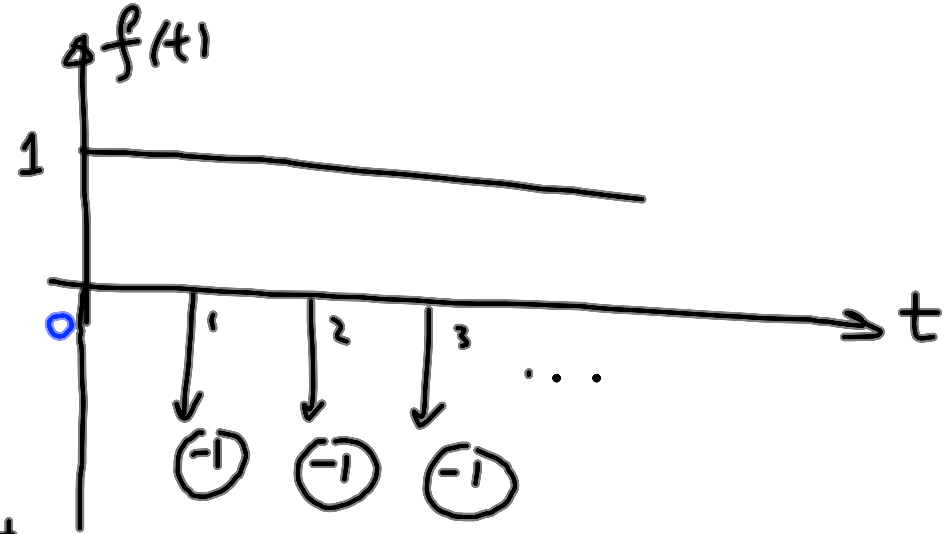
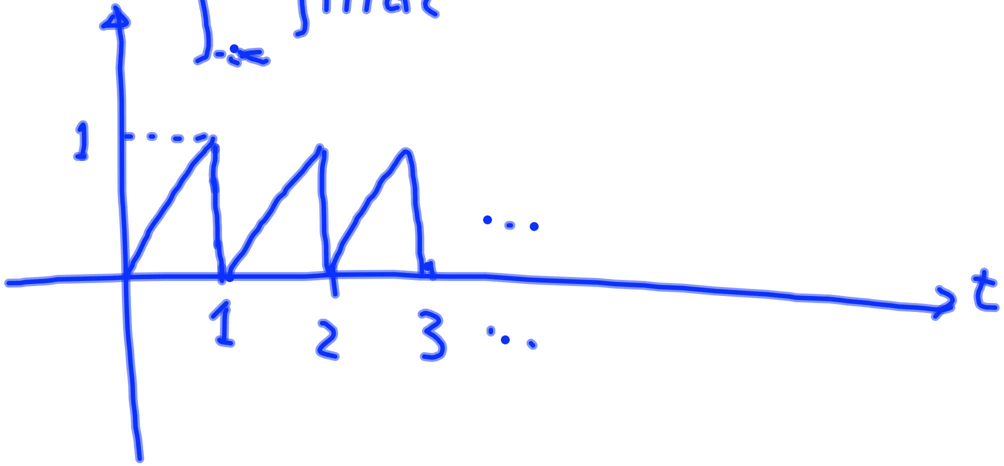
Ex 1

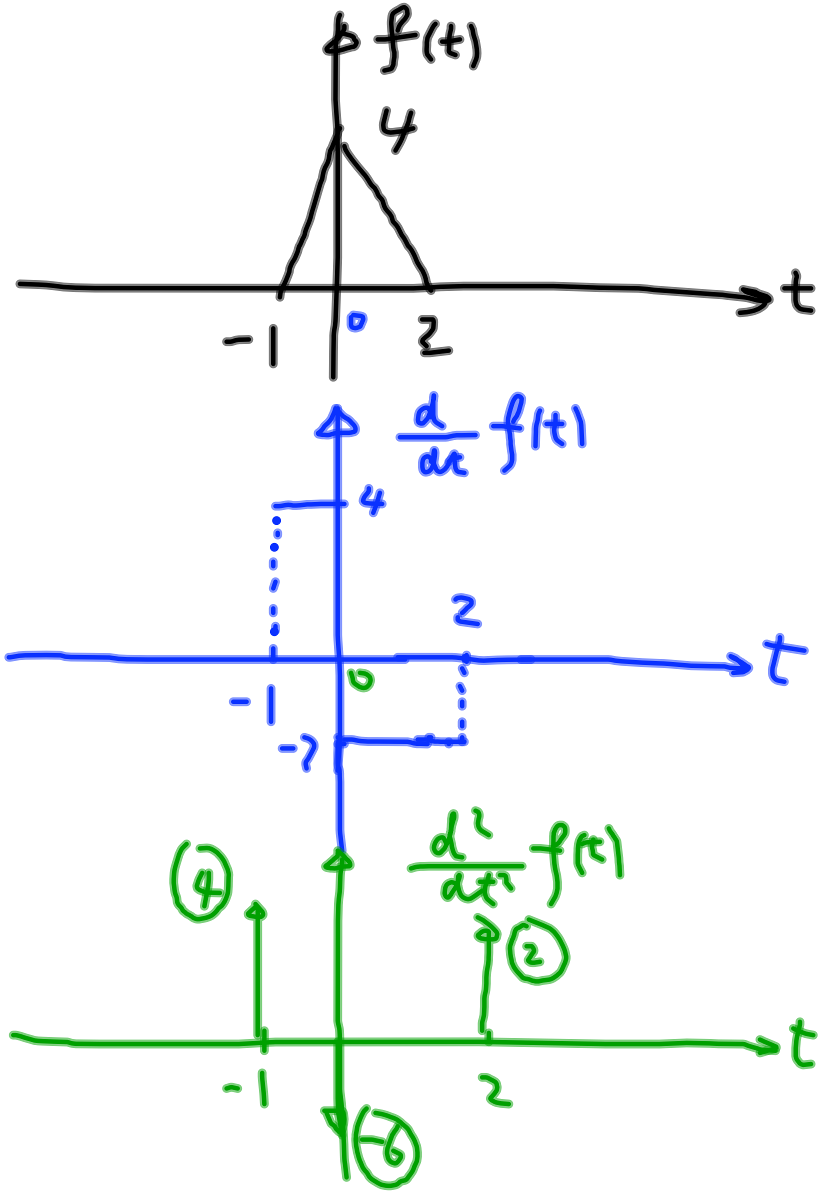


plot $\int_{-\infty}^{t} f(\tau)\,d\tau$

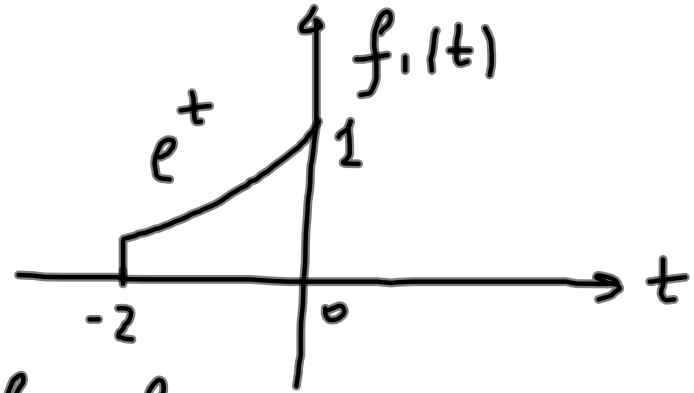
$\int_{-\infty}^{t} f(\tau)\,d\tau$

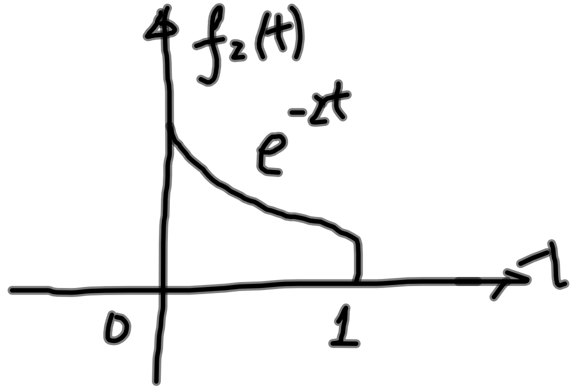


Ex 2:



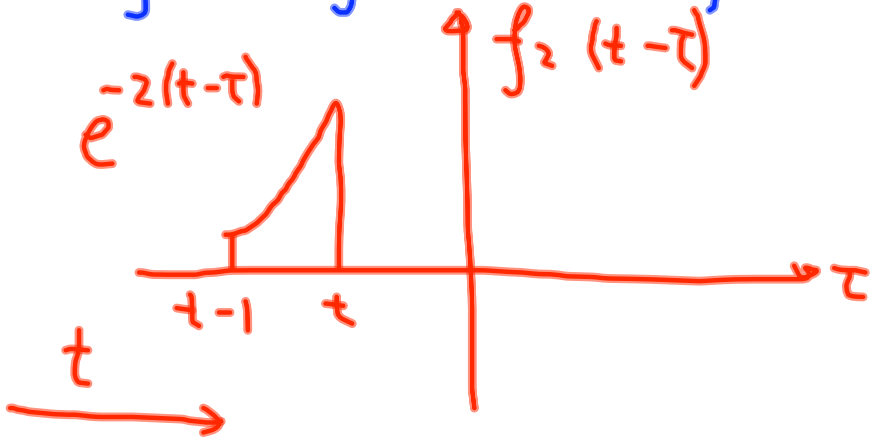
Ex 3

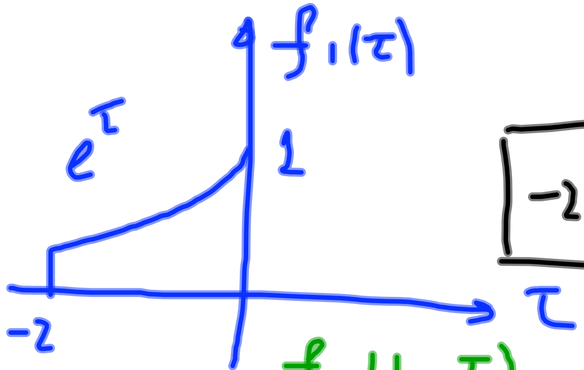




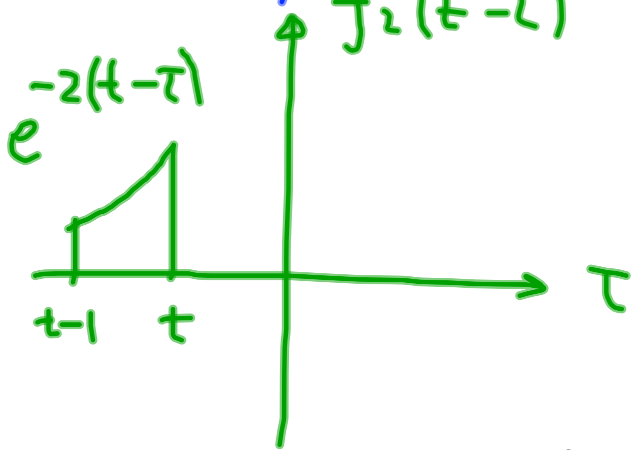
calculate $f_1 * f_2$

By the width property of convolution, we have $f_1(t) * f_2(t) \neq 0$ for $-2 \leq t \leq 1$

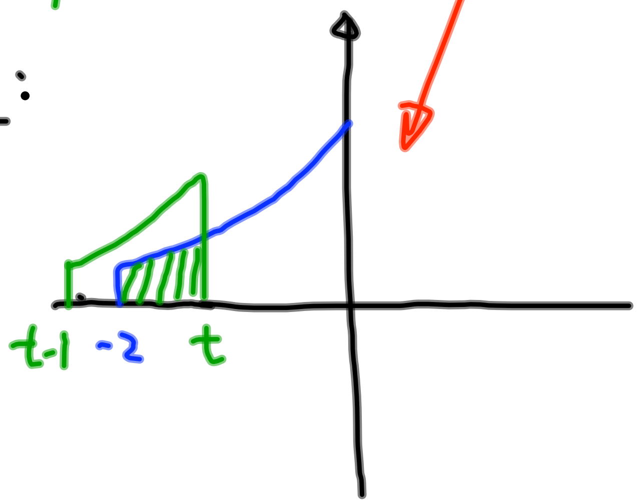




$$\boxed{-2 \le t \le 1}$$



Case 1:



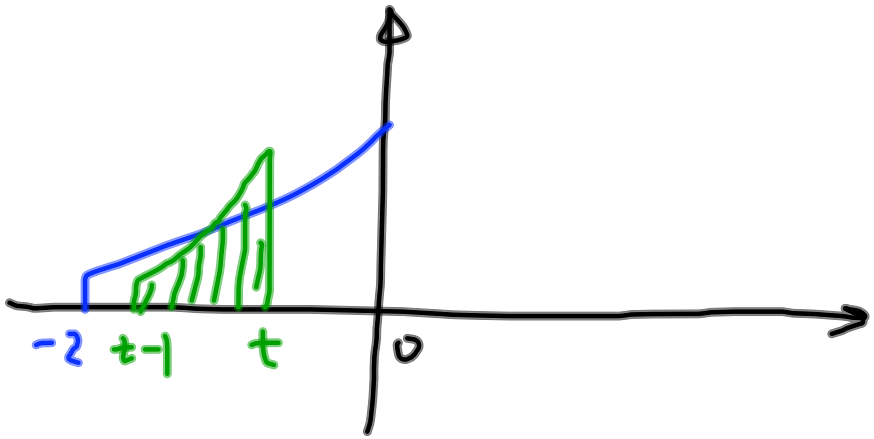
$$\begin{cases} t-1 \le -2 \\ t \ge -2 \end{cases} \Rightarrow \boxed{-2 \le t \le -1}$$

$$\int_{-2}^{t} \underbrace{e^{\tau}}_{f_1(\tau)} \cdot \underbrace{e^{-2(t-\tau)}}_{f_2(t-\tau)} \, d\tau$$

$$= e^{-2t} \int_{-2}^{t} e^{3\tau} \, d\tau$$

$$= \frac{1}{3} e^{-2t} \cdot e^{3\tau} \Big|_{-2}^{t} .$$

$$= \frac{1}{3} \left[e^{t} - e^{-2(t+3)} \right]$$



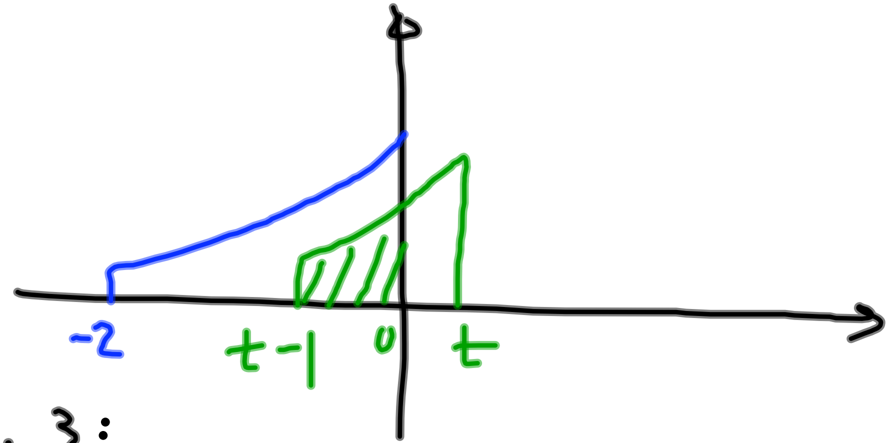
Case 2:

$$\begin{cases} t-1 \geq -2 \\ t \leq 0 \end{cases}$$

$$\Rightarrow \boxed{-1 \leq t \leq 0}$$

$$\int_{t-1}^{t} e^{\tau} e^{-2(t-\tau)} d\tau$$

$$= \frac{1}{3} e^{t} \left(1 - e^{-3}\right)$$



Case 3:

$$\begin{cases} t-1 \le 0 \\ t \ge 0 \end{cases}$$

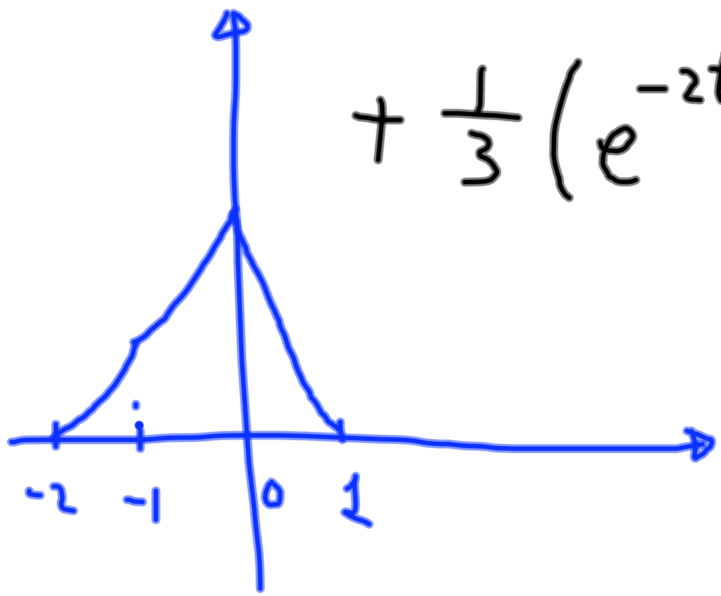
$$\Rightarrow \boxed{0 \le t \le 1}$$

$$\int_{t-1}^{0} e^{\tau} \cdot e^{-2(t-\tau)} \, d\tau = \frac{1}{3}\left(e^{-2t} - e^{t-3}\right)$$

$$\therefore f_1(t) * f_2(t) = \frac{1}{3}\left(e^{t} - e^{-2(t+3)}\right)\left[u(t+2) - u(t+1)\right]$$

$$+ \frac{(1-e^{-3})}{3} e^{t} \left[u(t+1) - u(t)\right]$$

$$+ \frac{1}{3}\left(e^{-2t} - e^{t-3}\right)\left[u(t) - u(t-1)\right]$$



Ex 4

The trigonometric Fourier series of aperiodic signal is given by

$$f(t) = 3 + \sqrt{3}\cos 2t + \sin 2t + \sin 3t - \frac{1}{2}\cos\left(5t + \frac{\pi}{3}\right)$$

(a) Sketh the trigonometric Fourier spectra

(b) " " exponential " "

(c) write the expontial Fourier series

Recall:

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \cos\left(n\omega_0 t + \theta_n\right)$$

$$= \sum_{n=-\infty}^{+\infty} D_n e^{jn\omega_0 t}$$

trigonometric spectra: $C_n \sim n\omega_0$, $\theta_n \sim n\omega_0$

exponential spectra: $|D_n| - n\omega_0$, $\measuredangle D_n \sim n\omega_0$

By inspection of $f(t)$, $\omega_0 = 1$

$$f(t) = \underbrace{3}_{a_0} + \underbrace{\sqrt{3}}_{a_2} \cos 2t + \underbrace{1}_{b_2} \cdot \sin 2t + \underbrace{1}_{b_3} \cdot \sin 3t$$

$$\underbrace{- \frac{1}{2} \cos\left(5t + \frac{\pi}{3}\right)}$$

$$\frac{1}{2} \cos\left(5t + \frac{\pi}{3} - \pi\right)$$

$$= \underbrace{\frac{1}{2}}_{C_5} \cos\left(5t \underbrace{- \frac{2\pi}{3}}_{\theta_5}\right)$$

$$\begin{cases} c_n = \sqrt{a_n^2 + b_n^2} \\ \theta_n = \tan^{-1}\left(\dfrac{-b_n}{a_n}\right) \\ c_0 = a_0 \end{cases}$$

$c_4 = 0$

$$\begin{cases} c_5 = \dfrac{1}{2} \\ \theta_5 = -\dfrac{2}{3}\pi \end{cases}$$

(a) trig. spectra :

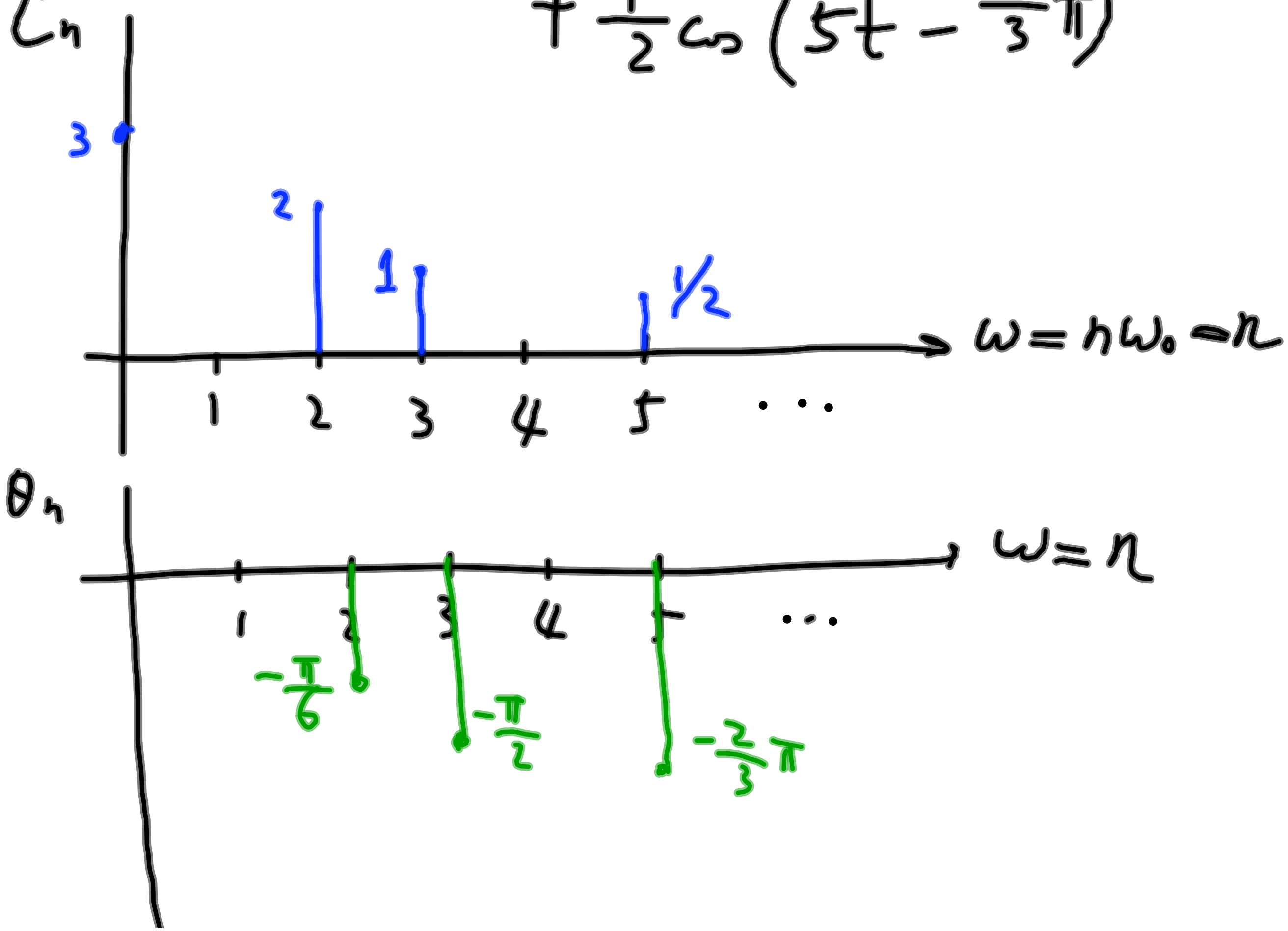
$c_0 = a_0 = 3$

$c_1 = 0$

$$\begin{cases} c_2 = 2 \\ \theta_2 = \tan^{-1}\left(\dfrac{-1}{\sqrt{3}}\right) = -\dfrac{\pi}{6} \end{cases}$$

$$\begin{cases} c_3 = 1 \\ \theta_3 = \tan^{-1}\left(\dfrac{-1}{0}\right) = -\dfrac{\pi}{2} \end{cases}$$

$$f(t) = 3 + 2\cos\left(2t - \frac{\pi}{6}\right) + \cos\left(3t - \frac{\pi}{2}\right) + \frac{1}{2}\cos\left(5t - \frac{2}{3}\pi\right)$$



(b) exp. spectra:

$$\begin{cases} |D_n| = |D_{-n}| = \frac{1}{2} C_n \\ \measuredangle D_n = -\measuredangle D_{-n} = \theta_n \\ D_0 = C_0 \end{cases}$$

$\therefore$ $D_0 = 3$

$|D_1| = |D_{-1}| = \emptyset$

$$\begin{cases} |D_2| = |D_{-2}| = 1 \\ \measuredangle D_2 = -\measuredangle D_{-2} = -\frac{\pi}{6} \end{cases}$$

$$\begin{cases} |D_3| = |D_{-3}| = \frac{1}{2} \\ \measuredangle D_3 = -\measuredangle D_{-3} = -\frac{\pi}{2} \end{cases}$$

$|D_4| = |D_{-4}| = 0$

$$\begin{cases} |D_5| = |D_{-5}| = \frac{1}{4} \\ \measuredangle D_5 = -\measuredangle D_{-5} = -\frac{2}{3}\pi \end{cases}$$

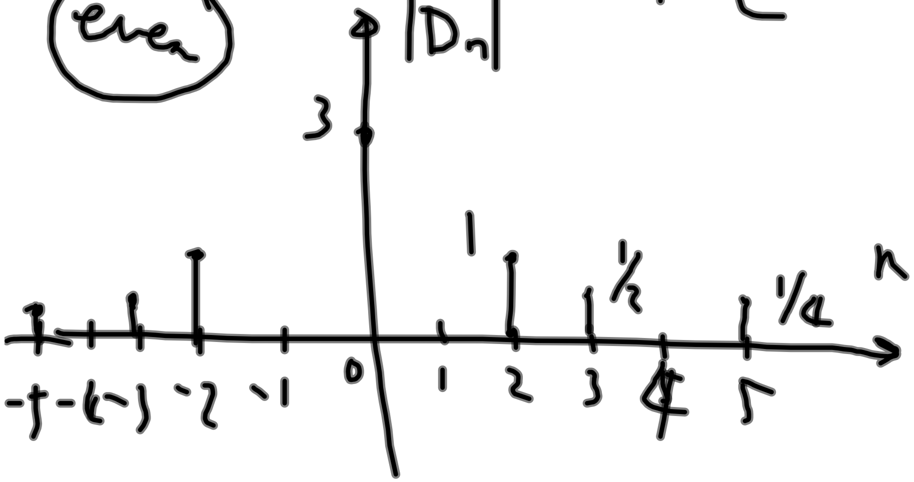
$$\therefore f(t) = 3 + \left[e^{j\left(2t - \frac{\pi}{6}\right)} + e^{-j\left(2t - \frac{\pi}{6}\right)} \right]$$

exp. series

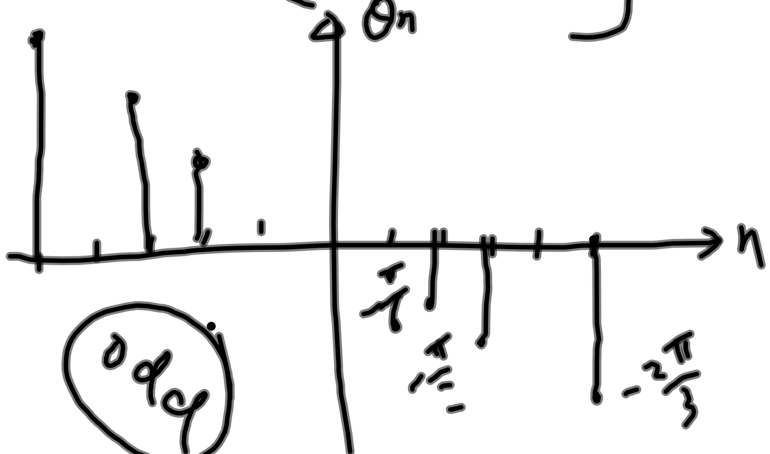
$$+ \frac{1}{2}\left[e^{j\left(3t - \frac{\pi}{2}\right)} + e^{-j\left(3t - \frac{\pi}{2}\right)} \right]$$

$$+ \frac{1}{4}\left[e^{j\left(5t - \frac{2}{3}\pi\right)} + e^{-j\left(5t - \frac{2}{3}\pi\right)} \right]$$

even



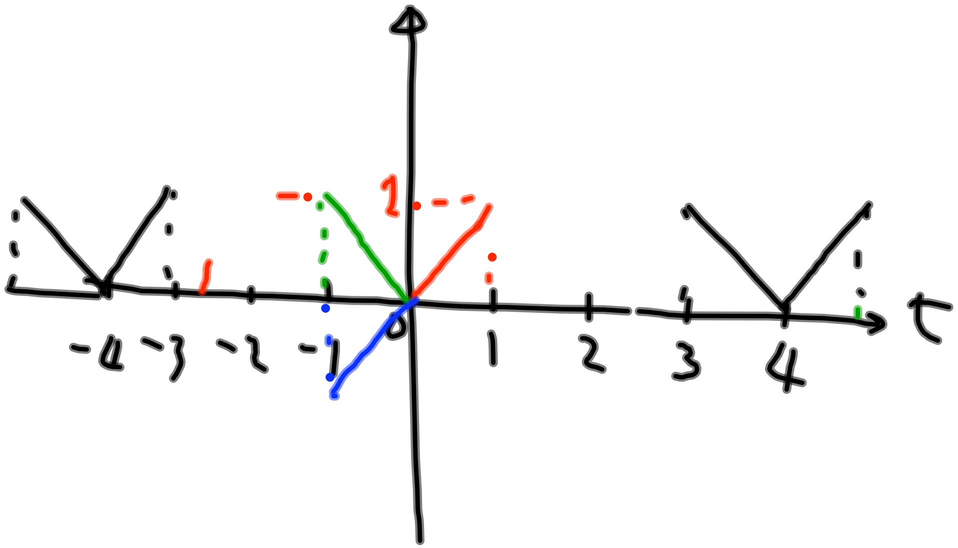
odd



$$f(t) = t, \quad 0 \le t \le 1.$$

① $\omega_0 = \pi/2$, cosine terms only

$T_0 = 4$ even



$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{\pi}{2} n\right) t$$

$$a_0 = \frac{2}{T_0} \int_0^1 t \, dt = \frac{1}{4}$$

$$a_n = \frac{4}{T_0}\int_0^1 t \cos n\omega_0 t \, dt$$

$$= \int_0^1 t \cos\left(\frac{\pi}{2} n t\right) dt$$

$$= \frac{2}{n\cdot\pi}\int_0^1 t \; d \sin\left(\frac{\pi}{2} n t\right).$$

$$= \frac{2}{n\cdot\pi}\left[\underbrace{t\cdot\sin\left(\frac{\pi}{2} n t\right)\Big|_0^1}_{\sin\left(\frac{\pi}{2} n\right)} \underbrace{- \int_0^1 \sin\left(\frac{\pi}{2} n t\right) dt}_{-\frac{2}{n\cdot\pi}\cos\left(\frac{\pi}{2} n t\right)\Big|_0^1} \right]$$

$$= \frac{2}{\pi \cdot n}\left[\sin\frac{n\pi}{2} + \frac{2}{\pi \cdot n}\cos\frac{\pi \cdot n}{2} - \frac{2}{\pi \cdot n}\right]$$

$$= \frac{2}{\pi \cdot n}\sin\frac{n\pi}{2} + \frac{4}{\pi^2 \cdot n^2}\left(\cos\frac{\pi n}{2} - 1\right)$$