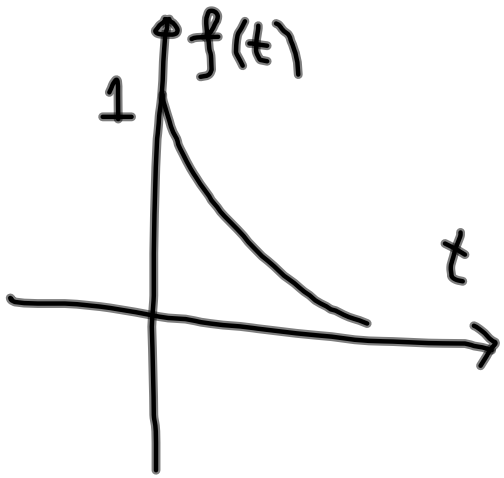
$$f(t) = f_e(t) + f_o(t)$$

$$f_e(t) = \frac{1}{2}\left(f(t) + f(-t)\right)$$

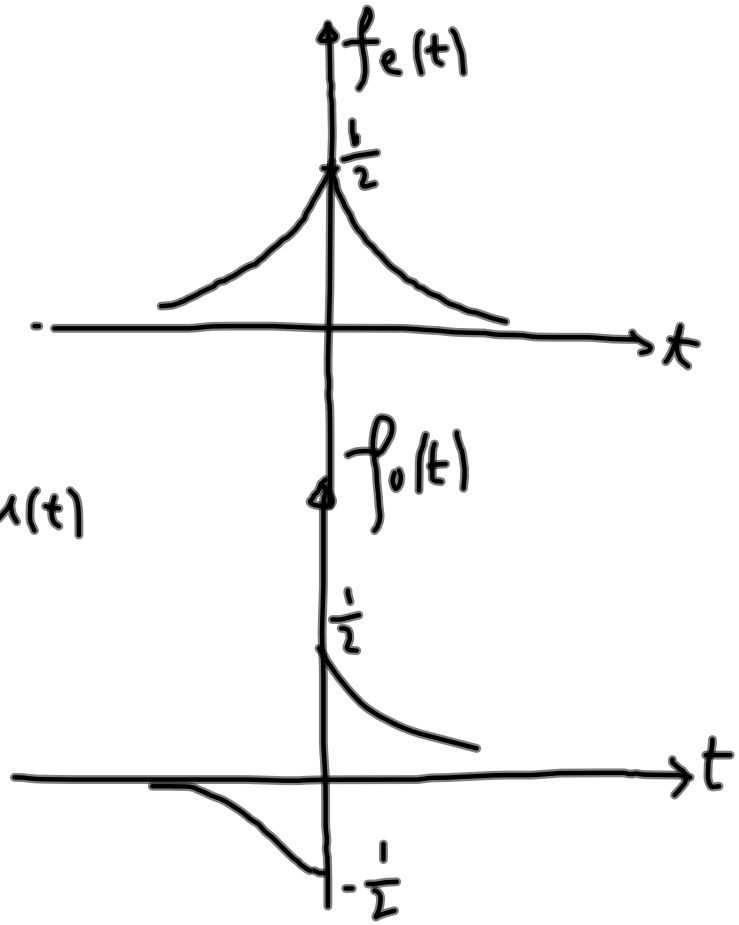
$$f_o(t) = \frac{1}{2}\left(f(t) - f(-t)\right)$$

<u>Ex:</u> $f(t) = e^{-at}u(t)$



$$f_o(t) = \frac{1}{2}\left[e^{-at}u(t) - e^{at}u(-t)\right]$$

$$f_e(t) = \frac{1}{2}\left[e^{-at}u(t) + e^{at}u(-t)\right]$$



Ex: $f(t) = e^{j\omega t} = \cos\omega t + j\sin\omega t$

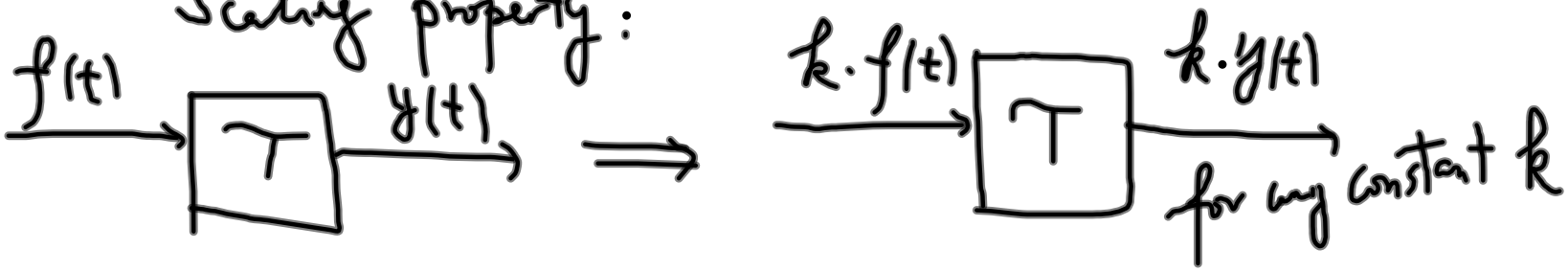
$$f_e(t) = \frac{1}{2}\left[e^{j\omega t} + e^{-j\omega t}\right] = \cos\omega t$$

$$f_o(t) = \frac{1}{2}\left[e^{j\omega t} - e^{-j\omega t}\right] = j\sin\omega t$$

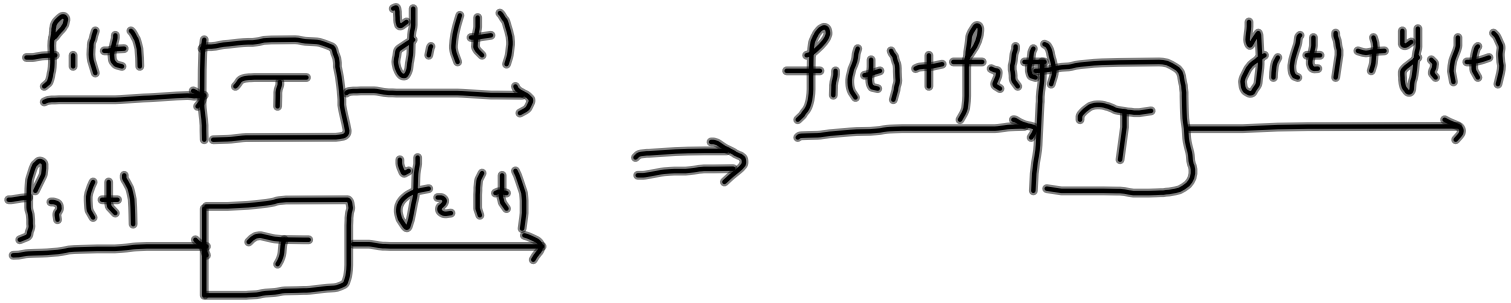
§1.5. Classifications of Systems

① linear vs Nonlinear systems

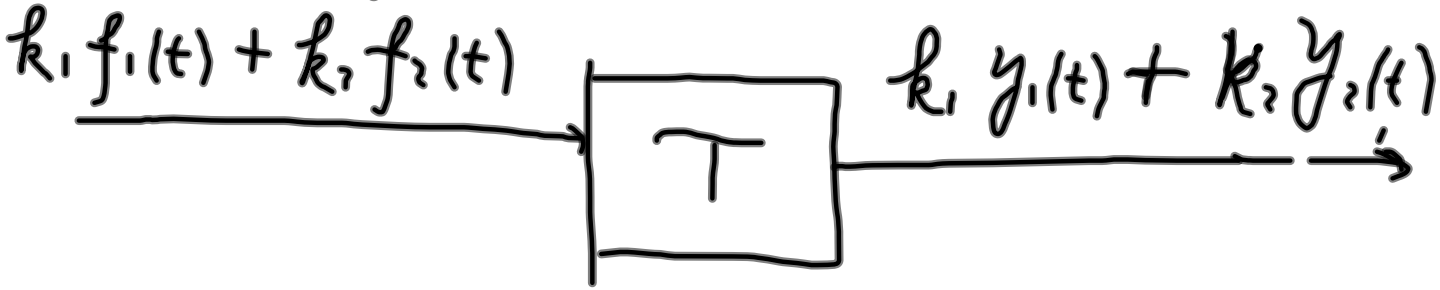
- Scaling property:



- additivity property:



* Combining the two properties: superposition property



Def: A system $T(\cdot)$ is $\boxed{\text{linear}}$ if for any input $f_1(t)$ & $f_2(t)$, given

$$T(f_1(t)) = y_1(t)$$
$$T(f_2(t)) = y_2(t)$$

then for any constants k_1 & k_2

$$T\left(k_1 f_1(t) + k_2 f_2(t)\right) = k_1 y_1(t) + k_2 y_2(t)$$

Def: A system is $\boxed{\text{nonlinear}}$ if it violates either the scaling property or the additivity property

Ex:

$$\longrightarrow \boxed{(\cdot)^2} \longrightarrow$$

$$f(t) \longrightarrow y(t) = f^2(t)$$

$$a \cdot f(t) \longrightarrow a^2 f^2(t) \neq a \cdot y(t)$$

$$f_1(t) \longrightarrow y_1(t) = f_1^2(t)$$

$$f_2(t) \longrightarrow y_2(t) = f_2^2(t)$$

$$f_1(t) + f_2(t) \longrightarrow \left(f_1(t) + f_2(t)\right)^2 \neq y_1(t) + y_2(t)$$

Ex:

$$\frac{dy(t)}{dt} + 3y(t) = f(t)$$

$f_1 \to y_1$:

$$\frac{dy_1(t)}{dt} + 3y_1(t) = f_1(t) \quad \times k_1$$

$f_2 \to y_2$:

$$\frac{dy_2(t)}{dt} + 3y_2(t) = f_2(t) \quad \times k_2$$

$$\Downarrow$$

$k_1 f_1 + k_2 f_2 \to k_1 y_1 + k_2 y_2$:

$$\frac{d(k_1 y_1 + k_2 y_2)}{dt} + 3(k_1 y_1 + k_2 y_2) \overset{?}{=} (k_1 f_1 + k_2 f_2)$$

linear ✓

$$\left(D^n + a_{n-1} D^{n-1} + \cdots + a_0\right) y(t)$$

$$= \left(b_m D^m + b_{m-1} D^{m-1} + \cdots + b_0\right) f(t)$$

$$D^n \triangleq \frac{d^n}{dt^n}$$

The system is $\boxed{\text{linear}}$ if $\{a_i\}$ & $\{b_i\}$ are constants or independent functions of t

On the other hand, if one of a_i or b_i is a function of $f(t)$ or $y(t)$, then it is $\boxed{\text{nonlinear}}$

Ex:

$$\frac{d}{dt} y(t) + t^2 y(t) = (2t+3) f(t)$$

output output input

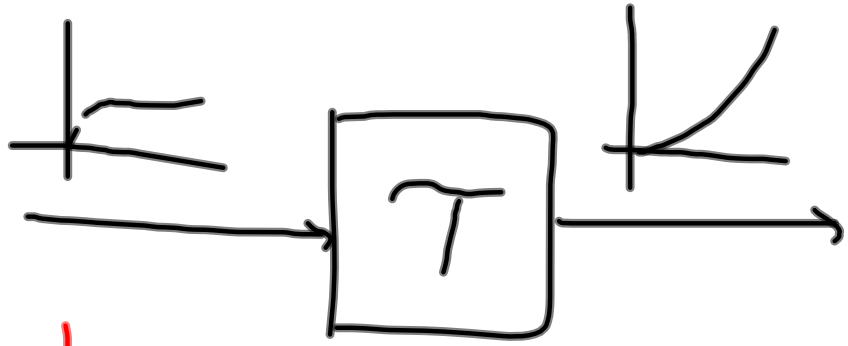
linear

Ex:

$$y(t) \cdot \frac{d}{dt} y(t) + 3y(t) = f(t)$$

non linear

② Time-invariant vs Time-varying system



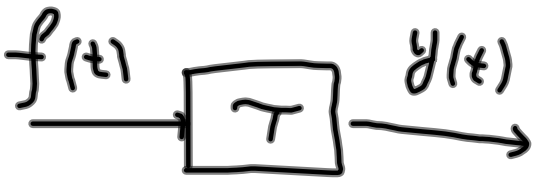
<u>Def.</u> A system $T(\cdot)$ is <u>time-invariant</u> if

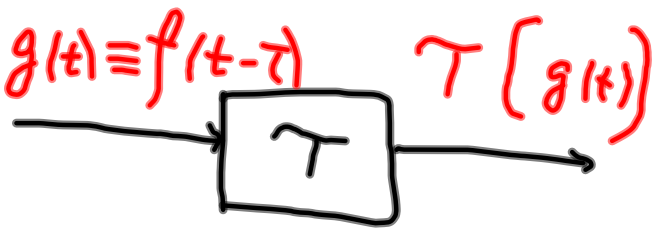
$$T(f(t)) = y(t)$$

$$\Longrightarrow T(f(t-\tau)) = y(t-\tau)$$

for all τ.

Def: A system $T(\cdot)$ is <u>time-varying</u> if it is NOT time-invariant.





$$\underline{y(t-\tau)} \overset{?}{=} \underline{T[g(t)]}$$

✓ ✓

Ex: $y(t) = \sin t \, f(t-2)$

① $y(t-\tau) = \boxed{\sin(t-\tau)} f(t-\tau-2)$

② $g(t) = f(t-\tau)$

$$T[g(t)] = \sin t \; g(t-2)$$
$$= \boxed{\sin t} f(t-\tau-2)$$

$\therefore T(\cdot)$ is time-varying!

Ex: $y(t) = f(t-2)$

① $y(t-\tau) = f(t-\tau-2)$

② $g(t) = f(t-\tau)$

$T[g(t)] = g(t-2)$

$= f(t-\tau-2)$

$\therefore$ time-invariant !

Ex: $y(t) = f(-t)$

① $y(t-\tau) = f(-(t-\tau))$

$= f(-t+\tau)$

② $g(t) \equiv f(t-\tau)$

$T[g(t)] = g(-t)$

$= f(-t-\tau)$

$\therefore T(\cdot)$ is time-varying

$$(D^n + a_{n-1} D^{n-1} + \cdots + a_0) y(t)$$
$$= (b_m D^m + b_{m-1} D^{m-1} + \cdots + b_0) f(t)$$

- The system is <u>time-invariant</u> if $\{a_i\}$ & $\{b_i\}$ are constants. $\longrightarrow$ Linear & Time-invariant (LTI)
- It is time-varying is some coeff. are functions of time

③ Instantaneous vs. dynamic systems

<u>Def</u>: A system is <u>instantaneous</u> (or <u>memoryless</u>) if $y(t)$ depends on $f(t)$ only, but it does NOT depend on $f(t+\tau)$ for any $\tau \neq 0$.

<u>Def</u>: A system is <u>dynamic</u> (or <u>with memory</u>) if $y(t)$ depends on $f(t+\tau)$ for some $\tau \neq 0$.

④ Causal vs non-causal systems

<u>Def:</u> A system is <u>causal</u> if for any t_0. $y(t_0)$ depends only on $f(t)$ for $t \le t_0$

$\Longrightarrow$ the present output depends on only the past & present input.

<u>Def:</u> A system is <u>non causal</u> if there is t_0, such that $y(t_0)$ depends on $f(t_0+\tau)$ for some $\tau > 0$. <u>ex.</u> $y(t) = f(t-2) + f(t+2)$

(5) Continuous-time vs. discrete-time systems

both input & output are cont-time signals.

both input & output are disc.-time signals

(6) Analog & Digital systems

input & output are analog signals

digital input & output signals