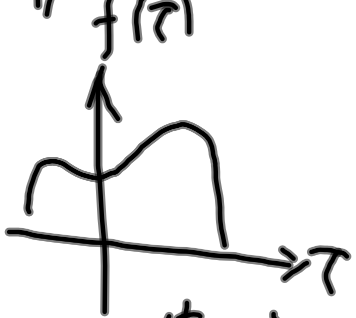
$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau)\, g(t-\tau)\, d\tau$$

1)



Ex:

$$\left.\begin{aligned} f(t) &= e^{-t}\, u(t) \\ h(t) &= e^{-2t}\, u(t) \end{aligned}\right\} y(t) = f(t) * h(t)$$

$$y(t) = \int_{-\infty}^{+\infty} f(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{+\infty} e^{-\tau} u(\tau)\, e^{-2(t-\tau)} u(t-\tau)\, d\tau$$

$$= \int_0^t e^{-\tau} \cdot e^{-2(t-\tau)}\, d\tau = e^{-2t}\int_0^t e^{\tau}\, d\tau = e^{-2t} \cdot e^{\tau}\Big|_0^t$$

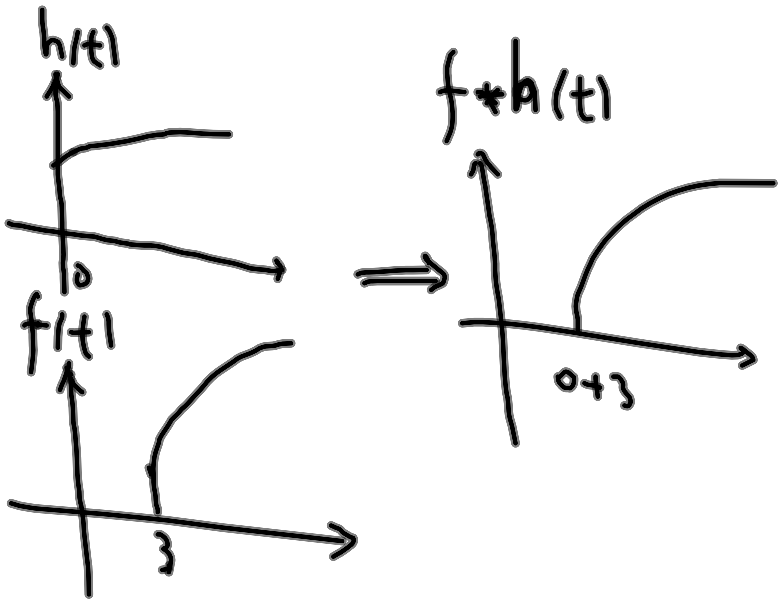
$$= e^{-2t}\left(e^{t}-1\right) = e^{-t} - e^{-2t} \quad t \geq 0$$

$$\Rightarrow y(t) = \left[e^{-t} - e^{-2t}\right] u(t)$$

Ex:

$$h(t) = e^{-t} u(t)$$

$$f(t) = e^{-2t} u(t-3)$$



$$h(t) * f(t) = \int f(\tau)\, h(t-\tau)\, d\tau$$

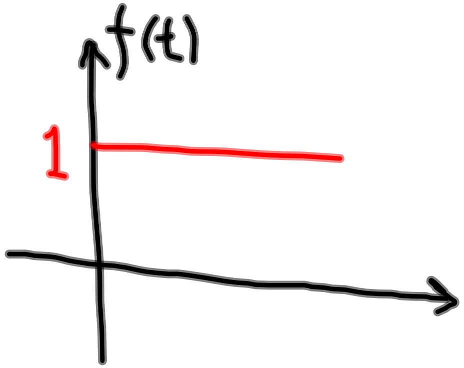
$$= \int_{-\infty}^{+\infty} e^{-2\tau} \underbrace{u(\tau-3)}_{\tau \geq 3}\, e^{-(t-\tau)} \underbrace{u(t-\tau)}_{\tau \leq t}\, d\tau$$

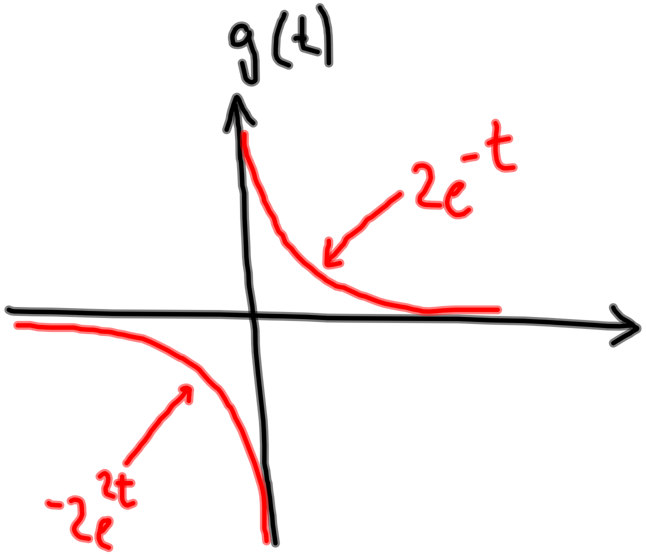
$$= \int_{3}^{t} e^{-2\tau} \cdot e^{-(t-\tau)} = e^{-t} \int_{3}^{t} e^{-\tau}\, d\tau$$

$$= e^{-t} \left(-e^{-\tau}\right)\Big|_{3}^{t} = e^{-t} \left[e^{-3} - e^{-t}\right]$$

$$h(t) * f(t) = \left[e^{-(t+3)} - e^{-2t} \right] u(t-3)$$

Ex:





$$f(t)=u(t)$$

$$g(t)=2e^{-t}u(t)+\left(-2e^{2t}\right)u(-t)$$

$$u(t)*\left[\underbrace{-2e^{2t}u(-t)}_{A}\underbrace{+2e^{-t}u(t)}_{B}\right]$$

$$B:\ u(t)*\left(2e^{-t}u(t)\right)=2\int_0^t e^{-(t-\tau)}d\tau=2e^{-t}\int_0^t e^{\tau}d\tau=2(1-e^{-t})u(t)$$

A: $u(t) * \left(-2e^{2t}u(-t)\right)$

$$= -2\int \underbrace{u(\tau)}_{\tau \geqslant 0} e^{2(t-\tau)} \underbrace{u(\tau - t)}_{\tau \geqslant t} d\tau$$

$$= \begin{cases} -2\int_t^{+\infty} e^{2(t-\tau)} d\tau = -1 & t \geqslant 0 \\ -2\int_0^{+\infty} e^{2(t-\tau)} d\tau = -e^{-2t} & t < 0 \end{cases}$$

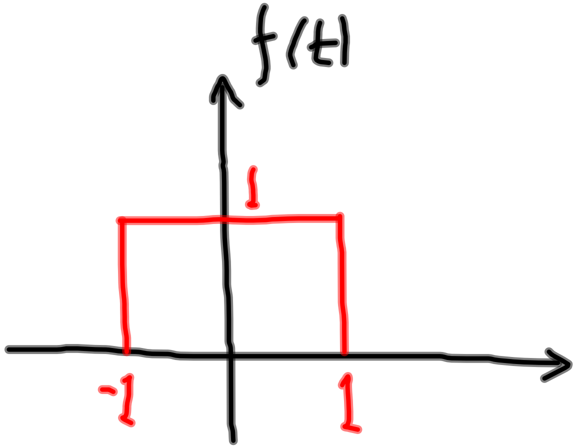
A + B :

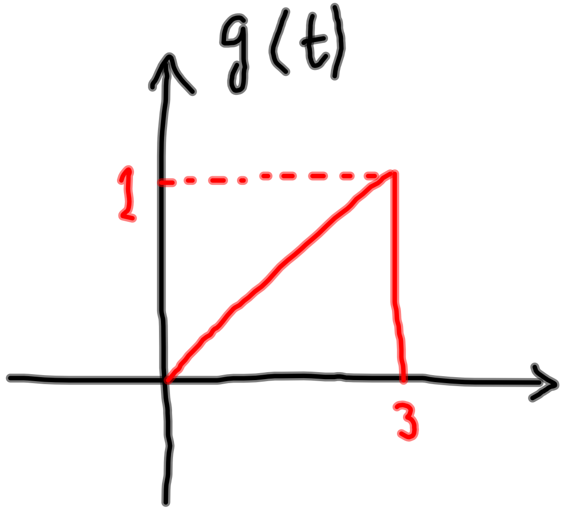
$$y(t) = (1 - 2e^{-t})u(t) - \underbrace{e^{2t}u(-t) - u(t)}$$

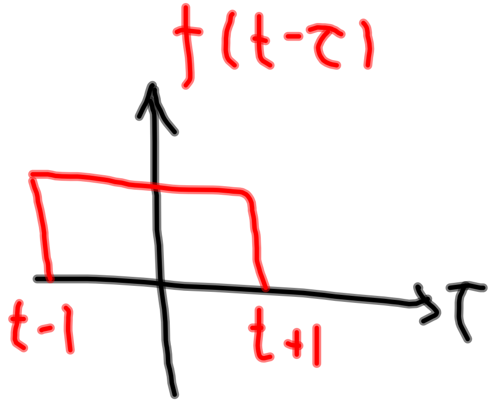
$$t \geq 0 \longrightarrow -1$$

$$t < 0 \longrightarrow e^{2t}u(-t)$$

Ex:

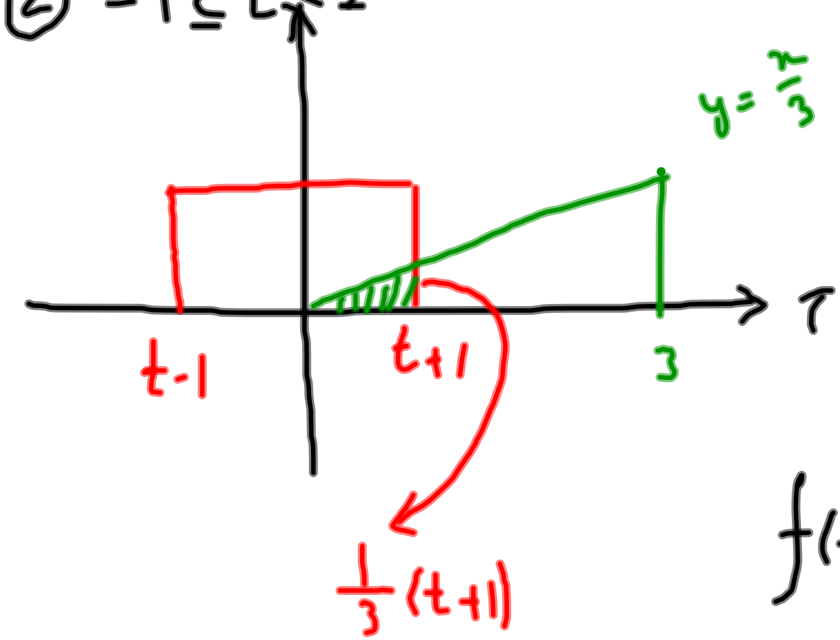






① $t < -1 \longrightarrow y(t) = 0$

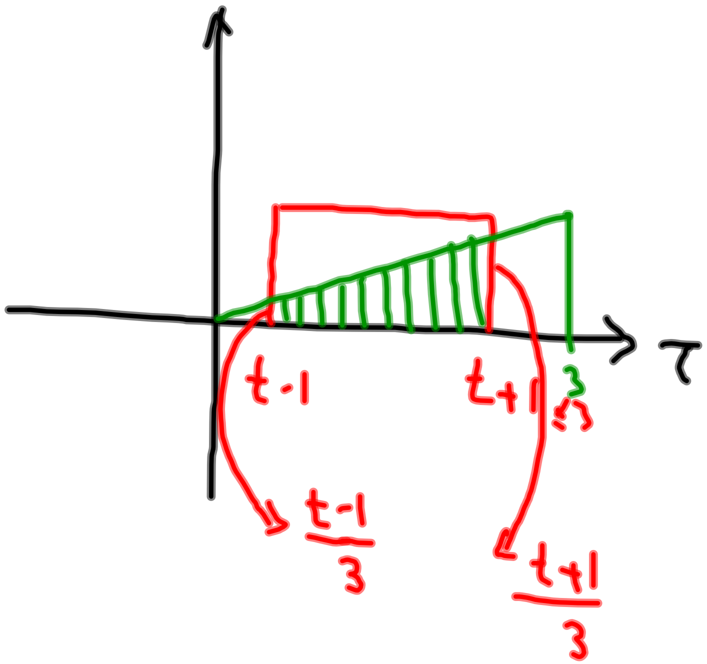
② $-1 \le t < 1$



$$f(t) * g(t) = \frac{1}{2} \cdot (t+1) \cdot \frac{t+1}{3}$$

$$= \frac{(t+1)^2}{6}$$

③ $1 \leq t \leq 2$

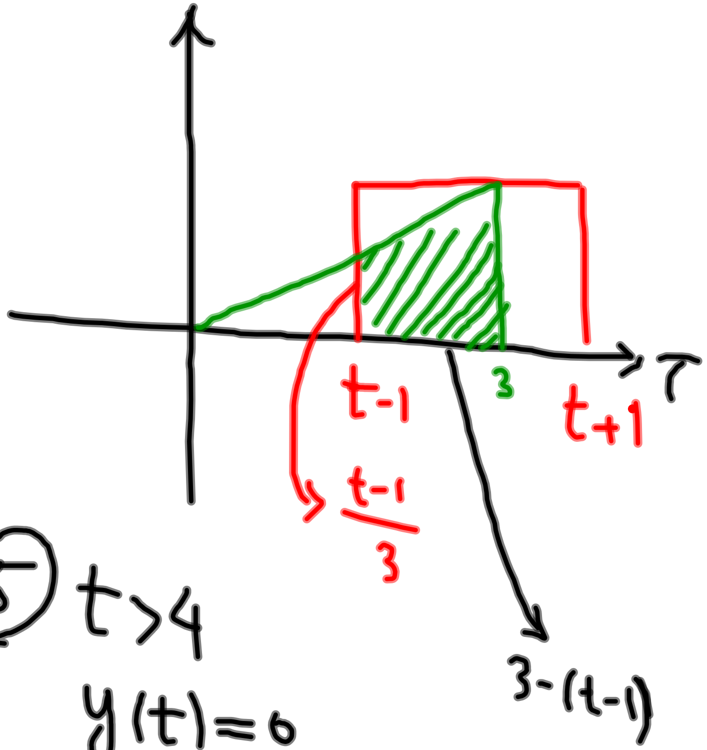


$$f(t) * g(t) = \left(\frac{1}{3}(t-1) + \frac{1}{3}(t+1) \right)$$

$$\times 2 \times \frac{1}{2}$$

$$= \frac{2}{3} t$$

④ $2 \leq t \leq 4$



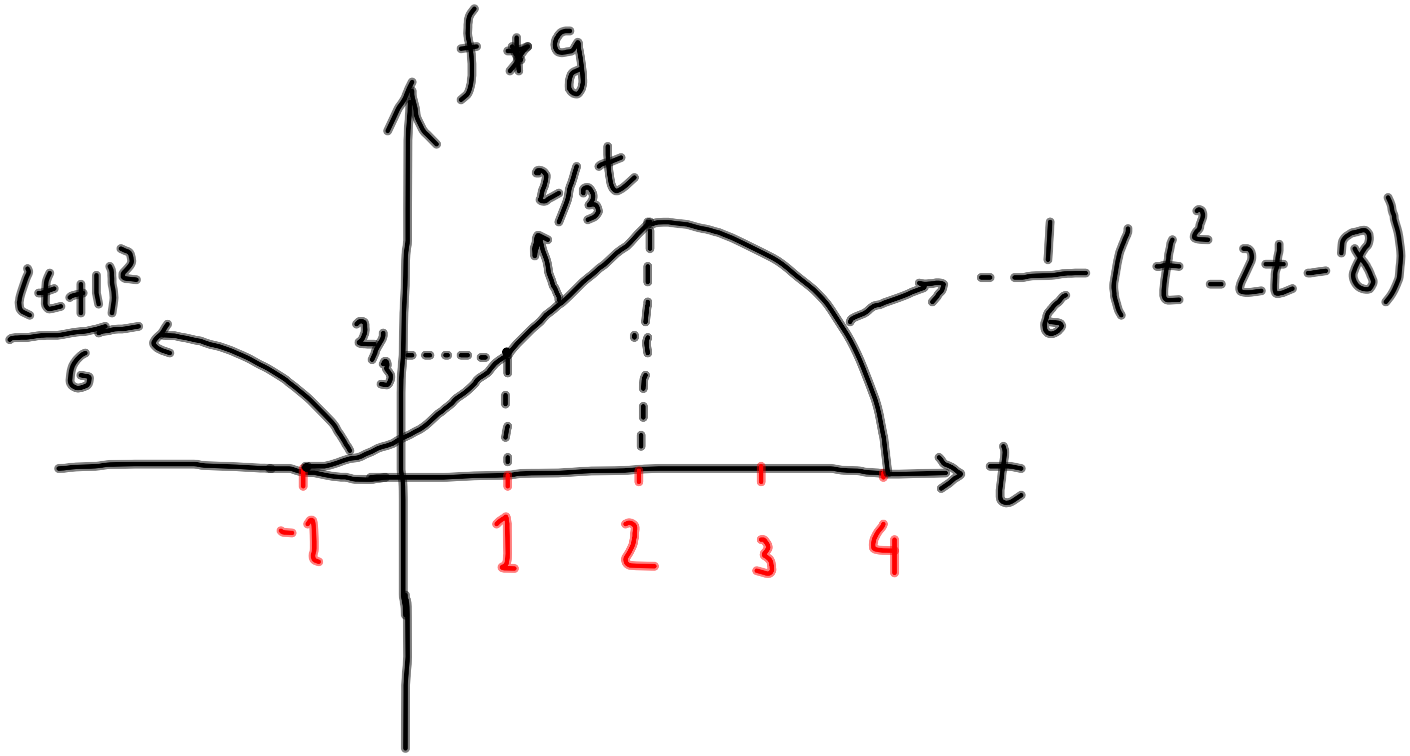
$$f(t) * g(t)$$

$$= \left[\frac{1}{3}(t-1)+1\right](3-(t-1)) \times \frac{1}{2}$$

$$= -\frac{1}{6}(t^2-2t-8)$$

⑤ $t > 4$

$y(t) = 0$



$f * g$
t
$\frac{(t+1)^2}{6}$
$2/3\,t$
$-\frac{1}{6}(t^2-2t-8)$
$2/3$
-1
1
2
3
4

1.7 Linear, Time Invariant System (LTI)

- System of Interest: TI- Continuous Time Systems.

- In Particular: linear Differential Systems.

$$\underbrace{\left(D^{n} + a_{n-1}D^{n-1} + \dots + a_1 D^{1} + a_0\right)}_{Q(D)} y(t) = \underbrace{\left(b_m D^{m} + \dots + b_1 D^{1} + b_0\right)}_{P(D)} f(t)$$

$\{a_n\}, \{b_n\}$ Constant

linearity:

$$T(f_1(t)) = y_1(t) \Rightarrow Q(D)\, y_1(t) = P(D) \cdot f_1(t)$$

$$T(f_2(t)) = y_2(t) \Rightarrow Q(D)\, y_2(t) = P(D)\, f_2(t)$$

$$T(k_1 f_1(t) + k_2 f_2) : Q(D)\left[k_1 y_1(t) + k_2 y_2(t)\right] = P(D)\left[k_1 f_1(t) + k_2 f_2(t)\right]$$

$$\Rightarrow T(k_1 f_1 + k_2 f_2) = k_1 y_1 + k_2 y_2$$

· In general: we're interested $m \leq n$

If $m > n$: The system behaves as an $(m-n)$th order Differentiator which will the high freq. Components of the noise.

· Zero Input Response:

System input $f(t)=0$

$\longrightarrow$ Result of internal System Conditions (Intial Cond.)

$$Q(D)\cdot y_{zi}(t)=0$$

Zero-State Response:

Sytem Response to the input when initial Condition is zero.

$$Q(D)\,y_{zs}(t) = P(D)\cdot f(t)$$

· Total Response:

Zero input Response + Zero State Response

$$Q(D)\left[y_{zi}(t) + y_{zs}(t)\right] = P(D) \cdot f(t)$$