Homework #2 Solutions
Signals and Systems – Columbia University – Fall 2007
Due 09/20/07

1.1-6

<u>**Note**</u>:

- <u>Signal Power</u> $P_f = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2\, dt$

$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} f(t)\cdot f(t)^*\, dt$

= mean squared value of f(t)

, where $f(t)^*$ is the complex conjugate of f(t)

- <u>rms</u> (root mean-squared value) of f(t) = $\sqrt{P_f}$

- <u>In case of f(t) being a sinusoid</u>, $\lim_{T\to\infty}\int_{-T/2}^{T/2} |f(t)|^2\, dt$ is bound within finite values because of cancellations of the positive and negative areas of a sinusoid(from Textbook), so if f(t) $=\sum_{n=1}^{N} C_n \cos(w_n t+\theta_n)$, then, $P_f=\frac{1}{2}\sum_{n=1}^{N} C^2{}_n$, where none of the two sinusoids have identical frequencies.

(c) $f(t) = (10+2\sin 3t)\cos 10t$ =10cos10t+2sin3tcos10t

After using Trigonometric Identities P47

= 10cos10t+sin13t-sin7t,

Using the note, we get P_f = 1/2*(100+1+1)=51, and rms $=\sqrt{P_f} = \sqrt{51}$

(f) f(t) = $e^{i\omega t}\cos w_0 t$

$|f(t)|^2 = f(t)f(t)^* = [e^{i\omega t}\cos w_0 t]\times[e^{-i\omega t}\cos w_0 t] = \cos^2 w_0 t$

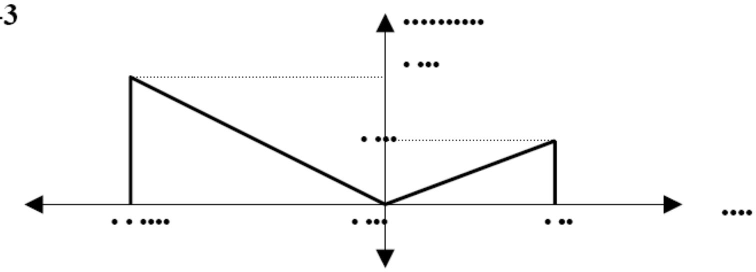
, where $f(t)^* = [e^{i\omega t}\cos w_0 t]^* = e^{-i\omega t}\cos w_0 t$

= 1/2*(1+cos2w_0t), by using sin(x)cos(y)=(sin(x-y)+sin(x+y))/2

So, $P_f = (1/4+1/4) = 1/2$, and rms = $\sqrt{P_f} = 1/\sqrt{2}$

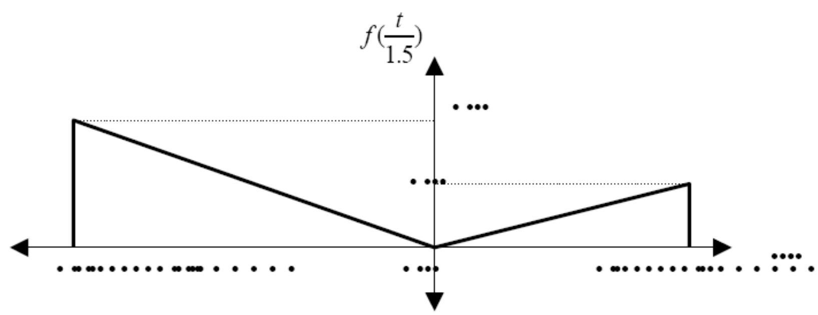
1.3-3

Fig. P1.3-3



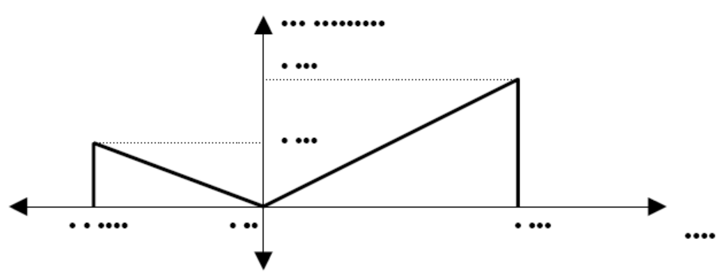
(b) $f(\frac{t}{1.5})$

Sol) It's f(t) expanded in time by factor 1.5



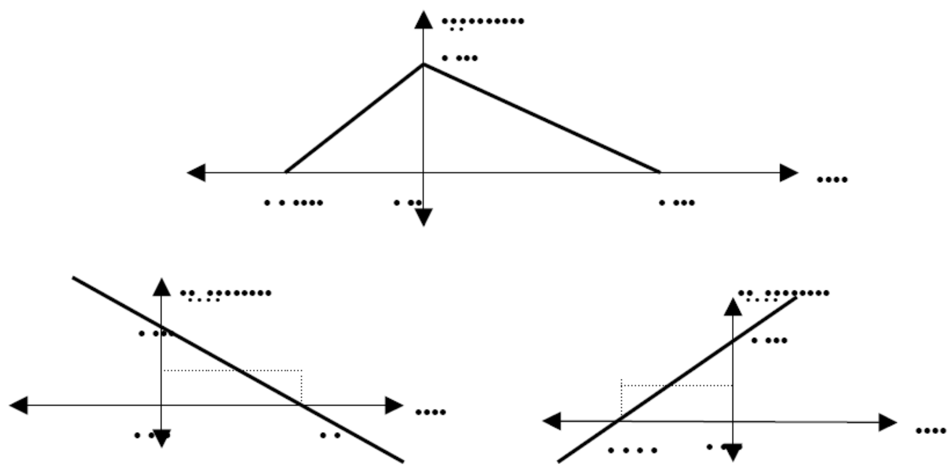
(c) f(-t)

Sol) time invert f(t), which is the mirror image of f(t) about the vertical axis.



1.4-2

1) Fig P1.4-2 (a)



$f_1(t) = f_{11}(t) + f_{12}(t)$

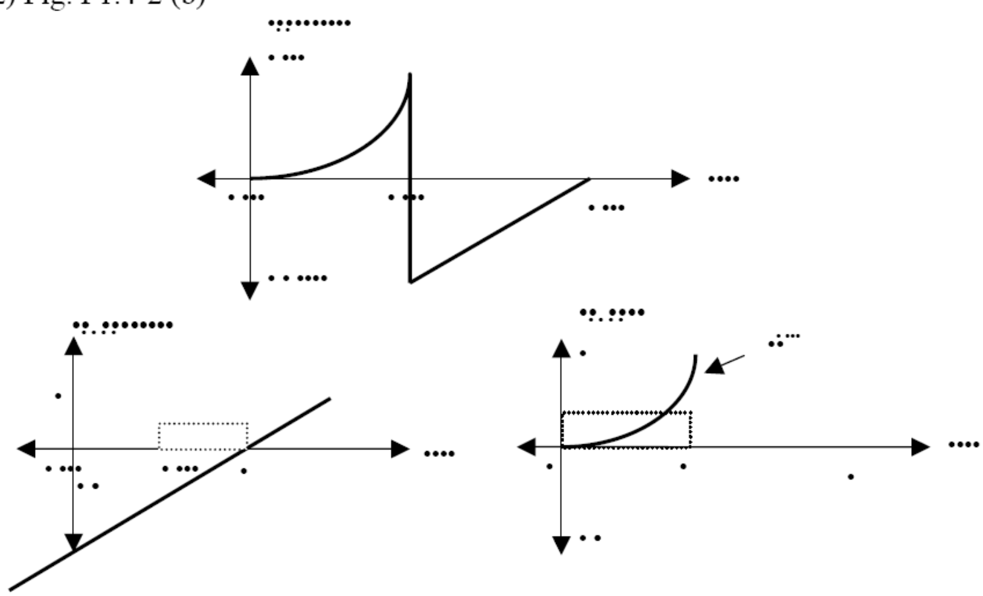
where, $f_{11}(t) = (-2t+4)[u(t)-u(t-2)]$

$f_{12}(t) = (4t+4)[u(t+1)-u(t)]$

$= (-2t+4)[u(t)-u(t-2)] + (4t+4)[u(t+1)-u(t)]$

$=(4t+4)u(t+1)-6tu(t)+(2t-4)u(t-2)$

2) Fig. P1.4-2 (b)



$f_2(t) = f_{21}(t) + f_{22}(t)$

where, $f_{21}(t) = (2t-8)[u(t-2)-u(t-4)]$

$f_{22}(t) = t^2[u(t)-u(t-2)]$

$= (2t-8)[u(t-2)-u(t-4)]+ t^2[u(t)-u(t-2)]$

$= t^2u(t)-(t^2-2t+8)u(t-2)-(2t-8)u(t-4)$

1.4-4

(a) $(\frac{\sin t}{t^2+2})\delta(t) = (\frac{\sin 0}{0^2+2})\delta(t) = 0\cdot\delta(t) = 0$

(d) $(\frac{\sin[\frac{\pi}{2}(t-2)]}{t^2+4})\delta(t-1) = (\frac{\sin[\frac{\pi}{2}(1-2)]}{1^2+4})\delta(t-1) = -\frac{1}{5}\delta(t-1)$

1.4-5

(b) $\int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)d\tau$, which is a function of variable t.

,where $\delta(t-\tau)$ = 0, at $\tau \neq t$, assuming $-\infty < t < \infty$.

So, $\int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)d\tau = \int_{-\infty}^{\infty} f(t)\delta(t-\tau)d\tau$

$= f(t)\cdot\int_{-\infty}^{\infty}\delta(t-\tau)d\tau$

$= f(t)$

(e) $\int_{-\infty}^{\infty}\delta(t+3)e^{-t}dt = \int_{-\infty}^{\infty}\delta(t+3)e^{3}dt$

$= e^3\int_{-\infty}^{\infty}\delta(t+3)dt$

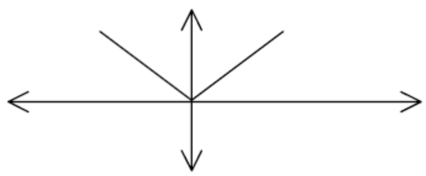
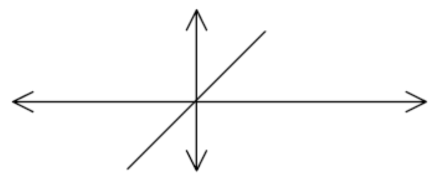
$= e^3$

1.5-1

$f(t) = f_e(t) + f_o(t)$ where $f_e(t) = 0.5[f(t) + f(-t)]$ and $f_o(t) = 0.5[f(t) - f(-t)]$.

(b) $f_e(t) = 0.5[tu(t) + -tu(-t)] = 0.5[tu(t) - tu(-t)]$

$f_o(t) = 0.5[tu(t) - -tu(-t)] = 0.5[tu(t) + tu(-t)]=0.5t$

(d) $f_e(t) = 0.5[\cos(w_0t)u(t) + \cos(-w_0t)u(-t)] = 0.5\cos w_0t$

$f_o(t) = 0.5[\cos(w_0t)u(t) - \cos(-w_0t)u(-t)]$

$= 0.5[\cos(w_0t)u(t) - \cos(w_0t)u(-t)]$

$=0.5\cos(w_0t)[u(t) -u(-t)]$