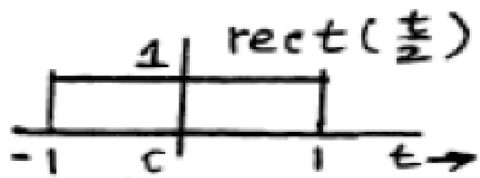
Signals and Systems

Homework #7

4.2-1

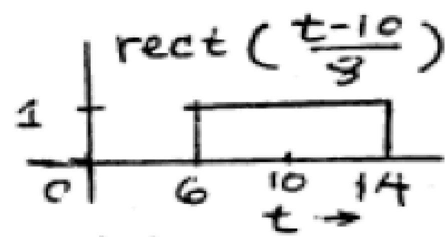
(a)

$$f(t) = rect\left(\frac{t}{2}\right)$$



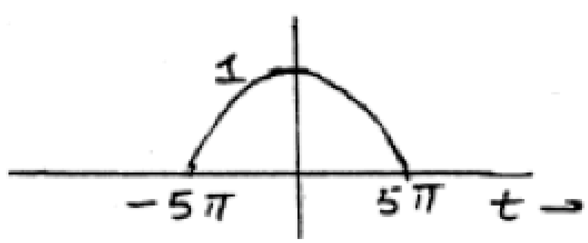
(c)

$$f(t) = rect\left(\frac{t-10}{8}\right)$$



(f)

$$f(t) = sinc\left(\frac{t}{5}\right) rect\left(\frac{t}{10\pi}\right)$$



4.2-3

$$f(t) = F^{-1}[rect(\frac{w-10}{2\pi})] = \frac{1}{2\pi}\int_{-\infty}^{\infty} rect(\frac{w-10}{2\pi}) e^{iwt}\, dt$$

$$= \frac{1}{2\pi}\int_{10-\pi}^{10+\pi} e^{iwt}\, dt = \frac{1}{i2\pi t} e^{iwt}\Big|_{10-\pi}^{10+\pi} = \frac{1}{i2\pi t}[e^{it(10+\pi)} - e^{it(10-\pi)}]$$

$$= \frac{1}{i2\pi t} e^{it10}[e^{it\pi} - e^{-it\pi}] = \frac{1}{i2\pi t} e^{it10}\, 2i\sin(t\pi)$$

$$= \frac{\sin(t\pi)}{\pi t} e^{it10} = \mathrm{sinc}(t\pi) e^{it10}$$

4.2.4

(a)

$$f(t) = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{-j\omega t_0} e^{j\omega t}$$

$$= \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega(t-t_0)} d\omega = \frac{1}{(2\pi) j(t-t_0)} e^{j\omega(t-t_0)}\Big|_{-\omega_0}^{\omega_0}$$

$$= \frac{\sin\omega_0(t-t_0)}{\pi(t-t_0)} = \frac{\omega_0}{\pi}\, \mathrm{sinc}(\omega_0(t-t_0))$$

(b)
$$f(t) = \frac{1}{2\pi}\left[\int_{-\omega_0}^{0} j e^{j\omega t} d\omega + \int_{0}^{\omega_0} -j e^{j\omega t} d\omega\right]$$

$$= \frac{1}{2\pi t} e^{j\omega t}\Big|_{-\omega_0}^{0} + \left(-\frac{1}{2\pi t} e^{j\omega t}\Big|_{0}^{\omega_0}\right)$$

$$= 1 - \frac{1}{2\pi t} e^{-j\omega_0 t} - \frac{1}{2\pi t} e^{j\omega_0 t} + 1$$

$$= \frac{1-\cos\omega_0 t}{\pi t}$$

4.3-2

Given Ft(f(t)) = F(w) = $\frac{1}{w^2}(e^{jw} - jwe^{jw} - 1)$,

1) $f_3 = f(t-1) + f(-t-1)$

   i) $f(t-1) \to F(w)e^{-jw}$ by time-shift property

   ii) $f(-t-1) \Rightarrow f(t) \to F(w)$

   - ⇨ $f(-t) \to F(-w)$ by scaling property
   - ⇨ $f(-(t+1)) \to F(-w)e^{jw}$ by time-shift property

   iii) Using the linearity of Fourier transform,

   $f_3 = f(t-1) + f(-t-1) \to F(w)e^{-jw} + F(-w)e^{jw}$

$$F(w)e^{-jw} + F(-w)e^{jw}$$

$$= \frac{1}{w^2}(e^{jw} - jwe^{jw} - 1)\, e^{-jw} + \frac{1}{w^2}(e^{-jw} + jwe^{-jw} - 1)\, e^{jw}$$

$$= \frac{1}{w^2}(1 - jw - e^{-jw} + 1 + jw - e^{jw})$$

$$= \frac{1}{w^2}(2 - 2\cos(w)) \quad = \frac{1}{w^2} 4 \sin^2\left(\frac{w}{2}\right)$$

$$= \text{sinc}^2\left(\frac{w}{2}\right)$$

2) $f_4(t) = f(t-1/2)+f(-(t+1/2)) = f(t-1/2) + f(-t-1/2)$

i) $f(t-1/2) \text{ -> } F(w)e^{-jwt/2}$ by time-shifting property

ii) $f(-t-1/2) \Rightarrow f(t) \text{ -> } F(w)$

$\Rightarrow f(-t) \text{ ->} F(-w)$ by scaling property

$\Rightarrow f(-(t+1/2)) \text{ -> } F(-w)e^{jw/2}$ by time-shift property

iii) Using the linearity of Fourier transform,

$f_4 = f(t-1/2) + f(-t-1/2) \text{ -> } F(w)e^{-jw/2} + F(-w)e^{iw/2}$

$$F(w)e^{-jw/2} + F(-w)e^{iw/2}$$

$$= \frac{1}{w^2}(e^{jw} - jwe^{jw} - 1)\, e^{-jw/2} + \frac{1}{w^2}(e^{-jw} + jwe^{-jw} - 1)\, e^{iw/2}$$

$$= \frac{1}{w^2}(e^{jw/2} - jwe^{jw/2} - e^{-jw/2} + e^{-jw/2} + jwe^{-jw/2} - e^{jw/2})$$

$$= \frac{-jw}{w^2}(e^{jw/2} - e^{-jw/2}) = \frac{2}{w}\sin(\frac{w}{2}) = \mathrm{sinc}(\frac{w}{2})$$

3) $f_5(t) = 1.5f(t/2-1) \Leftrightarrow f(t)\text{->}f(t-1)\text{->}f(t/2-1)\text{->}1.5f(t/2-1)$

i) $f(t) = F(w)$

ii) $f(t-1) = F(w)\, e^{-jw}$, by time-shifting property

iii) $f(t/2-1) = 2F(2w)\, e^{-2jw}$, by scaling property

iv) $1.5f(-t-1/2) \Rightarrow 3\, F(2w)\, e^{-2jw}$

$$3\, F(2w)\, e^{-2jw} = \frac{3}{(2w)^2}(e^{j2w} - j2we^{j2w} - 1)e^{-j2w}$$

$$= \frac{3}{4w^2}(1 - j2w - e^{-j2w})$$

4.3-10

(c) i) $f(t) = u(t+T)-u(t)-[u(t)-u(t-T)] = u(t+T)-2u(t)+u(t-T)$

Assume: $f(t) \leftrightarrow F(w)$

ii) $df/dt = \delta(t+T)-2\delta(t)+ \delta(t-T)$

iii) $df/dt \leftrightarrow jwF(w)$ by time-differentiation

iv) $df/dt = \delta(t+T)-2\delta(t)+ \delta(t-T) \leftrightarrow \exp(jwT)-2+\exp(-jwT) = jwF(w)$

,by time-shifting property

v) $F(w) = (1/jw)(2\cos wT-2) = (2j/w)(1-\cos wT)=(4j/w)(\sin^2(wT/2))$