HW Solutions
Signals and Systems E3801 HW#3

1.4-6 **(a)** Recall that the derivative of a function at the jump discontinuity is equal to an impulse of strength equal to the amount of discontinuity. Hence, df/dt contains impulses $4\delta(t+4)$ and $2\delta(t-2)$. In addition, the derivative is -1 over the interval $(-4, 0)$, and is 1 over the interval $(0, 2)$. The derivative is zero for $t < -4$ and $t > 2$. The result is sketched in Fig. S1.4-6a.

(b) Using the procedure in part **(a)**, we find d^2f/dt^2 for the signal in Fig. P1.4-2a as shown in Fig. S1.4-6b.

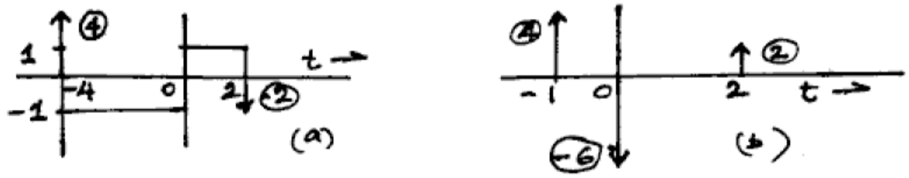


Fig. S1.4-6

1.4-7 **(a)** Recall that the area under an impulse of strength k is k. Over the interval $0 \le t \le 1$, we have

$$y(t) = \int_0^t 1\,dx = t \qquad 0 \le t \le 1$$

Over the interval $0 \le t < 3$, we have

$$y(t) = \int_0^1 1\,dx + \int_1^t (-1)\,dx = 2 - t \qquad 1 \le t < 3$$

At $t = 3$, the impulse (of strength unity) yields an additional term of unity. Thus,

$$y(t) = \int_0^1 1\,dx + \int_1^3 (-1)\,dx + \int_{3-\epsilon}^t \delta(x-3)\,dx = 1 + (-2) + 1 = 0 \qquad t > 3$$

(b)

$$y(t) = \int_0^t [1 - \delta(x-1) - \delta(x-2) - \delta(x-3) + \cdots]\,dx = tu(t) - u(t-1) - u(t-2) - u(t-3) - \ldots$$

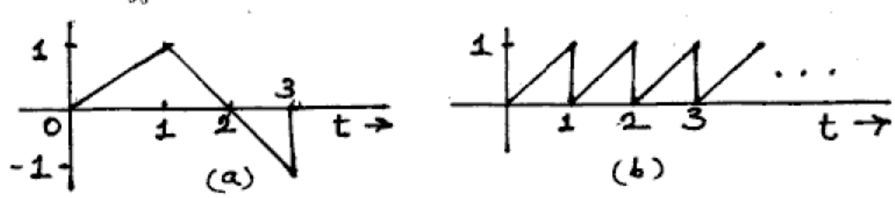


Fig. S1.4-7

1.7-1: c) Is non-linear (0 input gives non-zero output). e)is nonlinear h) is linear

1.7-2: d)is time varying f) is time invariant

(d) The system is time-varying. The input $f(t)$ yields the output $y(t) = tf(t)$. For the input $f(t-T)$, the output is $tf(t-T)$, which is not $tf(t)$ delayed by T. Hence the system is time-varying.

(f) The system is time-invariant. The input $f(t)$ yields the output $y(t)$, which is the square of the second derivative of $f(t)$. If the input is delayed by T, the output is also delayed by T. Hence the system is time-invariant.

2.4-11 **(a)** $y(t) = e^{-t}u(t) * e^{-2t}u(t) = (e^{-t} - e^{-2t})u(t)$

(c) $e^{-2t}u(t-3) = e^{-6}e^{-2(t-3)}u(t-3)$. Now from the result in part **(a)** and the shift property of the convolution [Eq. (2.34)]:

$y(t) = e^{-6}\left[e^{-(t-3)}u(t) - e^{-2(t-3)}\right]u(t-3)$

(d) $f(t) = u(t) - u(t-1)$. Now $y_1(t)$, the system response to $u(t)$ is given by

$$y_1(t) = e^{-t}u(t) * u(t) = (1 - e^{-t})u(t)$$

The system response to $u(t-1)$ is $y_1(t-1)$ because of time-invariance property. Therefore the response $y(t)$ to $f(t) = u(t) - u(t-1)$ is given by

$$y(t) = y_1(t) - y_1(t-1) = (1 - e^{-t})u(t) - [1 - e^{-(t-1)}]u(t-1)$$

The response is shown in Fig. S2.4-11.

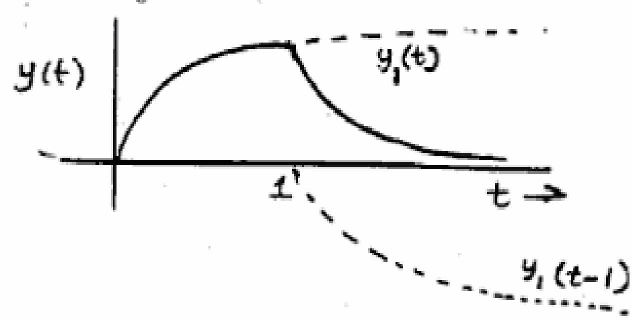


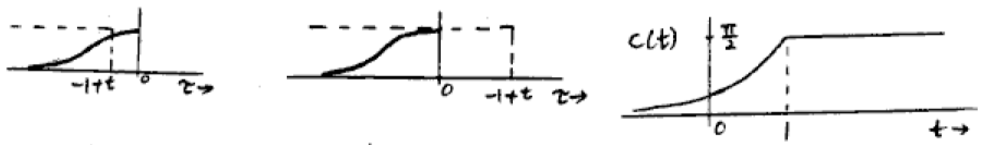
Fig. S2.4-11

2.4-16

(e)

$$c(t) = \int_{-\infty}^{-1+t} \frac{1}{\tau^2+1}\, d\tau = \tan^{-1}(t-1) + \frac{\pi}{2} \quad t \le 1$$

$$c(t) = \int_{-\infty}^{0} \frac{1}{\tau^2+1}\, d\tau = \tan^{-1}\tau \Big|_{-\infty}^{0} = \frac{\pi}{2} \quad t \ge 1$$



(e)

(h) $f_1(t) = e^t, \quad f_2(t) = e^{-2t}, \quad f_1(\tau) = e^\tau, \quad f_2(t-\tau) = e^{-2(t-\tau)}.$

$$c(t) = \int_{-1+t}^{0} e^\tau e^{-2(t-\tau)}\, d\tau = e^{-2t}\int_{-1+t}^{0} e^{3\tau}\, d\tau = \frac{1}{3}[e^{-2t} - e^{t-3}] \qquad 0 \le t \le 1$$

$$c(t) = \int_{-1+t}^{t} e^\tau e^{-2(t-\tau)}\, d\tau = e^{-2t}\int_{-1+t}^{t} e^{3\tau}\, d\tau = \frac{1}{3}[e^{t} - e^{t-3}] \qquad 0 \ge t \ge -1$$

$$c(t) = \int_{-2}^{t} e^\tau e^{-2(t-\tau)}\, d\tau = e^{-2t}\int_{-2}^{t} e^{3\tau}\, d\tau = \frac{1}{3}[e^{t} - e^{-2(t+3)}] \qquad -1 \ge t \ge -2$$

$$c(t) = 0 \qquad t \le -2$$