Signals And Systems
HW#10
Solutions

6.6-1

$$H(s) = \frac{s^2 + 2s}{s^3 + 8s^2 + 19s + 12} = \left(\frac{s}{s+1}\right)\left(\frac{s+2}{s+3}\right)\left(\frac{1}{s+4}\right) = \frac{-1/6}{s+1} - \frac{3/2}{s+3} + \frac{8/3}{s+4}$$

Also $$H(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

with $a_0 = 12$, $a_1 = 19$, $a_2 = 8$, and $b_0 = 0$, $b_1 = 2$, $b_2 = 1$. Figure S6.6-1 shows the canonical, series and parallel realizations.

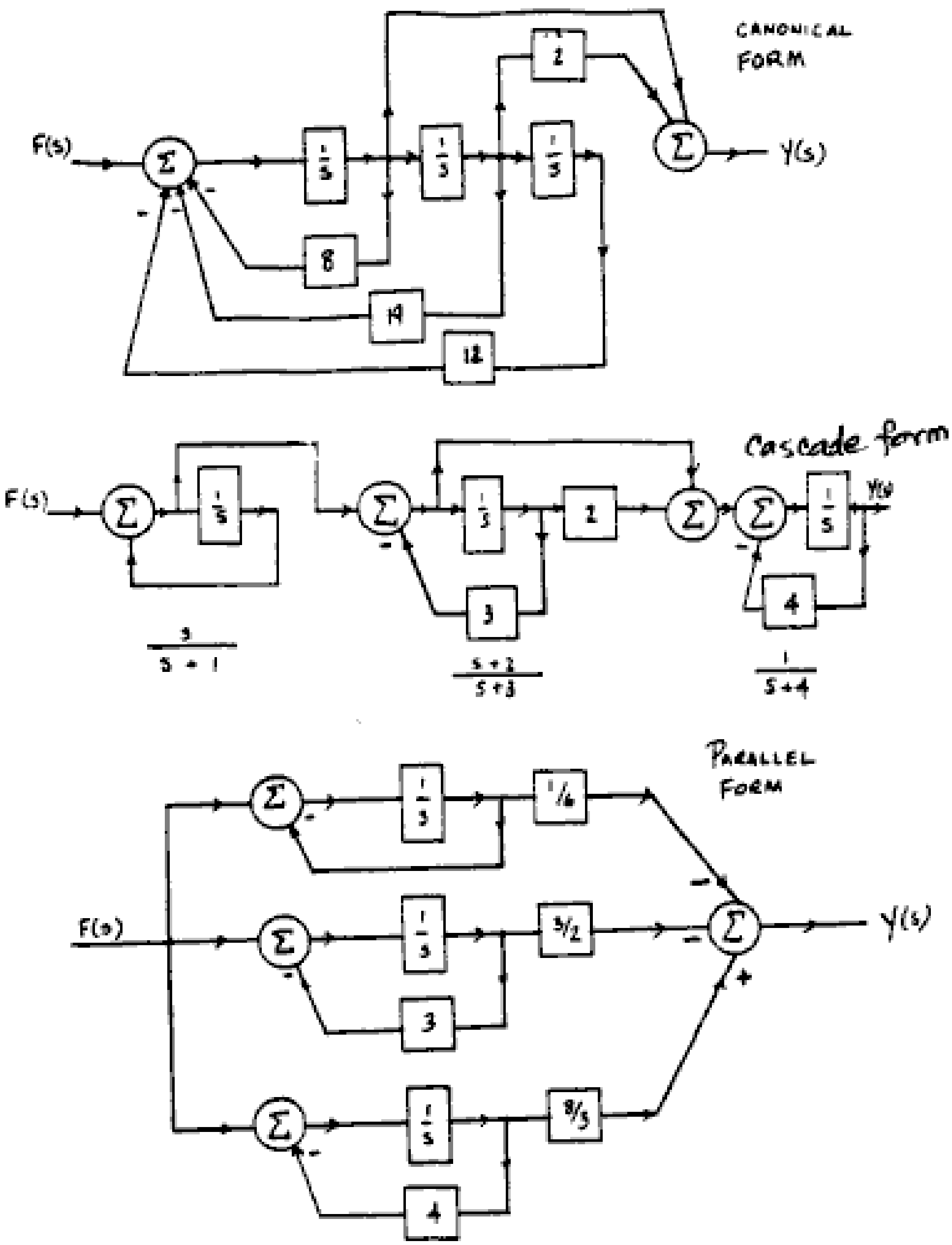
CANONICAL FORM
F(s)
Y(s)
8
19
12
2
Cascade form
$\frac{s}{s+1}$
$\frac{s+2}{s+3}$
$\frac{1}{s+4}$
3
4
PARALLEL FORM
1/6
3/2
8/3

Fig. S6.6-1

6.6-3

$$H(s) = \frac{2s+3}{5(s^4+7s^3+16s^2+12s)} = \frac{0.4s+0.6}{s^4-7s^3+16s^2+12s}$$

$$= \left(\frac{1}{s}\right)\left(\frac{1}{s+2}\right)\left(\frac{1}{s+2}\right)\left(\frac{0.4s+0.6}{s+3}\right) = \frac{\frac{1}{20}}{s} - \frac{\frac{1}{4}}{s+2} + \frac{\frac{1}{10}}{(s+2)^2} + \frac{\frac{1}{5}}{s+3}$$

Figure S6.6-3 shows a canonical, cascade and parallel realizations.

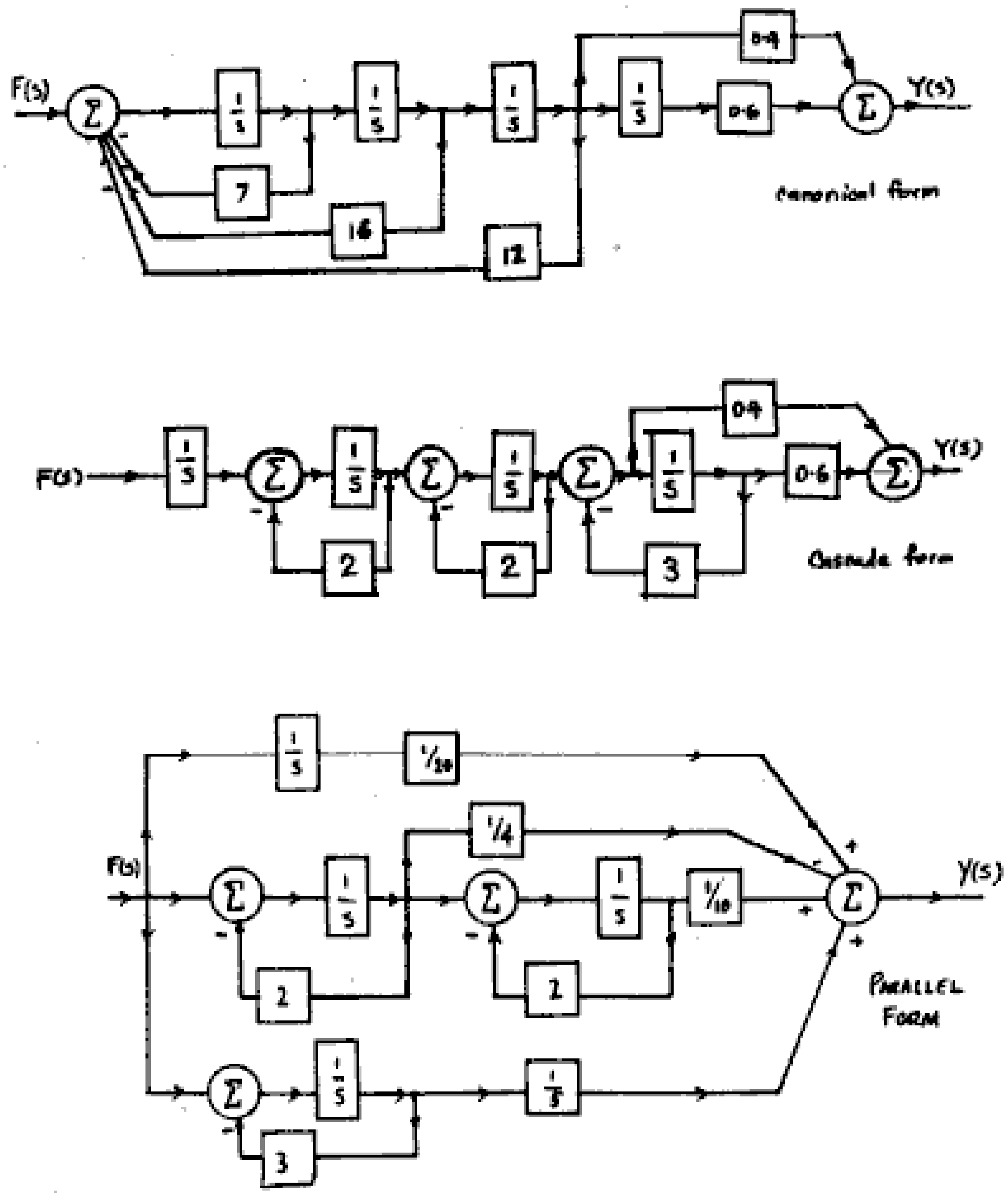

Fig. S6.6-3

13.2-6 Let us choose x_1, x_2 and x_3 as the outputs of the subsystem shown in the figure:

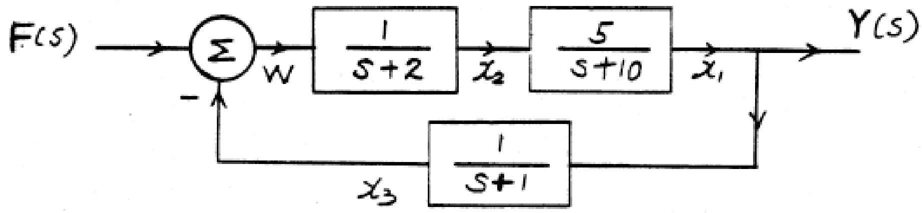

Figure S13.2-6

From the block diagram we obtain:

$$5x_2 = \dot{x}_1 + 10x_1 \Longrightarrow \dot{x}_1 = -10x_1 + 5x_2 \tag{1}$$

$$x_1 = \dot{x}_3 + x_3 \Longrightarrow \dot{x}_3 = x_1 - x_3 \tag{2}$$

$$w = \dot{x}_2 + 2x_2 \Longrightarrow \dot{x}_2 = w - 2x_2$$

$$\dot{x}_2 = -2x_2 - x_3 + f \tag{3}$$

From (1), (2) and (3) the state equations can be written as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -10 & 5 & 0 \\ 0 & -2 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} f$$

And the output equation is:

$$y = x_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

13.2-8

$$H(s) = \frac{3s + 10}{s^2 + 7s + 12}$$

Controller canonical form:
We can write the state and output equations straightforward from the transfer function $H(s)$. Thus we get:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -12 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f$$

$$y = \begin{bmatrix} 10 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

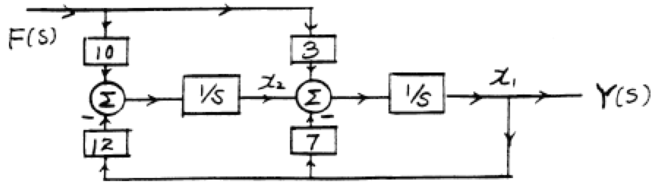

Figure S13.2-8a:observer canonical

Observer canonical form: In this case the block diagram can be drawn as shown in Fig. S6.10a.

$$\text{hence:} \qquad \dot{x}_1 = -7x_1 + x_2 + 3f$$

$$\dot{x}_2 = -12x_1 + 10f$$

or

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -7 & 1 \\ -12 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 10 \end{bmatrix} f$$

The output equation is:

$$y = x_1 = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The cascade form:

$$H(s) = \frac{3s + 10}{s^2 + 7s + 12} = \left(\frac{3s + 10}{s + 4}\right)\left(\frac{1}{s + 3}\right)$$

Hence we can write:

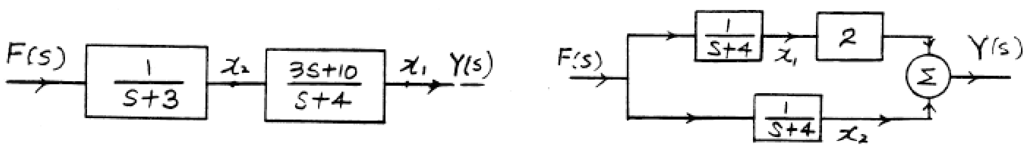

Figure S13.2-8b: cascade and parallel

$$\left.\begin{aligned} \dot{x}_1 + 4x_1 &= 3\dot{x}_2 + 10x_2 \\ \dot{x}_2 &= -3x_2 + f \end{aligned}\right\} \Longrightarrow \begin{aligned} \dot{x}_1 &= -4x_1 - 9x_2 + 10x_2 + 3f \\ \dot{x}_2 &= -3x_2 + f \end{aligned}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix} f$$

and

$$y = x_1 = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Parallel form:

$$H(s) = \frac{2}{s + 4} + \frac{1}{s + 3}$$

$$\begin{aligned} \dot{x}_1 &= -4x_1 + f \\ \dot{x}_2 &= -3x_1 + f \end{aligned} \Longrightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} f$$

And the output equation is:

$$y = 2x_1 + x_2 = [2 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

7.1-1

$$H(j\omega) = \frac{j\omega + 2}{(j\omega)^2 + 5j\omega + 4} = \frac{j\omega + 2}{(4 - \omega^2) + j5\omega}$$

$$|H(j\omega)| = \sqrt{\frac{\omega^2 + 4}{(4 - \omega^2)^2 + (5\omega)^2}} = \sqrt{\frac{\omega^2 + 4}{\omega^4 + 17\omega^2 + 16}}$$

$$\angle H(j\omega) = \tan^{-1}(\frac{\omega}{2}) - \tan^{-1}(\frac{5\omega}{4 - \omega^2})$$

(b) $f(t) = 10\sin(2t + 45°)$

$$y(t) = 10(\tfrac{\sqrt{2}}{5})\sin(2t + 45° - 45°) = 2\sqrt{2}\sin 2t$$

7.1-2

$$H(j\omega) = \frac{j\omega + 3}{(j\omega + 2)^2}$$

$$|H(j\omega)| = \frac{\sqrt{\omega^2 + 9}}{\omega^2 + 4} \quad \text{and} \quad \angle H(j\omega) = \tan^{-1}(\frac{\omega}{3}) - \tan^{-1}(\frac{\omega}{2})$$

(b) $f(t) = \cos(2t + 60°)\, u(t)$. Here $\omega = 2$

$$|H(j2)| = \tfrac{\sqrt{13}}{8} \quad \text{and} \quad \angle H(j2) = 33.69° - 90° = -56.31°$$

Therefore

$$y(t) = \tfrac{\sqrt{13}}{8}\cos(2t + 60° - 56.31°)u(t) = \tfrac{\sqrt{13}}{8}\cos(2t + 3.69°)u(t)$$

(d) $f(t) = e^{j3t}u(t)$

$$y(t) = H(j3)e^{j3t} = |H(j3)|e^{j[3t + \angle H(j3)]}u(t) = \tfrac{\sqrt{18}}{13}e^{j[3t - 67.62°]}u(t)$$