Signals and Systems
HW#6
Solutions

4.3-6 **Fig. (a)** The signal $f(t)$ in this case is a triangle pulse $\Delta(\frac{t}{2\pi})$ (Fig. S4.3-6) multiplied by $\cos 10t$.

$$f(t) = \Delta\left(\frac{t}{2\pi}\right)\cos 10t$$

Also from Table 4.1 (pair 19) $\Delta(\frac{t}{2\pi}) \Longleftrightarrow \pi\,\mathrm{sinc}^2(\frac{\pi\omega}{2})$ From the modulation property (4.41), it follows that

$$f(t) = \Delta\left(\frac{t}{2\pi}\right)\cos 10t \Longleftrightarrow \frac{\pi}{2}\left\{\mathrm{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \mathrm{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}$$

The Fourier transform in this case is a real function and we need only the amplitude spectrum in this case as shown in Fig. S4.3-6a.

Fig. (b) The signal $f(t)$ here is the same as the signal in Fig. (a) delayed by 2π. From time shifting property, its Fourier transform is the same as in part (a) multiplied by $e^{-j\omega(2\pi)}$. Therefore

$$F(\omega) = \frac{\pi}{2}\left\{\mathrm{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \mathrm{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}e^{-j2\pi\omega}$$

The Fourier transform in this case is the same as that in part (a) multiplied by $e^{-j2\pi\omega}$. This multiplying factor represents a linear phase spectrum $-2\pi\omega$. Thus we have an amplitude spectrum [same as in part (a)] as well as a linear phase spectrum $\angle F(\omega) = -2\pi\omega$ as shown in Fig. S4.3-6b. the amplitude spectrum in this case as shown in Fig. S4.3-6b.

Note: In the above solution, we first multiplied the triangle pulse $\Delta(\frac{t}{2\pi})$ by $\cos 10t$ and then delayed the result by 2π. This means the signal in Fig. (b) is expressed as $\Delta(\frac{t-2\pi}{2\pi})\cos 10(t-2\pi)$.

We could have interchanged the operation in this particular case, that is, the triangle pulse $\Delta(\frac{t}{2\pi})$ is first delayed by 2π and then the result is multiplied by $\cos 10t$. In this alternate procedure, the signal in Fig. (b) is expressed as $\Delta(\frac{t-2\pi}{2\pi})\cos 10t$.

This interchange of operation is permissible here only because the sinusoid $\cos 10t$ executes integral number of cycles in the interval 2π. Because of this both the expressions are equivalent since $\cos 10(t-2\pi) = \cos 10t$.

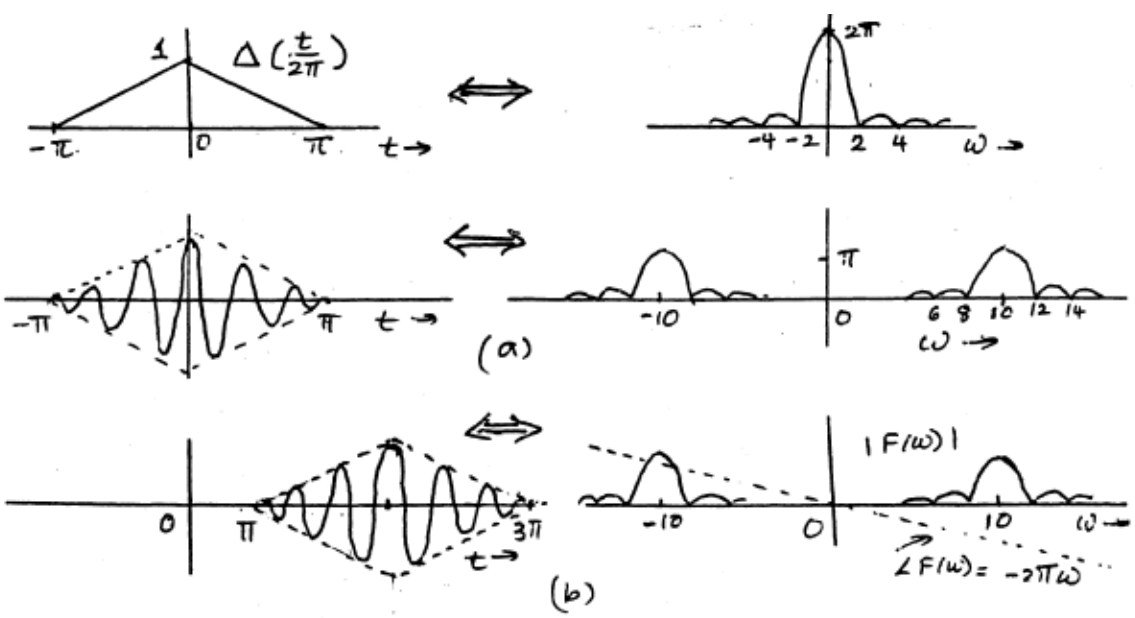

Fig. S4.3-6

4.3-7

(b)

$$F(\omega) = \triangle\left(\frac{\omega+4}{4}\right) + \triangle\left(\frac{\omega-4}{4}\right)$$

Also

$$\frac{1}{\pi}\text{sinc}^2(t) \Longleftrightarrow \triangle\left(\frac{\omega}{4}\right)$$

Therefore

$$f(t) = \frac{2}{\pi}\text{sinc}^2(t)\cos 4t$$

4.3-9 From the frequency convolution property, we obtain

$$f^2(t) \Longleftrightarrow \frac{1}{2\pi}F(\omega) * F(\omega)$$

Because of the width property of the convolution, the width of $F(\omega) * F(\omega)$ is twice the width of $F(\omega)$. Repeated application of this argument shows that the bandwidth of $f^n(t)$ is nB Hz (n times the bandwidth of $f(t)$).

4.6-4 Recall that

$$f_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_2(\omega)e^{j\omega t}\,d\omega \quad \text{and} \quad \int_{-\infty}^{\infty} f_1(t)e^{j\omega t}\,dt = F_1(-\omega)$$

Therefore

$$\begin{aligned}\int_{-\infty}^{\infty} f_1(t)f_2(t)\,dt &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f_1(t)[\int_{-\infty}^{\infty} F_2(\omega)e^{j\omega t}\,d\omega]\,dt \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_2(\omega)[\int_{-\infty}^{\infty} f_1(t)e^{j\omega t}\,dt]\,d\omega = \frac{1}{2\pi}\int F_1(-\omega)F_2(\omega)\,d\omega\end{aligned}$$

Interchanging the roles of $f_1(t)$ and $f_2(t)$ in the above development, we can show that

$$\int_{-\infty}^{\infty} f_1(t)f_2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega)F_2(-\omega)\,d\omega$$

4.6-5 Application of duality property [Eq. (4.31)] to pair 3 (Table 4.1) yields

$$\frac{2a}{t^2+a^2} \Longleftrightarrow 2\pi e^{-a|\omega|}$$

The signal energy is given by

$$E_f = \frac{1}{\pi}\int_0^\infty |2\pi e^{-a\omega}|^2\, d\omega = 4\pi \int_0^\infty e^{-2a\omega}\, d\omega = \frac{2\pi}{a}$$

The energy contained within the band (0 to W) is

$$E_W = 4\pi \int_0^W e^{-2a\omega}\, d\omega = \frac{2\pi}{a}[1 - e^{-2aW}]$$

If $E_W = 0.99E_f$, then

$$e^{-2aW} = 0.01 \quad \Longrightarrow W = \frac{2.3025}{a}\ \text{rad/s} = \frac{0.366}{a}\ \text{Hz}$$

4.7-1 (i) For $m(t) = \cos 1000t$

$$\begin{aligned}\varphi_{\text{DSB-SC}}(t) &= m(t)\cos 10,000t = \cos 1000t \cos 10,000t \\ &= \frac{1}{2}[\underbrace{\cos 9000t}_{\text{LSB}} + \underbrace{\cos 11,000t}_{\text{USB}}]\end{aligned}$$

(ii) For $m(t) = 2\cos 1000t + \cos 2000t$

$$\begin{aligned}\varphi_{\text{DSB-SC}}(t) &= m(t)\cos 10,000t = [2\cos 1000t + \cos 2000t]\cos 10,000t \\ &= \cos 9000t + \cos 11,000t + \frac{1}{2}[\cos 8000t + \cos 12,000t] \\ &= \underbrace{[\cos 9000t + \frac{1}{2}\cos 8000t]}_{\text{LSB}} + \underbrace{[\cos 11,000t + \frac{1}{2}\cos 12,000t]}_{\text{USB}}\end{aligned}$$

(iii) For $m(t) = \cos 1000t \cos 3000t$

$$\begin{aligned}\varphi_{\text{DSB-SC}}(t) &= m(t)\cos 10,000t = \frac{1}{2}[\cos 2000t + \cos 4000t]\cos 10,000t \\ &= \frac{1}{2}[\cos 8000t + \cos 12,000t] + \frac{1}{2}[\cos 6000t + \cos 14,000t] \\ &= \frac{1}{2}\underbrace{[\cos 8000t + \cos 6000t]}_{\text{LSB}} + \frac{1}{2}\underbrace{[\cos 12,000t + \cos 14,000t]}_{\text{USB}}\end{aligned}$$

This information is summarized in a table below. Figure S4.7-1 shows various spectra.

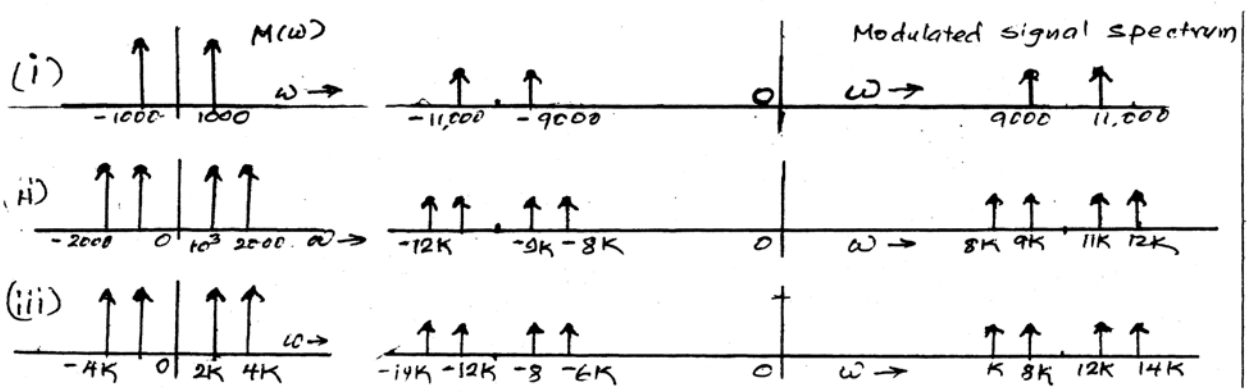

Fig. S4.7-1

case	Baseband frequency	DSB frequency	LSB frequency	USB frequency
i	1000	9000 and 11,000	9000	11,000
ii	1000	9000 and 11,000	9000	11,000
	2000	8000 and 12,000	8000	12,000
iii	2000	8000 and 12,000	8000	12,000
	4000	6000 and 14,000	6000	14,000