Ex:

(a) $(D+1)(D^2+4D+8)\,y(t) = (D-3)\,f(t)$

poles: -1

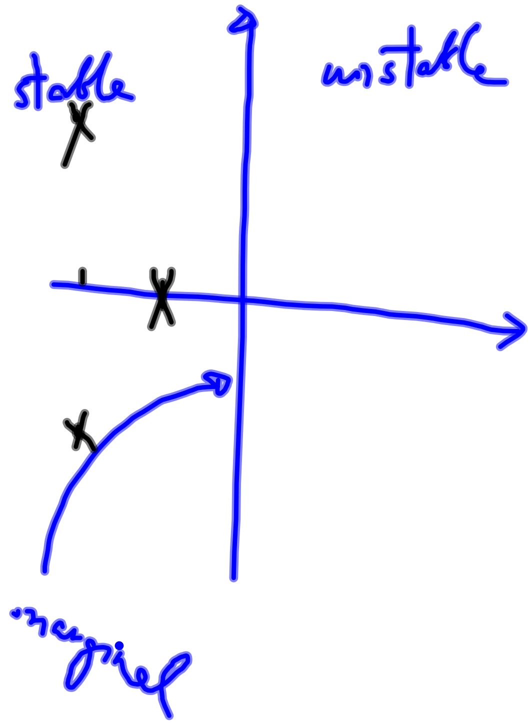
$$(D^2+4D+4)+4$$

$$= (D+2)^2 + 2^2$$

$$\Rightarrow -2 \pm 2j$$

stable

poles



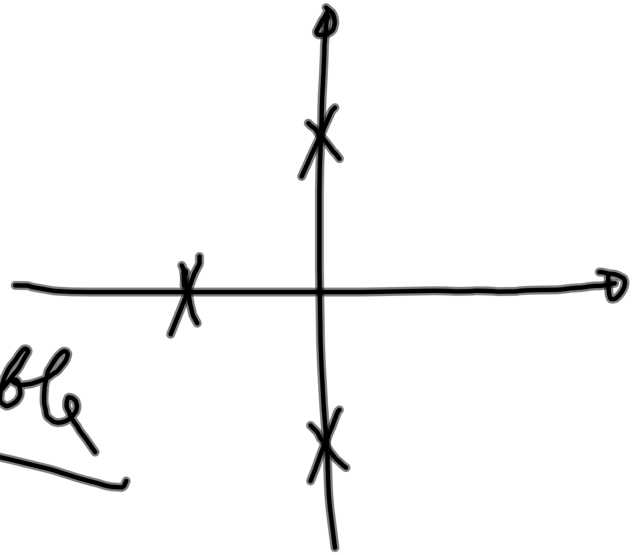
(b) $(D-1)(D^2+4D+8)\,y(t) = (D+2)\,f(t)$

poles: 1 unstable

(c) $(D+2)(D^2+4)\,y(t) = (D^2+D+1)\,f(t)$

poles: -2

$\pm 2j$

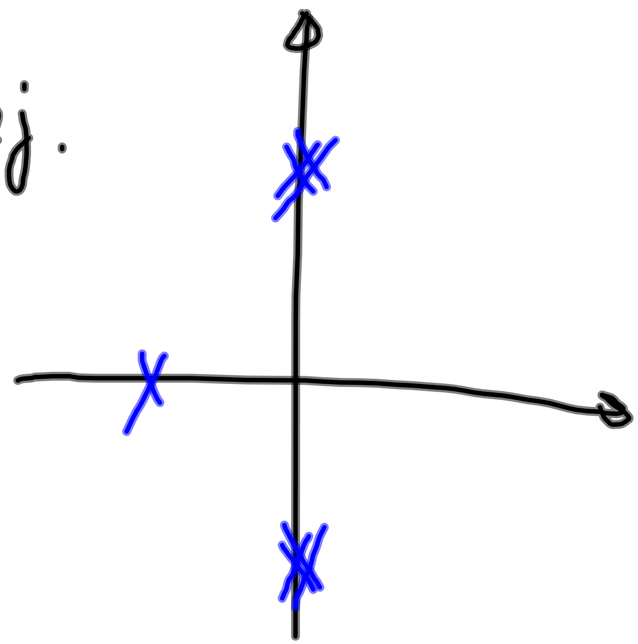


marginally stable

(d) $(D+1)\,(D^2+4)^2\,y(t) = (D^2+2D+1)\,f(t)$

poles : -1

$\pm 2j,\ \pm 2j.$



<u>unstable</u>

② BIBO stability

<u>Def:</u> A system is BIBO (bounded input bounded output) stable iff a bounded input always produces a bounded output.

- Condition for BIBO stability:

An LTI system is BIBO stable iff

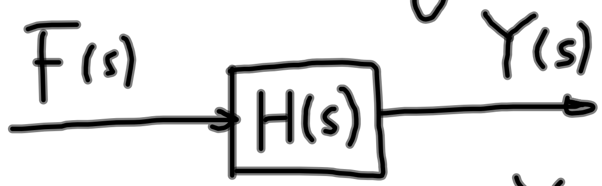
$$\int_{-\infty}^{+\infty} |h(t)|\,dt < \infty$$

③ Relationship between asymptotic stability & BIBO stability

- Asymptotic stable $\Longrightarrow$ BIBO stable
- Marginally stable $\Longrightarrow$ BIBO unstable
- $\underbrace{\text{BIBO stability}}_{\text{external description}}$ does NOT imply $\underbrace{\text{asymptotic stability}}_{\text{internal description}}$

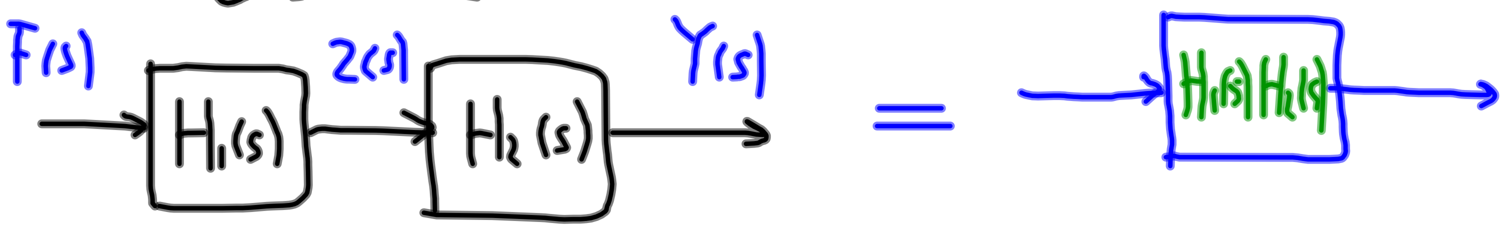
§ 4.5. Block Diagrams.

① Three elementary interconnections.



$$Y(s) = F(s) \cdot H(s).$$

- Cascade

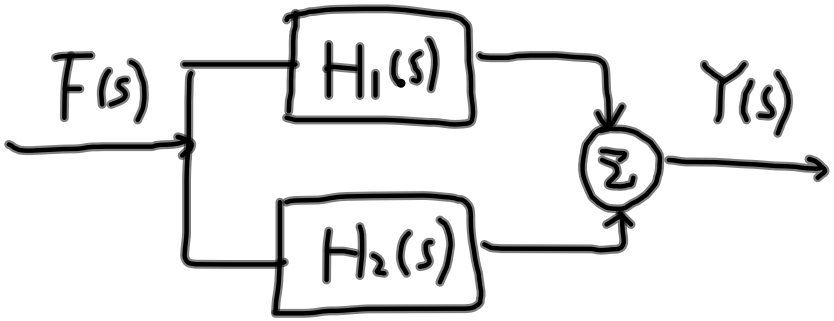


$$Z(s) = F(s) \cdot H_1(s)$$

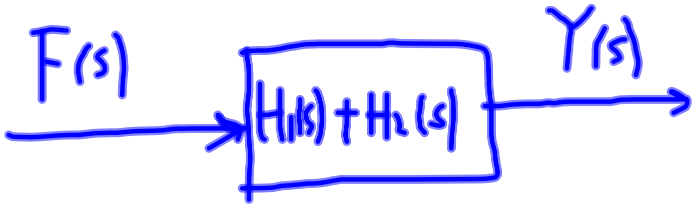
$$Y(s) = Z(s) \cdot H_2(s)$$

$$= F(s) \cdot \boxed{H_1(s) H_2(s)}$$

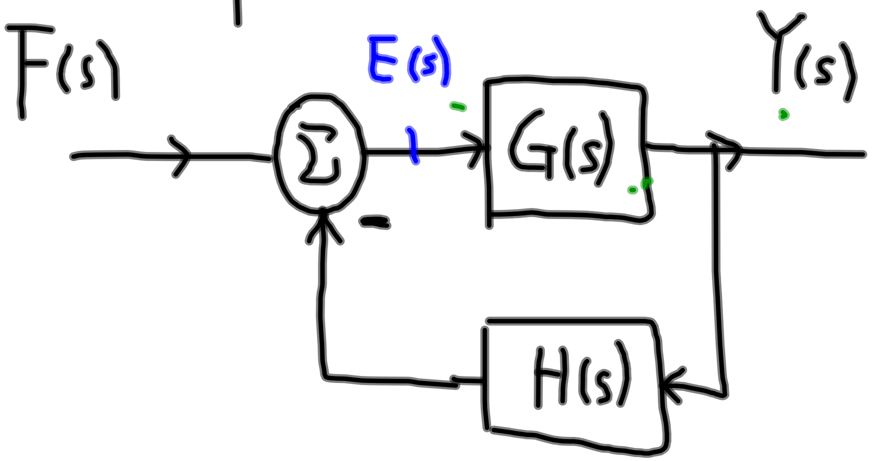
· parallel



$=$



· feed back



$$E(s) = F(s) - H(s) \cdot Y(s)$$

$$Y(s) = E(s) \cdot G(s)$$

$$\Longrightarrow \frac{Y(s)}{F(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

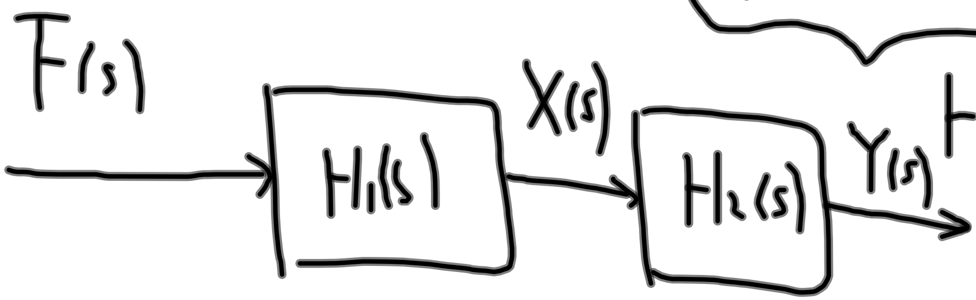
② System Realizations

a) Direct-form realization

<u>Ex:</u>

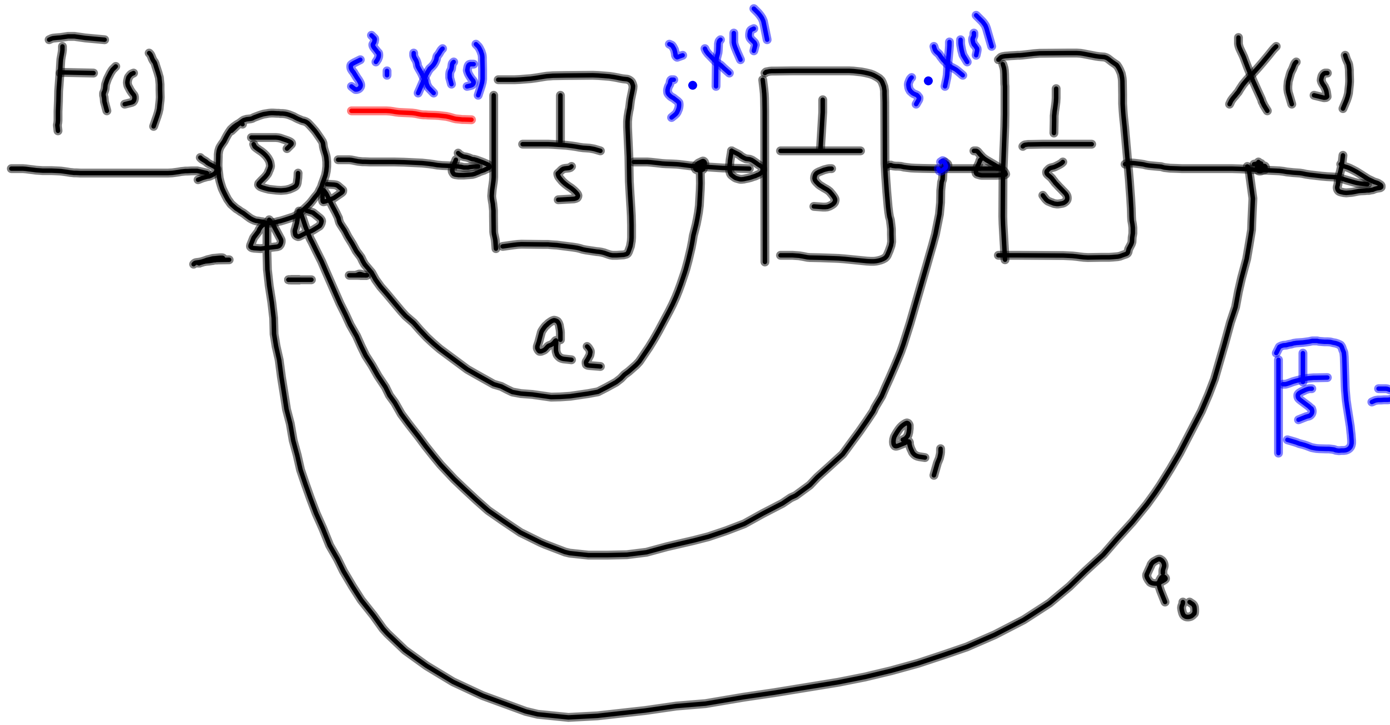
$$H(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

$$= \underbrace{\left(\frac{1}{s^3 + a_2 s^2 + a_1 s + a_0}\right)}_{H_1(s)} \underbrace{\left(b_3 s^3 + b_2 s^2 + b_1 s + b_0\right)}_{H_2(s).}$$



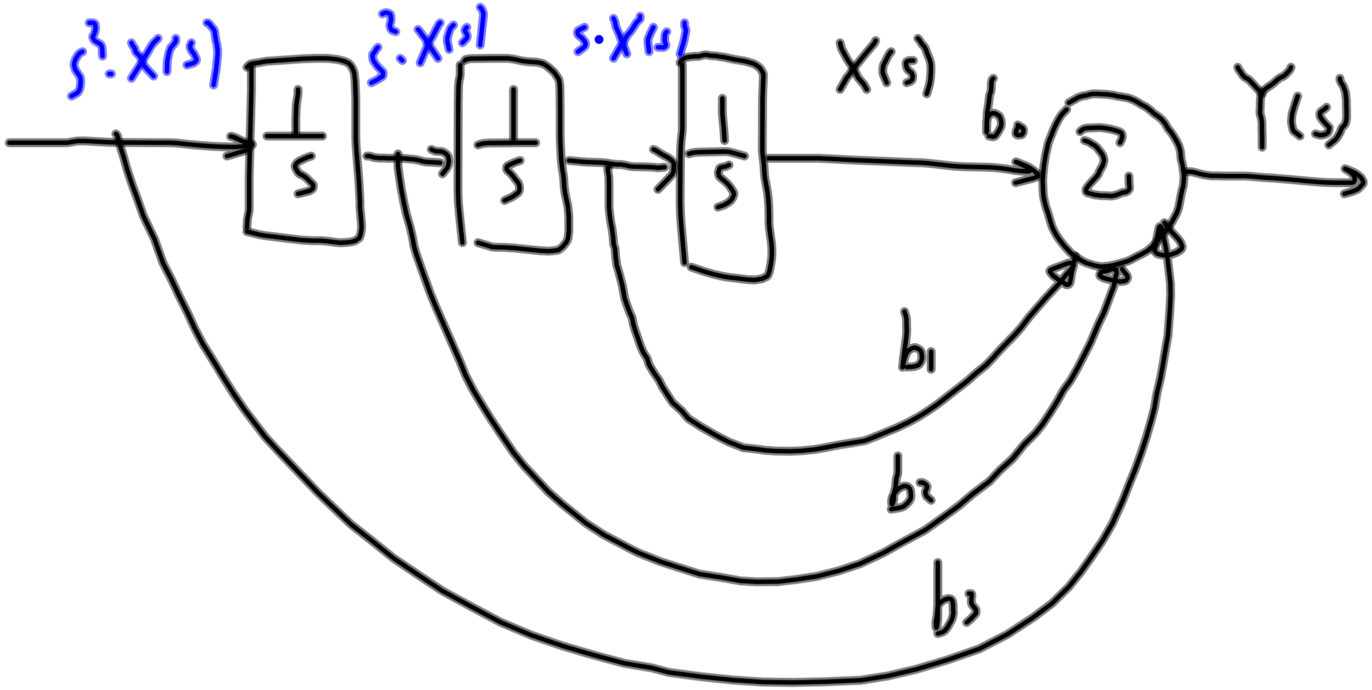
$$\frac{F(s)}{s^3 + a_2 s^2 + a_1 s + a_0} = X(s)$$

$$\Longrightarrow F(s) = \underline{s^3 \cdot X(s)} + a_2 s^2 \cdot X(s) + a_1 \cdot s \cdot X(s) + a_0 X(s)$$

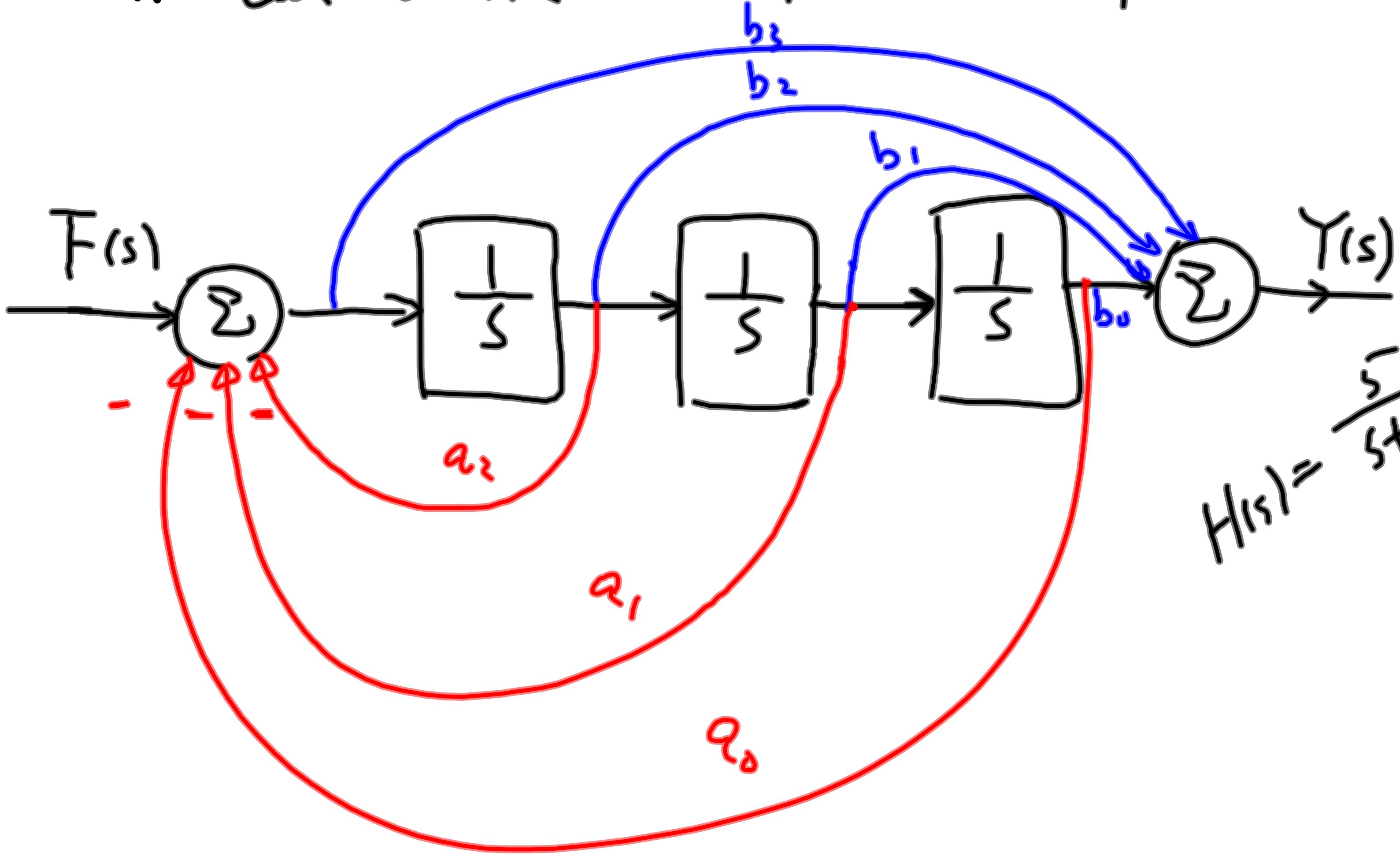


$$\boxed{\frac{1}{s}} = \boxed{\int}$$

$$X(s) \cdot \underbrace{\left(b_3 s^3 + b_2 s^2 + b_1 s + b_0\right)}_{H_2(s).} = Y(s)$$



$\therefore$ can combine $H_1(s)$ & $H_2(s)$ as follows:

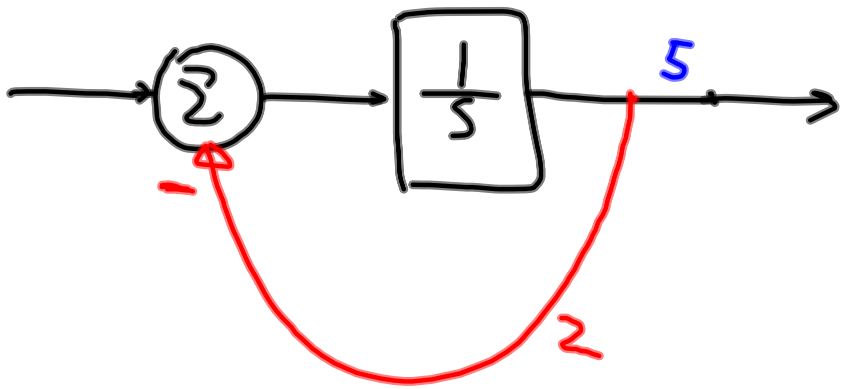


$H(s) = \frac{s}{s+2}$.

In general:
$$H(s) = \frac{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

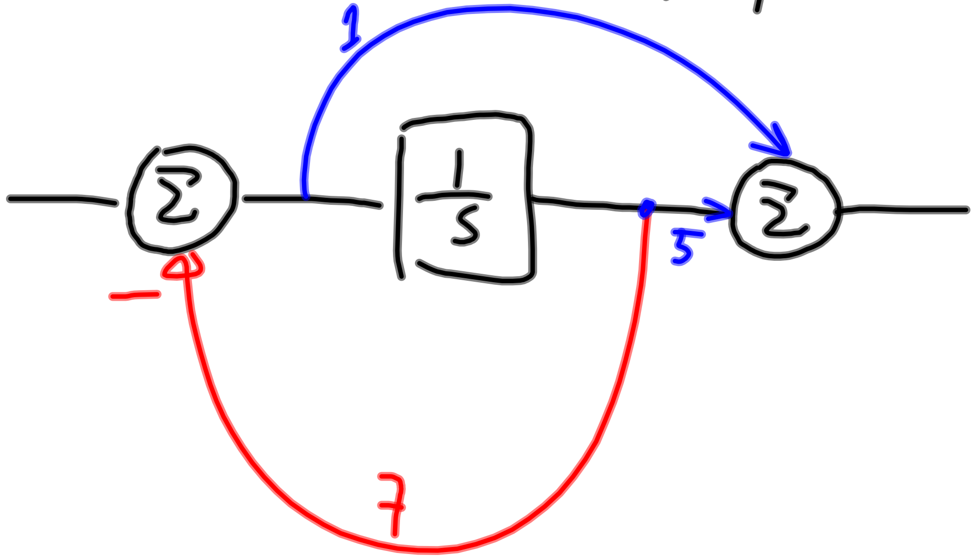
Need n integrators.

Ex:
$$H(s) = \frac{5}{s+2}$$



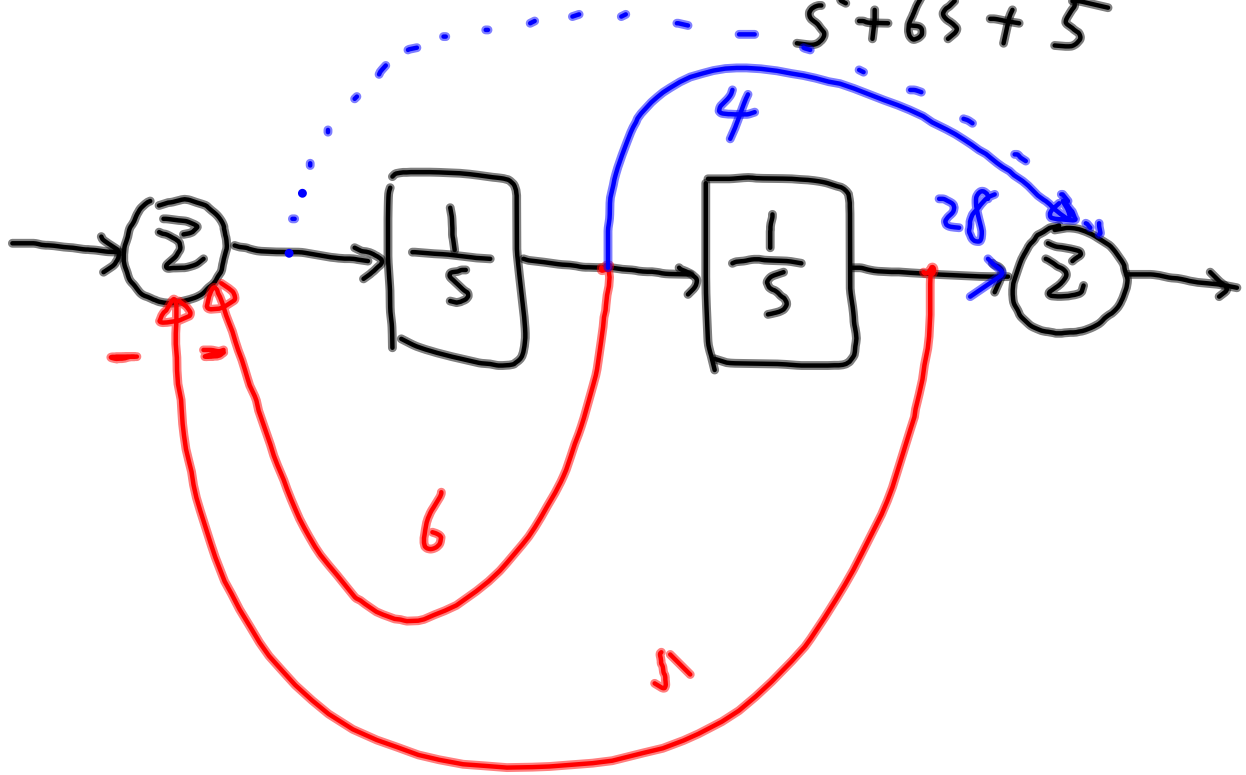
Ex:

$$H(s) = \frac{s+5}{s+7}$$



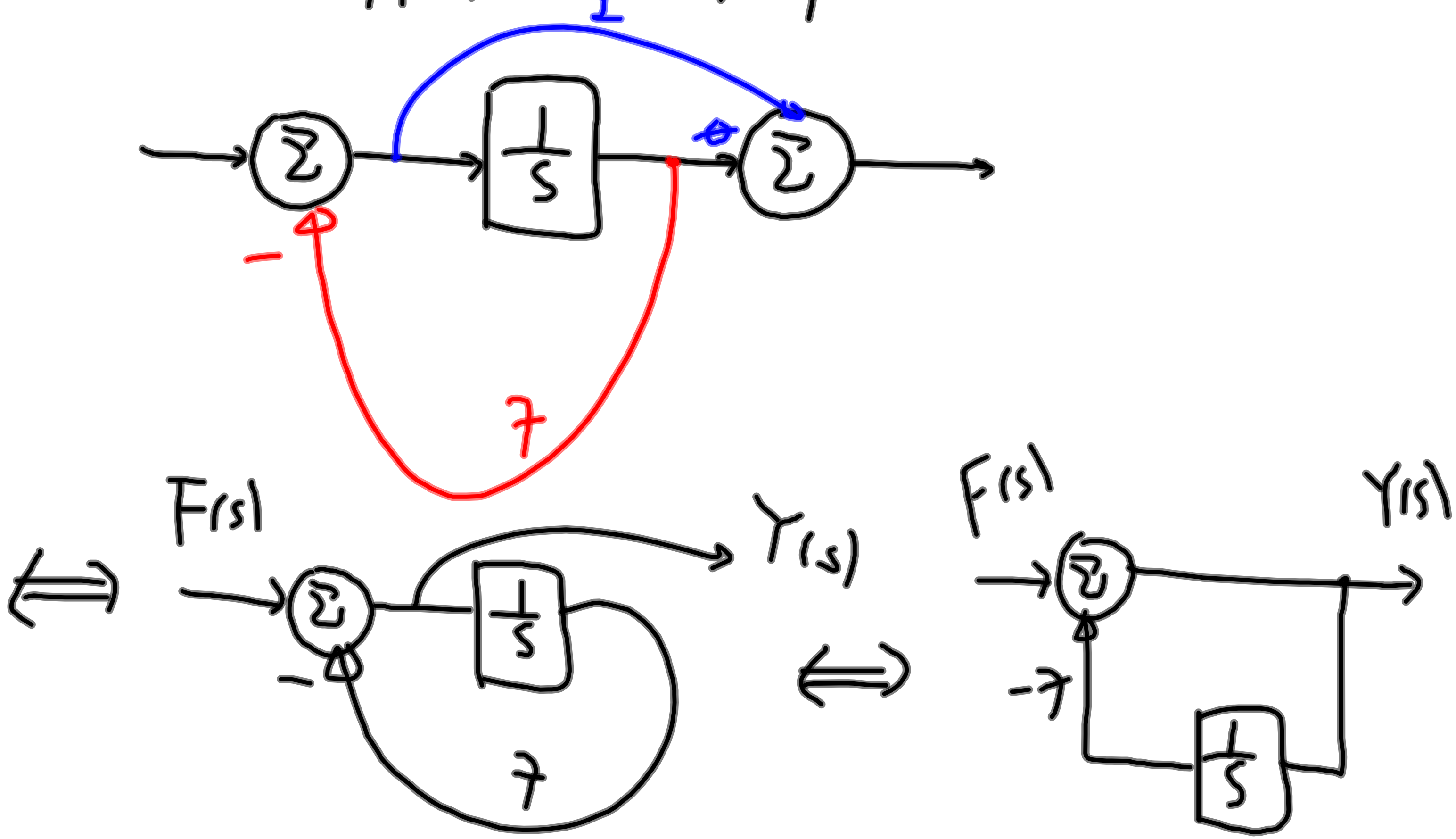
Ex:

$$H(s) = \frac{4s+28}{s^2+6s+5}$$



Ex:

$$H(s) = \frac{s}{s+7}$$



b) cascade & parallel realizations.

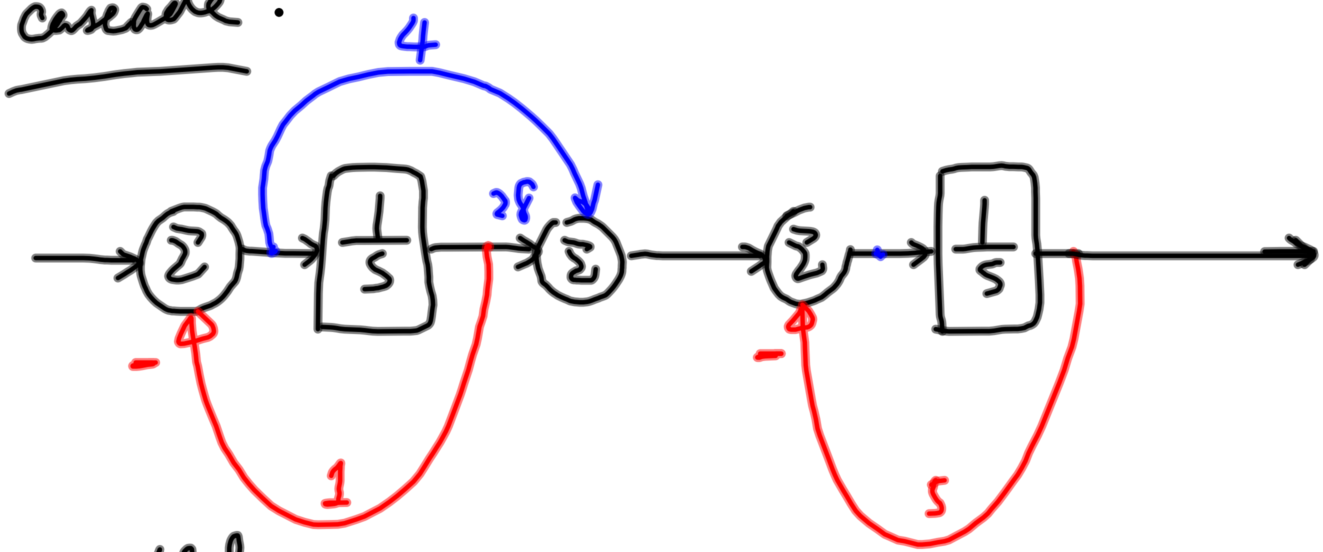
Ex:

$$H(s) = \frac{4s+28}{s^2+6s+5}$$

cascade : $= \dfrac{4s+28}{s+1} \cdot \dfrac{1}{s+5}$

parallel : $= \dfrac{6}{s+1} - \dfrac{2}{s+5}$

cascade :



parallel :