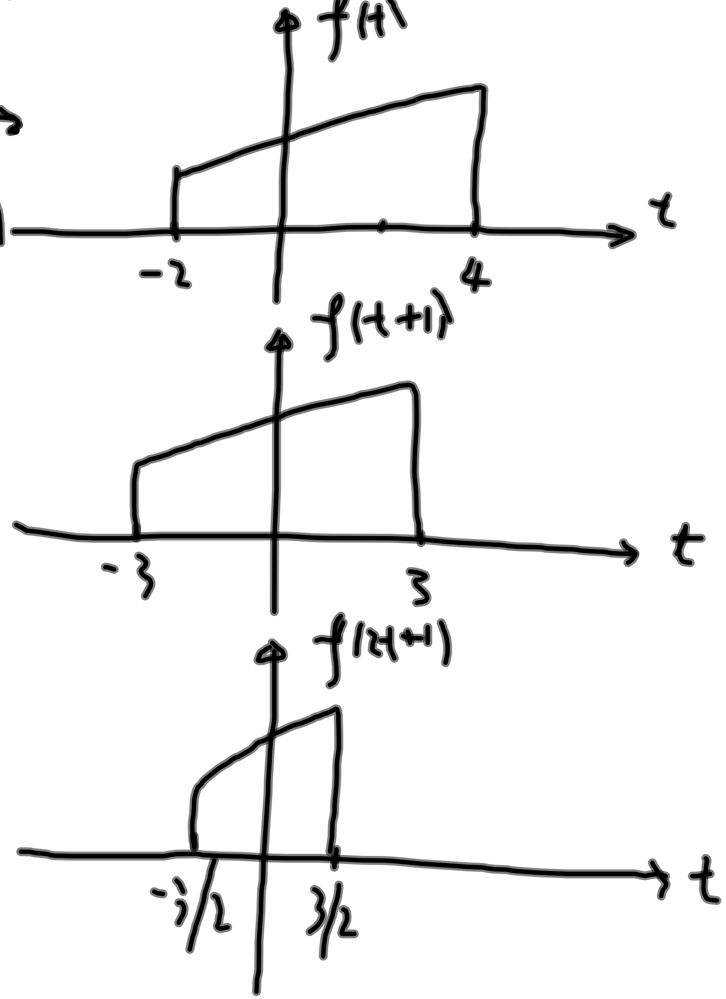
④ Combined Operations

$$f(t) \longrightarrow \phi(t) = f(at-b) \qquad a \neq 0$$

$\boxed{a > 0}$

Ex: $\longrightarrow$

$\phi(t) = f(2t+1)$



Method 1:

$$\begin{cases} g(t) = f(t-b) \\ \phi(t) = g(at) \end{cases}$$

Method 2:

$$h(t) = f(at)$$

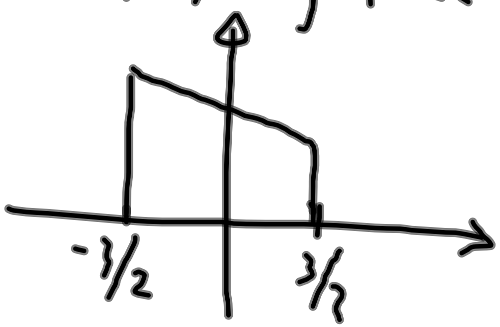
$$\phi(t) = h\left(t - \frac{b}{a}\right)$$

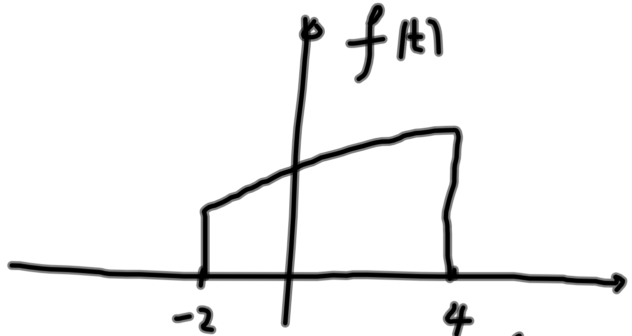
$$\boxed{a < 0}$$

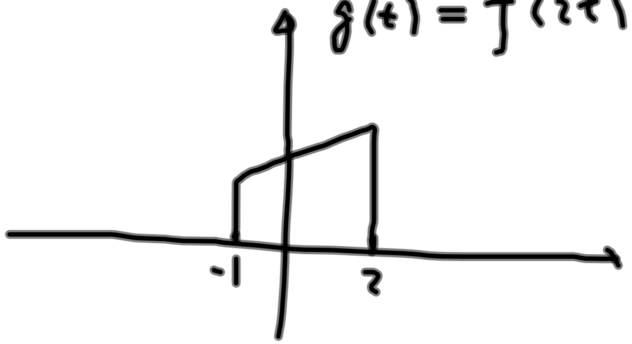
$$\psi(t) = f\left(|a|t - b\right)$$

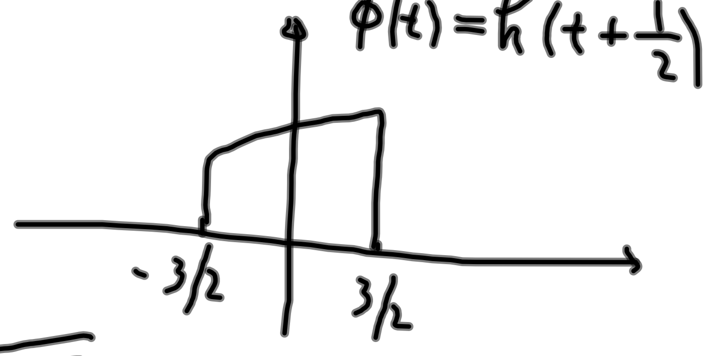
$$\phi(t) = \psi(-t)$$

<u>Ex:</u> $\phi(t) = f(-2t+1)$



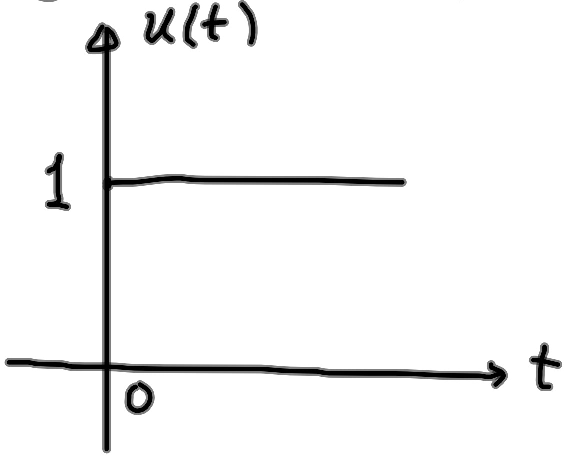






§1.4 Useful Signal Models

① Unit step function $u(t)$

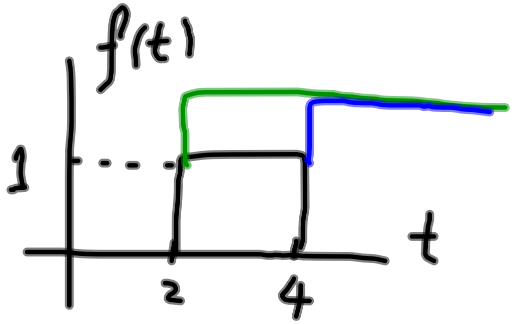


$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

- any causal signal $f(t) = f(t)u(t)$
- any anticausal signal $f(t) = f(t)u(-t)$

- $u(t)$ is useful in describing a signal w/ diff. expressions over diff intervals

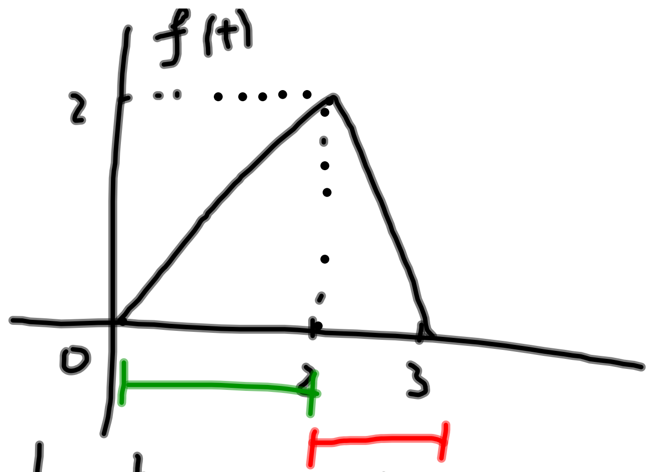
<u>Ex:</u>



$$f(t) = \begin{cases} 1 & 2 \le t \le 4 \\ 0 & t < 2 \text{ or } t > 4 \end{cases}$$

$$= u(t-2) - u(t-4)$$

Ex:

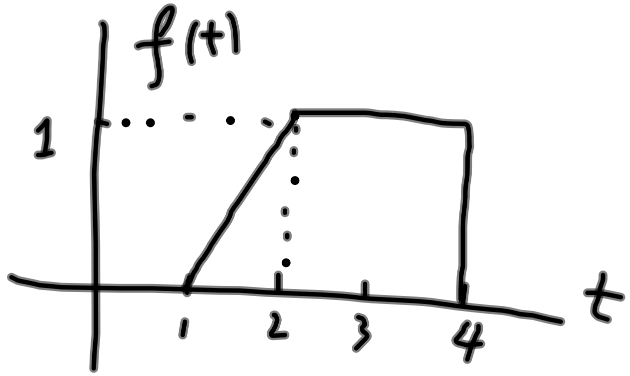


$$f(t) = \begin{cases} t, & 0 \le t \le 2 \\ -2(t-3), & 2 \le t \le 3 \\ 0 & o/w \end{cases}$$

$$= t\,\boxed{\big(u(t) - u(t-2)\big)} - 2(t-3)\,\boxed{\big(u(t-2) - u(t-3)\big)}$$

$$= t \cdot u(t) - 3(t-2)\,u(t-2) + 2(t-3)\,u(t-3)$$

Ex:



$$f(t) = (t-1)\left(u(t-1) - u(t-2)\right)$$

$$+ 1\cdot\left(u(t-2) - u(t-4)\right)$$

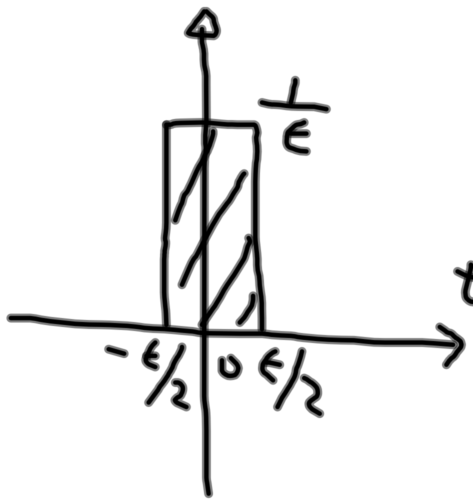
$$= (t-1)u(t-1) - (t-2)u(t-2)$$

$$- u(t-4).$$

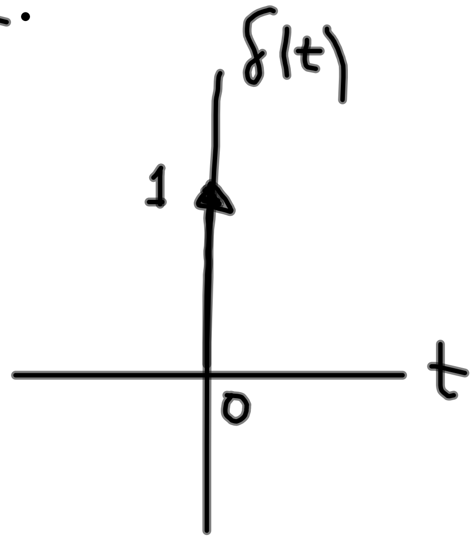
② Unit impulse function $\delta(t)$

<u>Def:</u>

$$\begin{cases} \delta(t) = 0, & t \neq 0 \\ \displaystyle\int_{-\infty}^{+\infty} \delta(t)\,dt = 1. \end{cases}$$

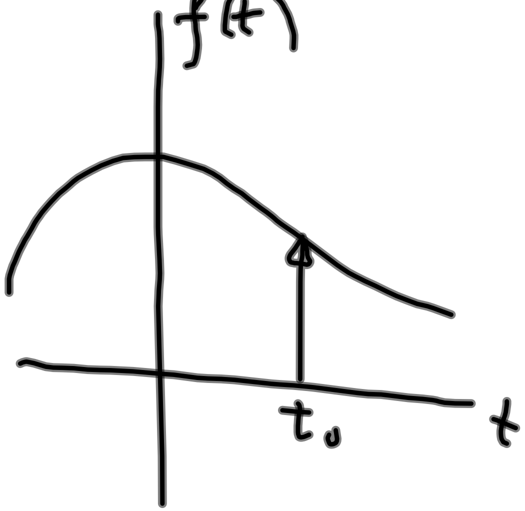


$\epsilon \to 0.$

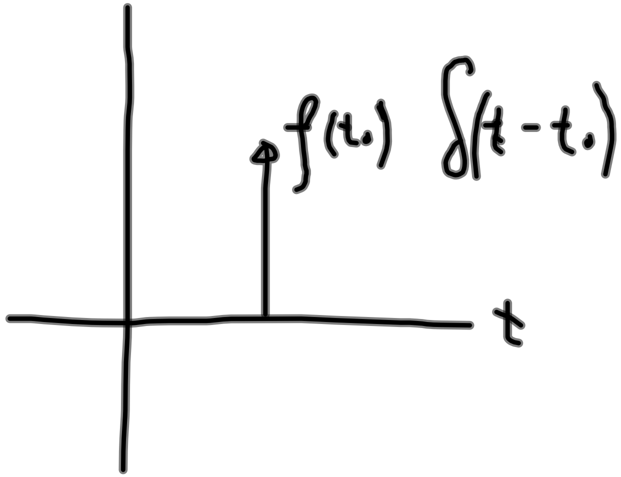


Property 1: If $f(t)$ is continuous at t_0.

then $\boxed{f(t)\cdot\delta(t-t_0) = f(t_0)\,\delta(t-t_0)}$



$\Longrightarrow$



Property 2: <u>sampling</u>

$$\boxed{\int_{-\infty}^{+\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)}$$

$$= \int_{-\infty}^{+\infty} f(t_0)\,\delta(t-t_0)\,dt$$

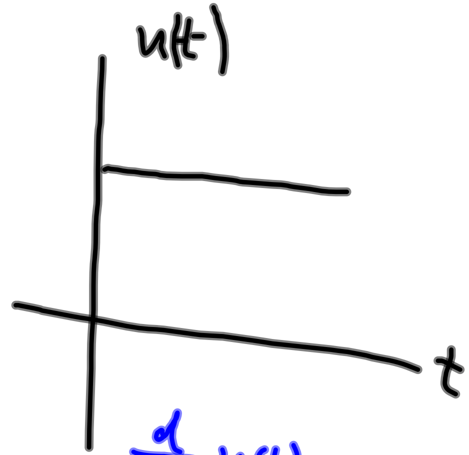
$$= f(t_0)\cdot\underbrace{\int_{-\infty}^{+\infty}\delta(t-t_0)\,dt}_{1.}$$

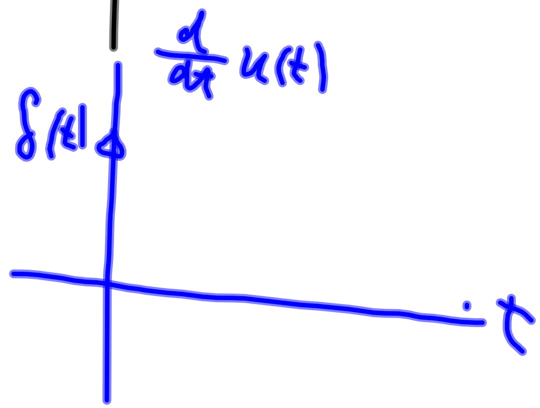
property 3 :

$$\boxed{\delta(t) = \frac{d}{dt} u(t)}$$

$$\because \int_{-\infty}^{t} \delta(\tau)\, d\tau = \begin{cases} 0. & t < 0 \\ 1 & t \geq 0 \end{cases}$$

$$= u(t)$$





Ex:

a) $$\frac{\omega^2+1}{\omega^2+9}\,\delta(\omega-1) = \frac{1+1}{1+9}\,\delta(\omega-1)$$
$$= \frac{1}{5}\,\delta(\omega-1)$$

b) $$\int_{-\infty}^{+\infty} \delta(t-2)\cos\frac{\pi t}{4}\,dt$$
$$= \cos\frac{\pi t}{4}\Big|_{t=2} = \phi.$$

③ Exponential function e^{st}

$s = \sigma + j\omega$ — complex freq.

$$e^{st} = e^{(\sigma + j\omega)t}$$

$$= e^{\sigma t} \cdot \underbrace{e^{j\omega t}}_{(\cos\omega t + j\sin\omega t)}$$

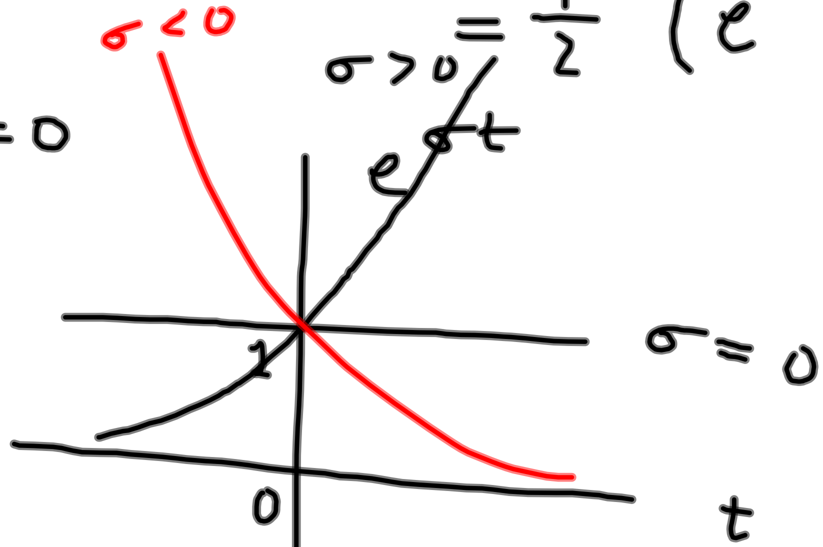
$$= e^{\sigma t} \cdot (\cos\omega t + j\sin\omega t)$$

$$e^{s^* t} = e^{\sigma t}(\cos\omega t - j\sin\omega t)$$

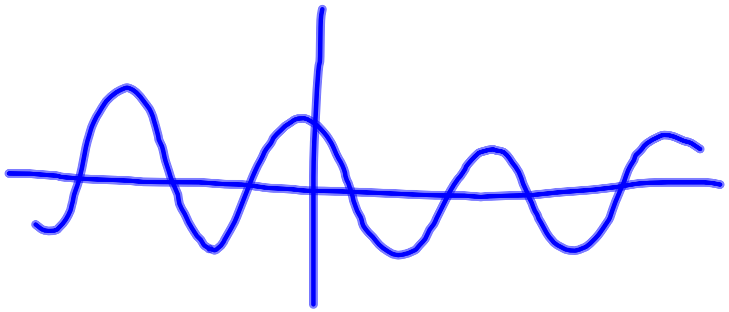
Ex: $\mathrm{Re}\{e^{st}\} = e^{\sigma t}\cos\omega t$

$= \frac{1}{2}\left(e^{st} + e^{s^*t}\right)$

a) $\omega = 0$

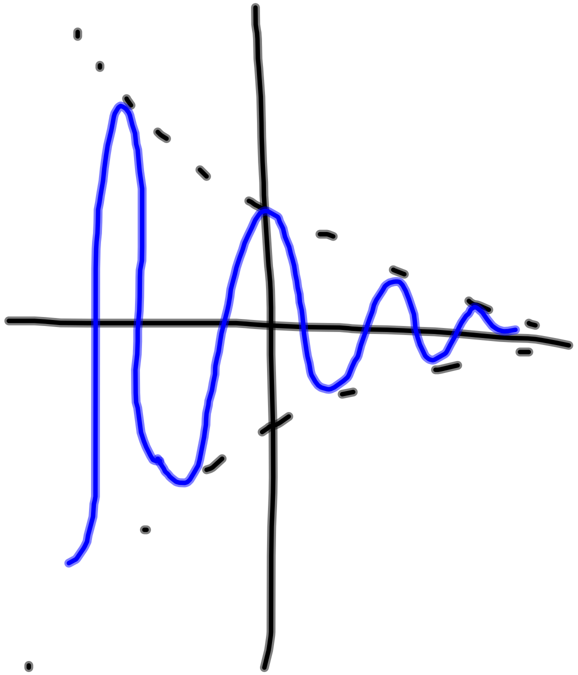


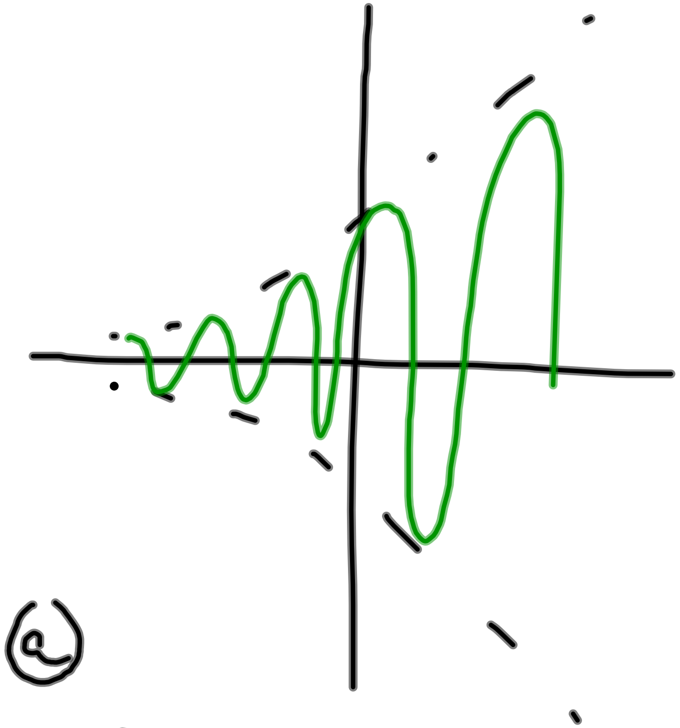
b) $\sigma = 0$

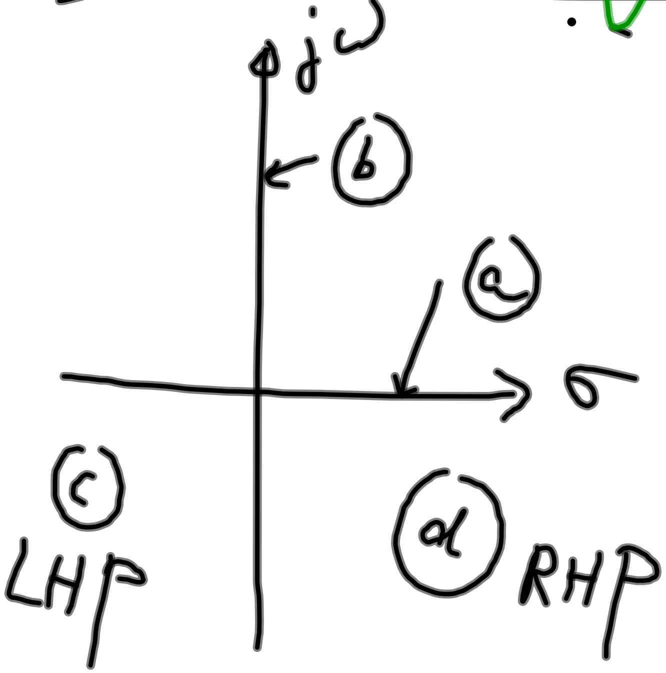


c) $\sigma < 0, \quad \omega \neq 0$

d) $\sigma > 0, \quad \omega \neq 0$

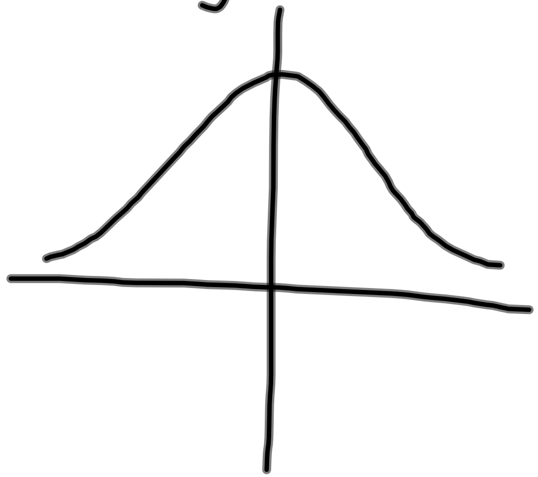






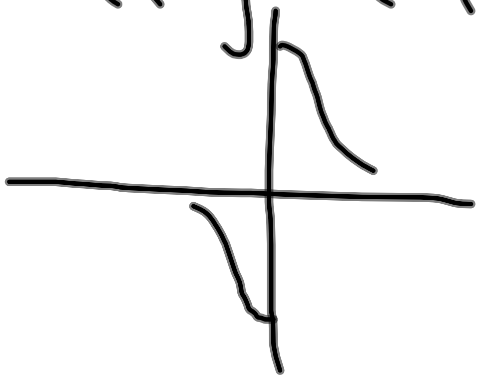
④ Even & odd functions

· even function: $f_e(-t) = f_e(t)$



symmetric about verticle axis

· odd function: $f_o(-t) = -f_o(t)$



anti-symmetric about verticle axis

Properties:

even × odd = odd

odd × odd = even

even × even = even

$$\int_{-a}^{a} f_o(t)\,dt = \emptyset$$

$$\int_{-a}^{a} f_e(t)\,dt = 2\cdot\int_{0}^{a} f_e(t)\,dt$$

(5) Even & odd Components of a signal

$$f(t) = f_e(t) + f_o(t)$$

$f_e(t) \downarrow$ even comp $f_o(t) \downarrow$ odd comp.

$$f_e(t) = \frac{1}{2}\left[f(t) + f(-t)\right]$$

$$f_o(t) = \frac{1}{2}\left[f(t) - f(-t)\right]$$