Ex: Find the freq. resp. for

(a) an ideal delay of T sec.

(b) an ideal differentiator

(c) an ideal integrator

(a) $$f(t) \longrightarrow \boxed{\delta(t-T)} \longrightarrow f(t-T)$$

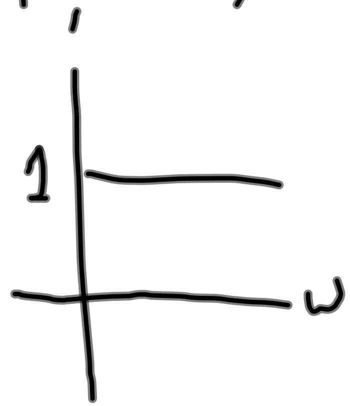
$$\therefore H(s) = e^{-sT}$$

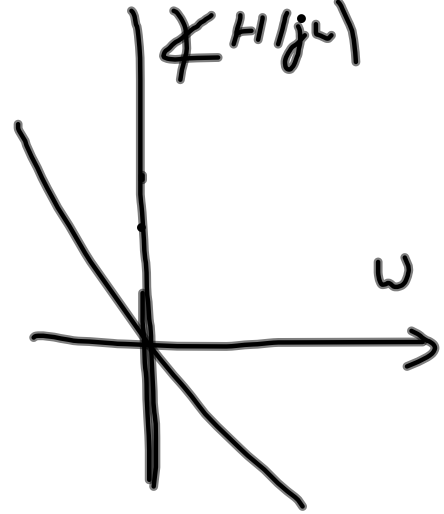
$$\Longrightarrow H(j\omega) = e^{-j\omega T}$$

$$\therefore |H(j\omega)| = 1$$

$$\angle H(j\omega) = -\omega T$$

$|H(j\omega)|$



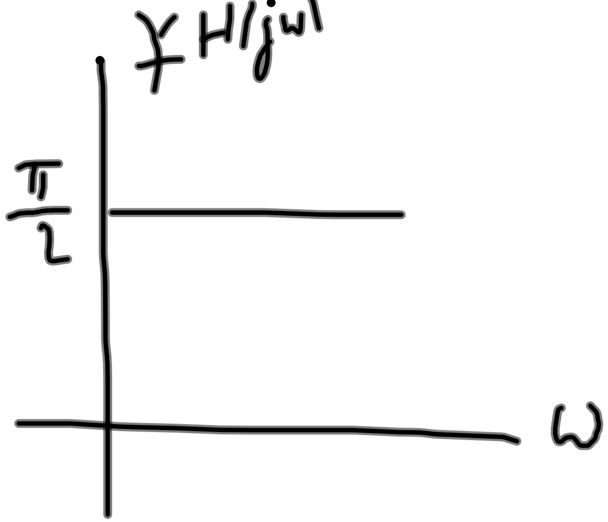


(b) $H(s) = s$

$H(j\omega) = j\omega = \omega \cdot e^{j\frac{\pi}{2}}$

$\therefore |H(j\omega)| = \omega$

$\angle H(j\omega) = \frac{\pi}{2}$

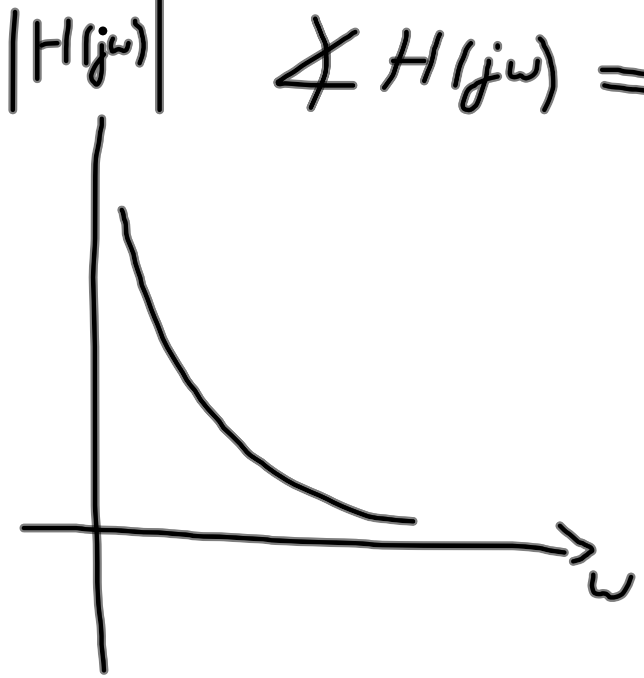


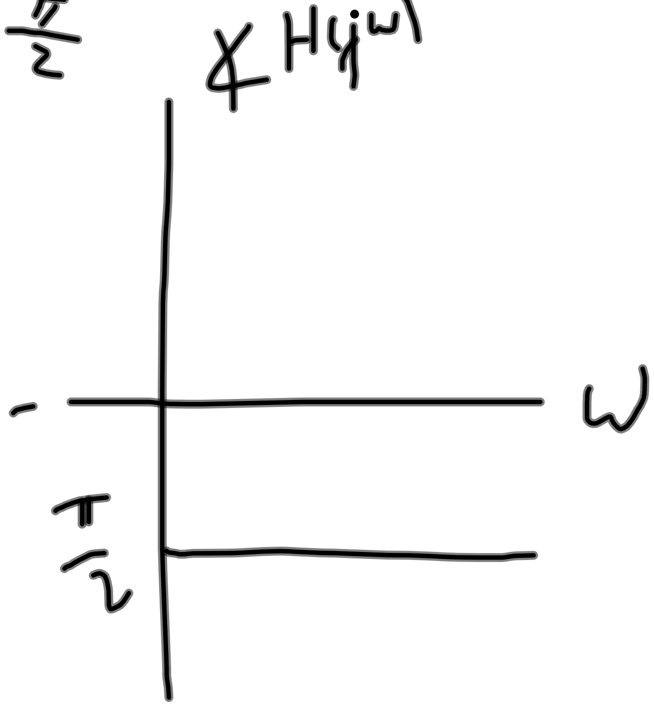
(c) $$H(s) = \frac{1}{s}$$

$$H(j\omega) = \frac{1}{j\omega}$$

$$\therefore \ |H(j\omega)| = \frac{1}{\omega}$$

$$\measuredangle H(j\omega) = -\frac{\pi}{2}$$





② Steady-state response to causal sinusoids

$$f(t) \longrightarrow \boxed{H(s) = \frac{P(s)}{Q(s)}} \longrightarrow y(t).$$

$$f(t) = e^{j\omega t} u(t)$$

$$F(s) = \frac{1}{s - j\omega}$$

$$\therefore Y_{zs}(s) = F(s) \cdot H(s) = \frac{P(s)}{\underbrace{(s-\lambda_1)\cdots(s-\lambda_n)}_{Q(s)}(s-j\omega)}$$

$$= \sum_{i=1}^{n} \frac{k_i}{s-\lambda_i} + \frac{\alpha}{s-j\omega}$$

$$\alpha = H(j\omega)$$

$$\therefore\ y(t) = \underbrace{\sum_{i=1}^{n} k_i e^{\lambda_i t} u(t)}_{y_{tr}(t)} + \underbrace{H(j\omega) e^{j\omega t} u(t)}_{\text{steady-state component } y_{ss}(t)}$$

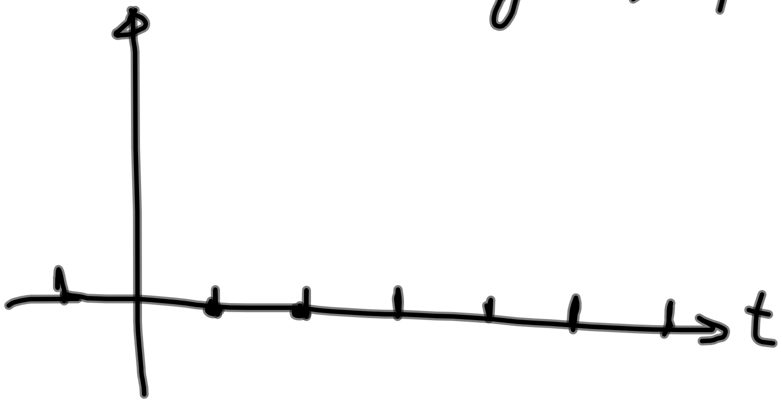
for asymptotically stable systems. $\lambda_i < 0$, the transient component $y_{tr}(t) \longrightarrow 0$

$$\therefore\; y_{ss}(t) = H(j\omega)\, e^{j\omega t} u(t)$$

Similarly, if the input is a causal sinusoid, $f(t) = \cos(\omega t + \theta)\, u(t)$ then the steady-state response is

$$y_{ss}(t) = |H(j\omega)| \cos\left(\omega t + \theta + \measuredangle H(j\omega)\right) u(t)$$

Chap. 5. Introduction to Discrete-time Signals & Systems.



§ 5.1. Useful discrete-time signal models.

① discrete-time impulse func. $\delta(k)$:

$$\delta(k) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

② unit-step $u(k)$.

$$u(k) = \begin{cases} 1, & k = 0, 1, 2, 3, \cdots \\ 0, & k = -1, -2, \cdots \end{cases}$$

③ exponential: $f(k) = \gamma^k$

④ discrete-time sinusoid $\longrightarrow f(k) = A \cdot \cos(\Omega \cdot k + \theta)$.

$k = 0, \pm 1, \pm 2, \cdots$

recall continuous-time sinusoid:

$$f(t) = A\cos(\omega t + \theta), \quad -\infty < t < \infty$$

$\omega = 2\pi \cdot f$

radian freq. (rad/s)

freq. (Hz), $f = \frac{1}{T}$.

<u>Properties:</u>

(a) $f(t)$ is periodic: $T = \frac{2\pi}{\omega}$

$f(t+T) = f(t)$

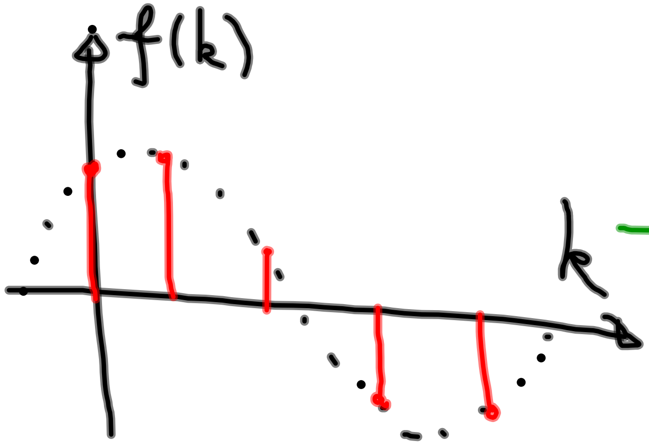
(b) different freq's give diff. signals

(c) Increase f. $\Rightarrow$ higher rate of oscillation, i.e., more oscillation periods are included in a given interval

for discrete-time sinusoids:

$$f(k) = A\cos(\Omega \cdot k + \theta)$$

Ω ← rad/sample



$$\Omega = 2\pi \cdot F$$

F ← freq: cycles/sample

properties:

(a) $f(k)$ is periodic iff its freq.

$$F = \frac{\Omega}{2\pi} \text{ is a rational number}$$

$$F = \frac{k}{N}$$

$$\Longrightarrow f(k+N) = f(k)$$

(b) $f_1(k) = A\cos(\Omega_1 k + \theta)$

$f_2(k) = A\cos(\Omega_2 k + \theta)$

if $\underline{\Omega_1 = \Omega_2 + 2\pi \cdot n}$

$\Longrightarrow f_1(k) = f_2(k)$.

$\therefore$ Only need to consider $|\omega| \leq \pi$.

or $\underbrace{|F| \leq \frac{1}{2}}_{\text{fundamental freq. range for discrete-time signals}}$

(c) highest rate of oscillation

$|\omega| = \pi$ or $|F| = 1/2$

Ex: $f(k) = 3\cos\left(5k + \frac{\pi}{6}\right)$

$\Omega = 5.\quad F = \frac{\Omega}{2\pi} = \frac{5}{2\pi} \longrightarrow$ aperiodic

Ex: $f(k) = e^{j\left(\frac{2\pi}{3}n + \frac{9}{2}\right)}$

$\Omega = \frac{2\pi}{3}.\quad F = \frac{\Omega}{2\pi} = \frac{1}{3}$ period $N = 3$.

- signal energy:

$$E_f := \sum_{k=-\infty}^{+\infty} |f(k)|^2$$

- signal power:

$$P_f = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{k=-N}^{N} |f(k)|^2$$

Useful Signal operations:

① time-shift:

$$f(n) \longrightarrow f(n-k)$$

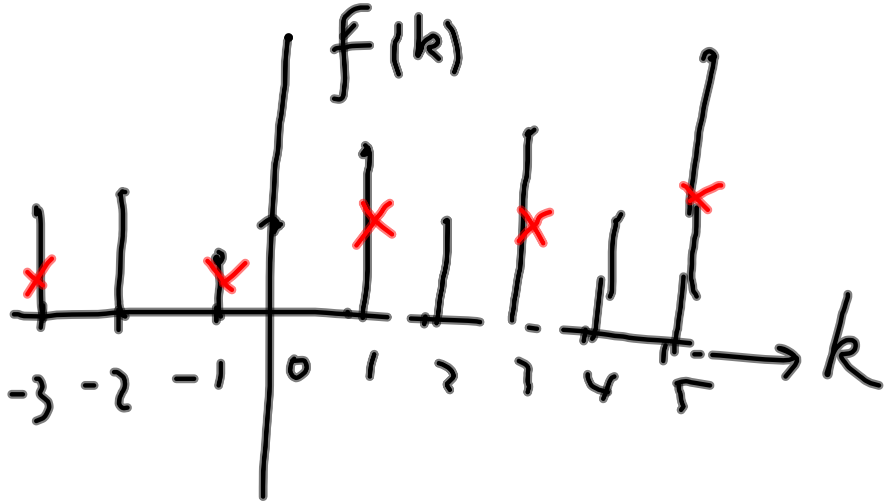
② time-reversal

$$f(n) \longrightarrow f(-n)$$

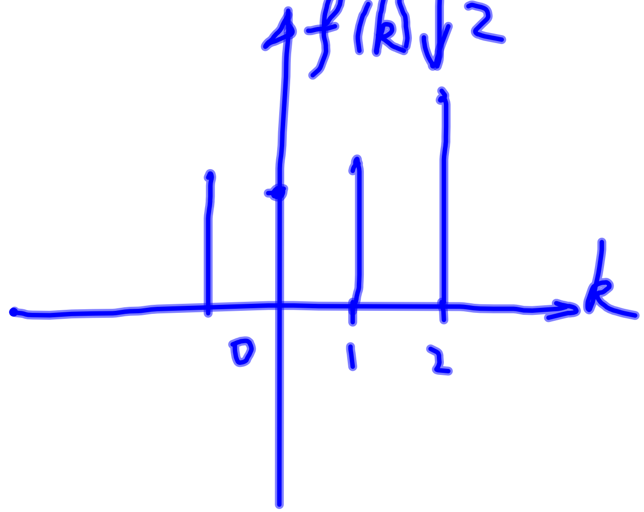
③ $\underbrace{\text{down-sampling}}$ / $\underbrace{\text{up-sampling}}$

$$f(n) \xrightarrow{\downarrow k} f(kn) \qquad f(n) \uparrow k$$

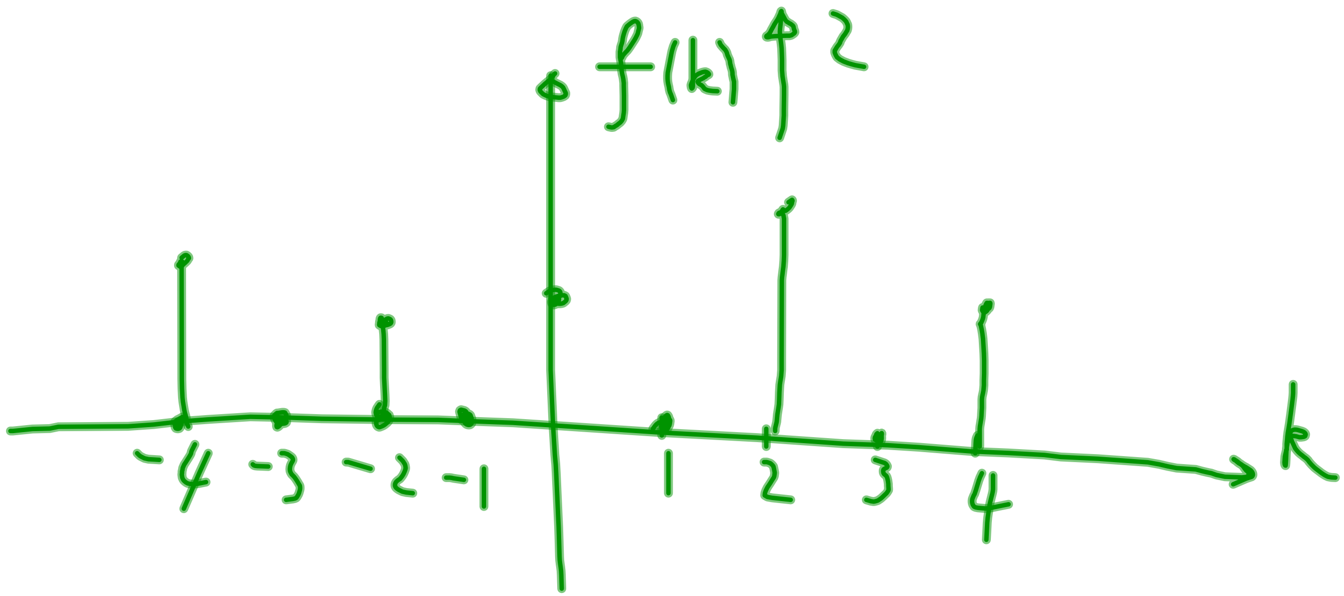
compressing expanding



f(k)
k
-3 -2 -1 0 1 2 3 4 5



f(k)↓2
k
0 1 2



f(k)↑2
k
-4 -3 -2 -1 1 2 3 4

Discrete-time convolution:

$$f(k) * h(k) = \sum_{m=-\infty}^{+\infty} f(m)\, h(k-m)$$

properties are the same as continuous-time convolution

Difference Eqn's to model discrete-time LTI systems:

$$a_0 y(k) + a_1 y(k-1) + \cdots + a_n y(k-n)$$

$$= b_0 f(k) + b_1 f(k-1) + \cdots + b_n f(k-n)$$

$$y(k) = \underbrace{y_{zs}(k)}_{f(k) * h(k)} + \underbrace{y_{zi}(k)}_{\text{initial conditions}}$$

unilateral

Z-transform (corresponding to Laplace).

$$F(z) = \mathcal{Z}\{f(k)\} = \sum_{k=0}^{\infty} f(k) z^{-k}$$

$$z = e^{j\omega} \Rightarrow F(\omega) = \sum_{k=0}^{\infty} f(k) e^{-j\omega k}.$$

discrete-time Fourier transform

sqr(x) * 10

grade = max{20% HW + 80% F, 20% HW + 20% M1 + 20% M2 + 40% F, }