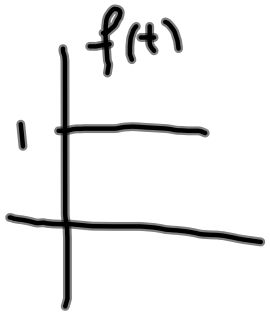
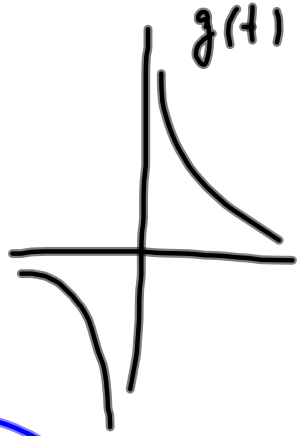




$$z(t) = f(t) \times g(t) \checkmark$$

$$y(t) = f(t) * g(t)$$

$t = 100$

$y(100)$

$$f(t) * g(t)$$

$$= (1 - 2e^{-t})u(t) - e^{2t}u(-t)$$

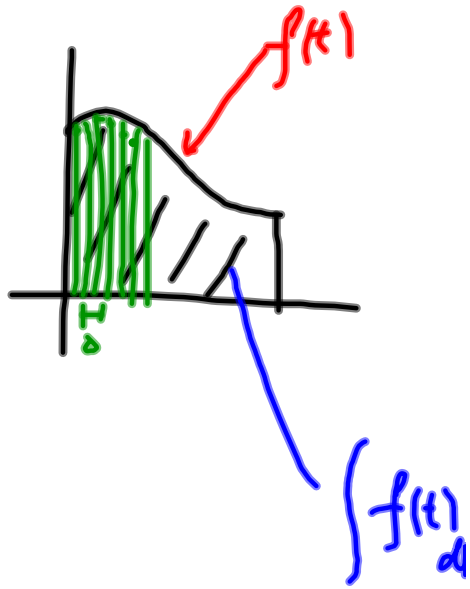
$$\cancel{-u(t)}$$

right notation:

$$y(t) = (f * g)(t)$$

§ 1.8 zero-state response

$$f(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow y(t) = \boxed{y_{zs}(t)} + y_{zi}(t)$$



① The unit impulse response

<u>Recall:</u>

$$f(t) = f(t) * \delta(t)$$

$$= \int_{-\infty}^{+\infty} f(\tau)\,\delta(t-\tau)\,d\tau$$

$$\cong \sum_{n=-\infty}^{+\infty} f(n\cdot\Delta\tau)\,\delta(t-n\Delta\tau)\cdot\Delta\tau$$

$$y(t) = T\left[f(t)\right]$$

$$= T\left[\sum_{n=-\infty}^{+\infty} \underbrace{f(n\cdot\Delta\tau)}_{\text{const}} \underbrace{\delta(t-n\cdot\Delta\tau)}_{\text{func. of } (t)} \cdot \underbrace{\Delta\tau}_{\text{const}}\right]$$

linearity $\rightarrow$

$$= \sum_{n=-\infty}^{+\infty} f(n\cdot\Delta\tau)\cdot\Delta\tau\cdot \underbrace{T\left[\delta(t-n\Delta\tau)\right]}_{h(t-n\cdot\Delta\tau)}$$

time-invariance $\rightarrow$

$$= \sum_{n=-\infty}^{+\infty} f(n\cdot\Delta\tau)\cdot h(t-n\cdot\Delta\tau)\cdot\Delta\tau$$

Def: $h(t) \overset{\text{def}}{=} T\left[\delta(t)\right]$

$$\rightarrow \int_{-\infty}^{+\infty} f(\tau)\, h(t-\tau)\, d\tau = f(t) * h(t)$$

Def: unit impulse response

$$h(t) \overset{\text{def}}{=} T[\delta(t)]$$

i.e., the system response to an impulse input $\delta(t)$ applied at $t=0$, with all initial conditions zero at $t=0^-$.

$$Q(D)\cdot y(t) = P(D)\cdot f(t)$$

methods for calculating $h(t)$ given an LTI system:

- time-domain — very tedious
- transform-domain — easy ✓

$$y_{zs}(t) = f(t) * h(t)$$

— zero-state response.

$y_{zi}(t) =$ func. of <u>initial conditions</u>

<u>Initial Conditions:</u>

- input $f(t)$ starts at $t=0$

  conditions immediately before $t=0$:

  $$y(0^-),\ \dot{y}(0^-),\ \ddot{y}(0^-),\ \ldots$$

conditions immediately after $t=0$:

$$y(0^+),\quad \dot{y}(0^+),\quad \ddot{y}(0^+),\ \ldots$$

- typically, for total response:

$$y(0^-) \neq y(0^+),\quad \dot{y}(0^-) \neq \dot{y}(0^+),\ \ldots$$

- for zero-input response:

$$y_{zi}(0^-) = y_{zi}(0^+)$$
$$\dot{y}_{zi}(0^-) = \dot{y}_{zi}(0^+)$$
$$\ldots$$

Chapter 2: Fourier series representations of Periodic signals

§2.1. Signal Representation by orthogonal signal set

① Real-valued signals

- <u>Def</u>: orthogonal signal set:

$$\left\{ x_1(t),\ x_2(t),\ \cdots,\ x_N(t),\ t \in [t_1, t_2] \right\}$$

is a set of <u>orthogonal</u> signals if

$$\int_{t_1}^{t_2} x_m(t)\, x_n(t)\, dt = \begin{cases} 0, & m \neq n \\ \mathcal{E}_n, & m = n \end{cases}$$

If $\mathcal{E}_n = 1$ for all n, then the set is <u>orthonormal</u>.

An orthogonal set can always be normalized by replacing $x_n(t)$ by $\dfrac{x_n(t)}{\sqrt{\mathcal{E}_n}}$ for all n.

- Approximate any signal $f(t)$, $t \in [t_1, t_2]$ by an orthogonal set:

$$f(t) \cong \sum_{n=1}^{N} c_n x_n(t), \quad t \in [t_1, t_2]$$

coeff.

<u>error:</u>

$$e(t) = f(t) - \sum_{n=1}^{N} c_n x_n(t)$$

choose the coeff's $\{c_1, c_2, \cdots, c_N\}$ to minimize the energy of $e(t)$.

$$\min_{c_1, c_2, \cdots, c_N} \mathcal{E}_e = \int_{t_1}^{t_2} e(t)^2 \, dt$$

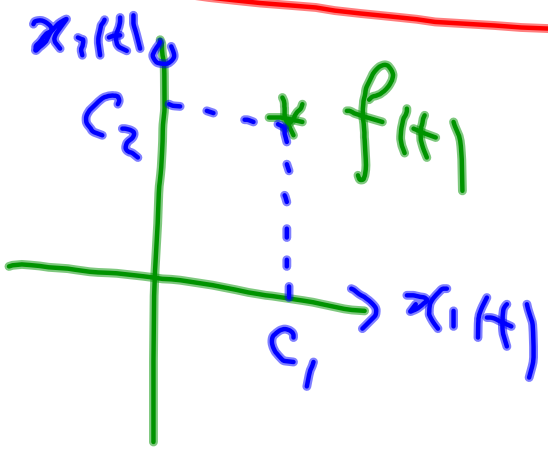
$$= \int_{t_1}^{t_2} \left(f(t) - \sum_{n=1}^{N} c_n x_n(t) \right)^2 dt$$

$$= \underbrace{\int_{t_1}^{t_2} f(t)^2 \, dt}_{E_f} - 2 \sum_{n=1}^{N} c_n \int_{t_1}^{t_2} f(t) \, x_n(t) \, dt$$

$$+ \underbrace{\int_{t_1}^{t_2} \left(\sum_{n=1}^{N} c_n x_n(t) \right)^2 dt}_{\sum_{n=1}^{N} c_n^2 \underbrace{\int_{t_1}^{t_2} x_n(t)^2 \, dt}_{\mathcal{E}_n} + 2 \sum_{n=1}^{N} \sum_{\substack{m=1 \\ m \neq n}}^{N} c_n c_m \underbrace{\int_{t_1}^{t_2} x_m(t) \cdot x_n(t) \, dt}_{0}}$$

$$\frac{\partial \mathcal{E}_e}{\partial c_n} = 2 \cdot \mathcal{E}_n \cdot c_n - 2 \int_{t_1}^{t_2} f(t)\, x_n(t) = \emptyset$$

$$\Rightarrow \boxed{c_n = \frac{1}{\mathcal{E}_n} \underbrace{\int_{t_1}^{t_2} f(t)\, x_n(t)\, dt}_{n=1,2,\cdots,N.}}$$



projection of $f(t)$ on $x_n(t)$.

- min error energy:

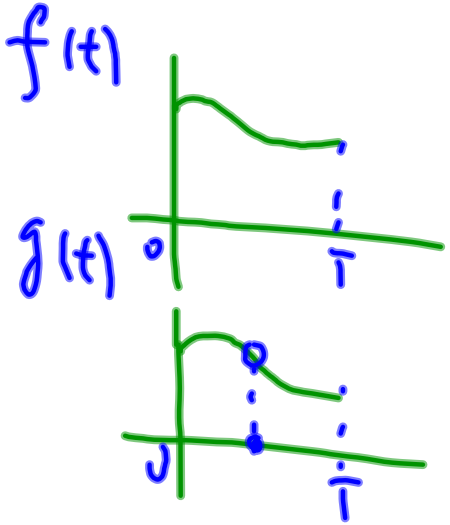
$$\boxed{\mathcal{E}_e = E_f - 2\sum_{n=1}^{N} c_n \cdot \underbrace{\int_{t_1}^{t_2} f(t)\, x_n(t)\, dt}_{c_n \cdot \mathcal{E}_n} + \sum_{n=1}^{N} \mathcal{E}_n c_n^2 = E_f - \sum_{n=1}^{N} c_n^2 \cdot \mathcal{E}_n.}$$

- When the set $\{x_n(t)\}$ is such that the error energy $\mathcal{E}_e \to 0$ as $N \to \infty$ for every member of some particular class

We say the set $\{x_n(t)\}$ is $\boxed{\text{complete}}$ on $[t_1, t_2]$ for that class of $f(t)$, and $\{x_n(t)\}$ is called a set of $\boxed{\text{basis signals}}$.

for complete set $\{x_n(t)\}$, we write

$$f(t) = \sum_{n=1}^{\infty} c_n x_n(t), \quad t \in [t_1, t_2]$$



NOT an equality in conventional sense!

$\mathcal{E}_e \to 0$

$f(t) = g(t), \quad t \in [0, T]$ ✗

$\int_0^T (f(t) - g(t))^2 \, dt = 0$ ✓