Review

I. Signals

① Time-domain signal operations

- time-shifting
- time-scaling
- time-reversal

$$\left.\begin{array}{l}\text{time-shifting}\\ \text{time-scaling}\\ \text{time-reversal}\end{array}\right\} f(at+b)$$

- convolution $f_1 * f_2$

② Fourier series for periodic signals

- Trigonometric series

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right)$$

$$\omega_0 = \frac{2\pi}{T_0}$$

$$\begin{cases} a_0 = \dfrac{1}{T_0} \displaystyle\int_{T_0} f(t)\,dt \\ a_n = \dfrac{2}{T_0} \displaystyle\int_{T_0} f(t) \cos n\omega_0 t\,dt \\ b_n = \dfrac{2}{T_0} \displaystyle\int_{T_0} f(t) \sin n\omega_0 t\,dt \end{cases}$$

- compact form: $$f(t) = C_0 + C_n \cos(n\omega_0 t + \theta_n)$$

$$\begin{cases} C_n = \sqrt{a_n^2 + b_n^2} \\ \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right) \\ C_0 = a_0 \end{cases}$$

- Exponential series:

$$f(t) = \sum_{n=-\infty}^{+\infty} D_n e^{jn\omega_0 t}$$

$$D_n = \frac{1}{T_0} \int_{T_0} f(t) e^{-jn\omega_0 t} dt$$

Relationship:

$$D_n = \frac{1}{2} C_n \cdot e^{j\theta_n} = \frac{1}{2}(a_n - j b_n)$$

$$D_{-n} = \frac{1}{2} C_n e^{-j\theta_n} = \frac{1}{2}(a_n + j b_n)$$

$$D_0 = C_0 = a_0$$

or

$$C_n = 2\left|D_n\right|$$

$$\theta_n = \measuredangle D_n$$

$$C_0 = D_0$$

Properties:

linearity

time-shifting $\quad x(t-t_0) \longleftrightarrow X_n \cdot e^{-jn\omega_0 t_0}$

time-reversal $\quad x(-t) \longleftrightarrow X_{-n}$

time-scaling $\quad x(at) = \sum_{n=-\infty}^{+\infty} X_n e^{jn(\omega_0 a)t}$

multiplication

$$x(t)\,y(t) \longleftrightarrow \sum_{l=-\infty}^{+\infty} X_l Y_{n-l}$$

conjugation $\quad x(t)^* \longleftrightarrow X_{-n}^*$

Parseval's

$$\frac{1}{T_0}\int_{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |X_k|^2$$

Symmetries :

$x(t)$: even $\longrightarrow$ $b_n = 0$

odd $\longrightarrow$ $a_n = 0$

real $\longrightarrow$ $X_{-n} = X_n^*$ $\longrightarrow$ $\begin{cases} \text{magnitude is even} \\ \text{phase is odd} \end{cases}$

real & even $\longrightarrow$ X_n is real & even

real & odd $\longrightarrow$ X_n is imag & odd

$$x_e(t) \longleftrightarrow \mathrm{Re}\{X_n\}$$

$$x_o(t) \longleftrightarrow j\,\mathrm{Im}\{X_n\}$$

③ Fourier Transform

$$\begin{cases} F(\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t}\, dt \\ f(t) = \mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)\, e^{j\omega t}\, d\omega \end{cases}$$

$$\delta(t) \longleftrightarrow 1$$

$$1 \longleftrightarrow 2\pi \cdot \delta(\omega)$$

$$e^{-at}\, u(t) \longleftrightarrow \frac{1}{a+j\omega}, \quad a>0$$

$$\frac{W}{\pi} \operatorname{sinc}(Wt) \longleftrightarrow \operatorname{rect}\left(\frac{\omega}{2W}\right)$$

· periodic signal:

$$f(t) = \sum_{n=-\infty}^{+\infty} D_n\, e^{jn\omega_0 t}$$

$$F(\omega) = 2\pi \cdot \sum_{n=-\infty}^{+\infty} D_n\, \delta(\omega - n\omega_0)$$

Properties:

- linearity
- time & freq. shifting:

$$x(t-t_0) \longleftrightarrow X(\omega)\, e^{-j\omega t_0}$$

$$x(t)\, e^{j\omega_0 t} \longleftrightarrow X(\omega-\omega_0)$$

$$x(t)\cos\omega_0 t \longleftrightarrow \frac{1}{2}\left[X(\omega-\omega_0)+X(\omega+\omega_0)\right]$$

- time-freq. duality:

$$x(t) \longleftrightarrow X(\omega) \implies X(t) \longleftrightarrow 2\pi \cdot x(-\omega)$$

- scaling : $x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$
- reversal : $x(-t) \longleftrightarrow X(-\omega)$
- convolution : $x_1(t) * x_2(t) \longleftrightarrow X_1(\omega) \cdot X_2(\omega)$

  $x_1(t) \cdot x_2(t) \longleftrightarrow \frac{1}{2\pi} \cdot X_1(\omega) * X_2(\omega)$
- differentiation : $\frac{d^n}{dt^n} x(t) \longleftrightarrow (j\omega)^n \cdot X(\omega)$
- conjugation : $x(t)^* \longleftrightarrow X(-\omega)^*$
- Parseval's : $\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \cdot \int_{-\infty}^{+\infty} |X(\omega)|^2 d\omega$

Symmetries:

$x(t)$: real $\rightarrow$ $X(-\omega) = X^*(\omega)$

$\rightarrow \begin{cases} |X(\omega)| \text{ is even} \\ \measuredangle X(\omega) \text{ is odd} \end{cases}$

real & even $\rightarrow$ $X(\omega)$ is real & even

real & odd $\rightarrow$ $X(\omega)$ is imag & odd.

$$x_e(t) \longleftrightarrow Re\{X(\omega)\}$$

$$x_o(t) \longleftrightarrow jIm\{X(\omega)\}$$

④ Sampling Theorem

- sampling: $\bar{f}(t) = f(t) \cdot \delta_{T_s}(t)$

$$\bar{F}(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} F(\omega - n\omega_s)$$

- Nyquist Thm:

  for perfect reconstruction: $F_s \geq 2B$ or $T_s \leq \frac{1}{2B}$

- Aliasing & anti-aliasing filtering

⑤ Laplace transform

- $$F(s) = \mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

$s = \sigma + j\omega$

ROC: $Re\{s\} > \sigma_0$

$\delta(t) \longleftrightarrow 1$

$u(t) \longleftrightarrow \frac{1}{s}$ $\quad Re\{s\} > 0$

$e^{at}u(t) \longleftrightarrow \frac{1}{s-a}$ $\quad Re\{s\} > Re\{a\}$

Inverse: use partial fractional expansion

✓ make the rational func. <u>proper</u>
by long division before doing
partial fractional expansion

<u>Properties:</u>

- linearity
- shifting $x(t-t_0)\,u(t-t_0) \leftrightarrow X(s)e^{-st_0}, \; t_0>0$

$$x(t)e^{s_0 t} \leftrightarrow X(s-s_0)$$

· scaling.

$$x(at) \longleftrightarrow \frac{1}{a} X\left(\frac{s}{a}\right), \quad a > 0.$$

· convolution:

$$x_1(t) * x_2(t) \longleftrightarrow X_1(s) \cdot X_2(s)$$

· differentiation:

$$\frac{d\,x(t)}{dt} \longleftrightarrow s \cdot X(s) - x(0^-)$$

$$\frac{d^2 x(t)}{dt} \longleftrightarrow s^2 \cdot X(s) - s\,x(0^-) - \dot{x}(0^-)$$

$$-t\,x(t) \longleftrightarrow \frac{d}{ds} F(s)$$

II. Systems

① Classifications:

- linear vs nonlinear
- time-invariant vs time-varying.
- Instantaneous vs dynamic.
- causal vs. noncausal

② Representations & Realizations of LTI systems

- input-output relationship via differential eqn's.
- impulse response : $h(t)$
- transfer function : $H(s)$
- state-space form :
- block diagram

with initial conditions

- realizations:
  - direct-form
  - cascade-form
  - parallel-form

3 System Analysis

$$Q(D)\, y(t) = P(D) \cdot f(t)$$

- transfer function:

$$H(s) = \frac{P(s)}{Q(s)}.$$

impulse response:

$$h(t) = \mathcal{L}^{-1}\left\{ H(s) \right\}$$

- zero-state response:

$$Y_{zs}(s) = H(s) \cdot F(s)$$

$$y_{zs}(t) = \mathcal{L}^{-1}\{Y_{zs}(s)\}$$

- zero-input response:

$$Y_{zi}(s) = \frac{g(y(0^-), \dot{y}(0^-), \cdots, y^{(n-1)}(0^-))}{Q(s)}$$

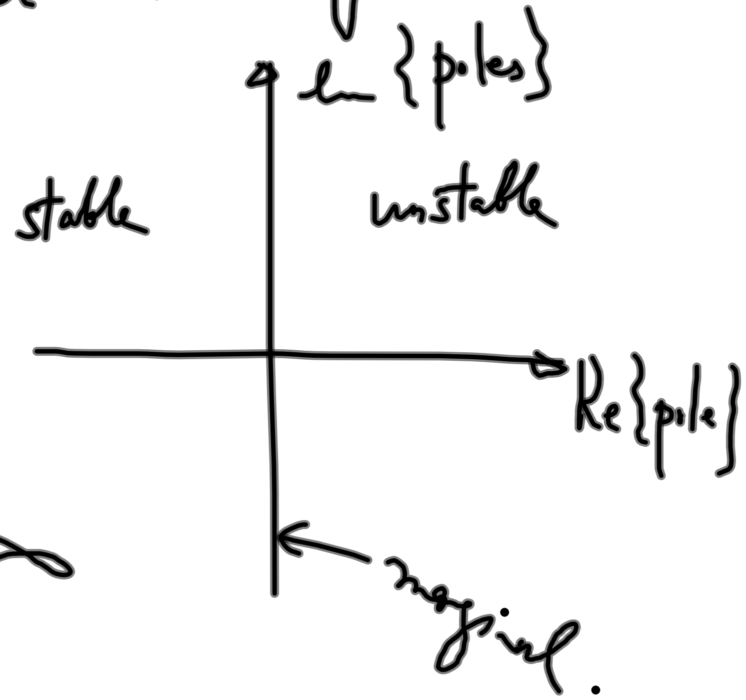
$$y_{zi}(t) = \mathcal{L}^{-1}\{Y_{zi}(s)\}$$

- Total response:

$$y(t) = y_{zs}(t) + y_{zi}(t)$$

④ System Stability:

- Asymptotic stability: determined by locations of poles



- BIBO stability:

$$\int_{-\infty}^{+\infty} |h(t)|\,dt < \infty$$

- Asymptotically stable $\Longrightarrow$ BIBO stable

unstable
marginally stable $\Longrightarrow$ BIBO unstable

⑤ Frequency Analysis for LTI system

$$\xrightarrow{f(t)} \boxed{H(s)} \xrightarrow{y(t)}$$

- $f(t) = e^{st}$. $y(t) = H(s) \cdot e^{st}$

- $f(t) = \cos(\omega_0 t + \theta)$

$$y(t) = \left|H(j\omega_0)\right| \cos\left(\omega_0 t + \theta + \angle H(j\omega_0)\right)$$

- $f(t) = \cos(\omega_0 t + \theta) u(t)$

$$y_{ss}(t) = \left|H(j\omega_0)\right| \cos\left(\omega_0 t + \theta + \angle H(j\omega_0)\right)$$