADS

- An input *a* is valid for a set of state C if $\delta(si, a) \neq \delta(sj, a)$ OR $\lambda(si, a) \neq \lambda(sj, a)$. Difference between PDS and ADS: validity w.r.t. all sets v.s. validity w.r.t. a particular set

- Algorithm for determining existence:

• Maintain a partition π , representing partition of initial states that can be distinguished

• Each time consider a valid input *a* for a block *B* in π , split the block if: 1) two states in B has different outputs, or 2) two states has same output but mapped to different blocks. If states can be partitioned into discrete sets, the ADS exist for this FSM.

Example: $\pi = \{\{s_1, s_2\}, s_3\} B = \{s_1, s_2\}$ if $\lambda(s_1, a) = \lambda(s_2, a)$ but $\delta(s_1, a) = s_1 and \delta(s_2, a) = s_3 \rightarrow \text{split} : \pi = \{\{s_1\}, \{s_2\}, \{s_3\}\}$ (discrete sets)

• Complexity: straightforward $O(pn^2)$, can be improved to O(pnlogn) (divide & conquer)

2. Construction of ADS

- Consists of two steps: 1) constructing splitting tree (ST), 2) constructing ADS from ST
- ST: internal nodes u_i = a set of states associated with sequence p. Edges are output symbols.
- Three types of valid input for a block *B* in current partition:
 - i) At least two states produce different outputs,
 - ii) All states produce same output but at least two of them are mapped to (next state belongs to) different blocks,
 - iii) All states produce same output and mapped to same block.
- Implication graph $G_{\pi} => I/O$ relationship between blocks

- Splitting Tree Algorithm:
 - Initialization: root labeled the set *S* of all states.
 - ST Expansion: (best illustrate with example..)

For type i) input: associate input symbol with current node *u*. Each child of *u* corresponds to a set of states with same output.

For type ii) input: *v* is lowest node contains the set $\delta(B, a)$, then associate string *a*-str(*v*). Each child *w* of *v*, if label(*w*) and $\delta(B, a)$ have common element => create a child of *u* labeled with $B \cap \delta^{-1}(w, a)$.

For type iii) input: find a path in G_{π} to another block has type i) or ii) input and expand the tree as for type ii)

- Example: (found a mistake?) block u_4 has type ii) input b instead of type ii), (page 11)
- ADS algorithm using ST:
 - Initialization: I = C

• Construct ADS as follows until certain leaf is reached: find lowest node u whose label contains set C, apply str(u) and update I and C.

• Example: a->aba-> ba

