§ 1.1. Signal Size

- **Signal**: A function of some independent variables (e.g., time, space)

  <u>Ex:</u> telephone signal $f(t)$

  television signal $f(x, y, t)$

- **System**: An entity that processes a set of signals (inputs) to yield another set of signals (outputs)

$$\xrightarrow{\text{input signal}} \boxed{\text{system}} \xrightarrow{\text{output signal}}$$

system realizations: hardware: electrical, mechanical, hydraulic

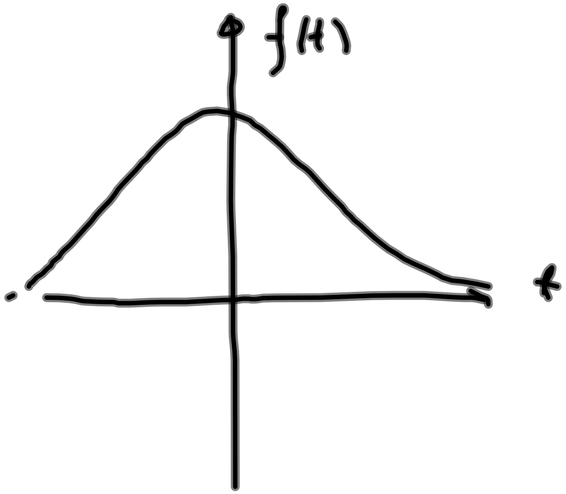
software: algorithms

① Signal Energy

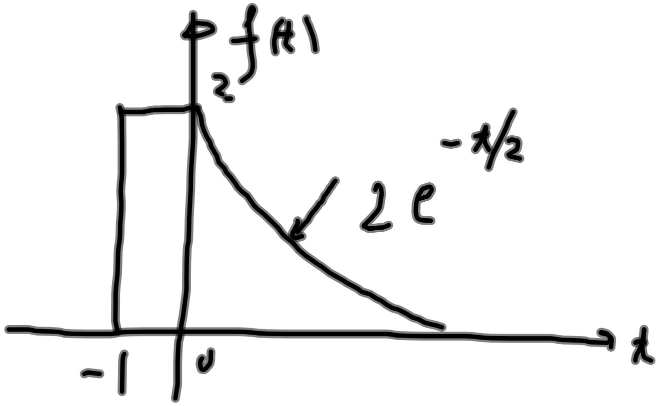
- $\bar{E}_f = \int_{-\infty}^{+\infty} f^2(t)\,dt$ — real-valued signals
- $\bar{E}_f = \int_{-\infty}^{+\infty} |f(t)|^2\,dt$ — complex-valued signals

Necessary condition for $\bar{E}_f$ to be finite:

$$|f(t)| \longrightarrow 0 \quad \text{as} \quad |t| \to \infty$$



Ex.



$$E_f = \int_{-\infty}^{+\infty} f(t)^2\,dt$$

$$= \underbrace{\int_{-1}^{0} 2^2\,dt}_{4} + \underbrace{\int_{0}^{\infty} \left(2e^{-t/2}\right)^2 dt}_{4\underbrace{\int_0^\infty e^{-t}\,dt}_{-e^{-t}\big|_0^\infty}}$$

$$= 8$$

② Signal Power

$$P_f = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 \, dt$$

— mean-squared value of $f(t)$

$\sqrt{P_f}$: root mean-squared value of $f(t)$

- If $f(t)$ is periodic with period T, then

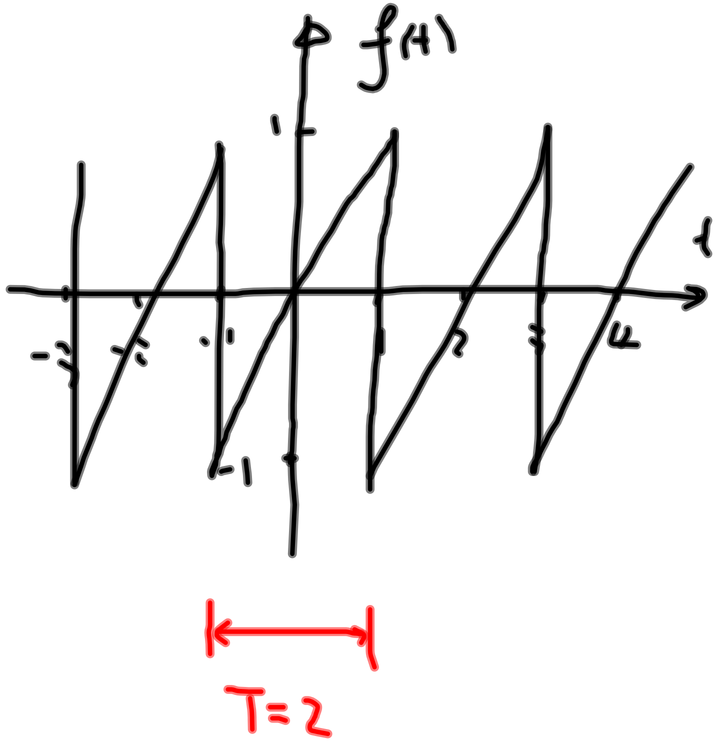
$$P_f = \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 \, dt$$

$$P_f = \frac{1}{2}\int_{-1}^{1} f(t)^2\,dt$$

$$= \frac{1}{2}\underbrace{\int_{-1}^{1} t^2\,dt}_{\frac{1}{3}t^3\Big|_{-1}^{1} = \frac{2}{3}}$$

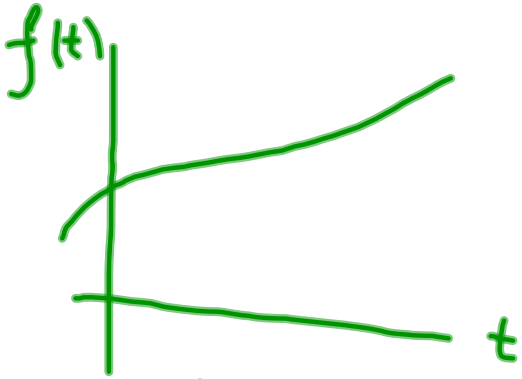
$$= \frac{1}{3}$$

rms : $\sqrt{P_f} = \frac{1}{\sqrt{3}}$

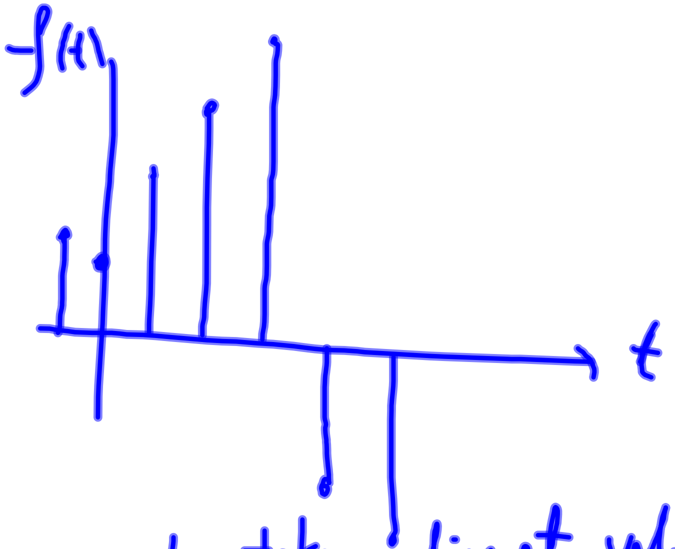


§ 1.2 Classifications of Signals

① Continuous-time vs discrete-time signals



t takes any values



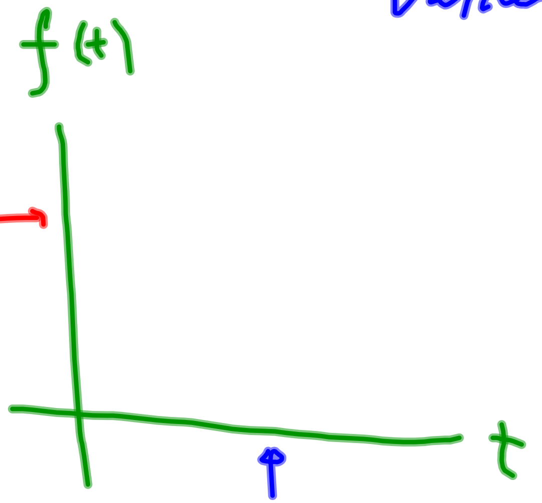
t takes discrete values

② Analog vs digital Signals

Analog: f(t) takes any value

digital: f(t) takes discrete values

Analog vs digital →



continuous-vs. discrete-time

③ Periodic vs Aperiodic Signals

- $f(t)$ is periodic if for some $T > 0$

$$f(t) = f(t+T) \quad \text{for all } t$$

- The smallest value of such T is the period of $f(t)$

- A signal is aperiodic if no such T exists.

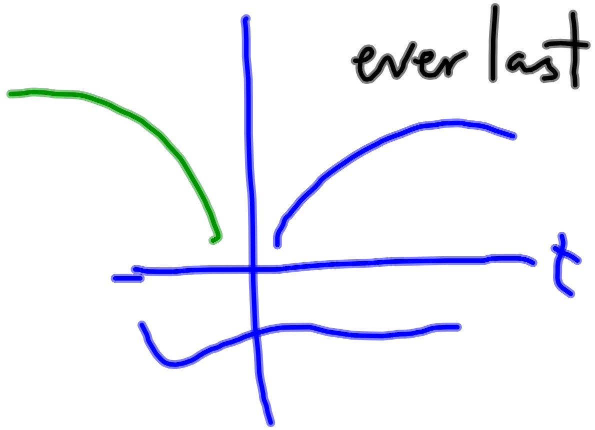
- A periodic signal is everlasting, i.e., the signal exists over $-\infty < t < \infty$

④ Causal vs Non-causal signals

- <u>causal</u> signal: $f(t) = 0$ for all $t < 0$
- non-causal signal: $f(t) \neq 0$ for some $t < 0$
- anti-causal signal: $f(t) = 0$ for all $t > 0$

everlasting $\Longrightarrow$ noncausal

$\cancel{\Longleftarrow}$



(5) Energy vs. Power signals

- $f(t)$ is an <u>energy signal</u> if $E_f < \infty$
- $f(t)$ is a <u>power signal</u> if $0 < P_f < \infty$

$$\left.\begin{array}{l}\text{Energy signal} \Longrightarrow \text{power} = 0 \\ \text{Power signal} \Longrightarrow \text{energy} = \infty\end{array}\right\}$$

$\Longrightarrow$ A signal can NOT be both!

• A signal can be <u>neither</u> energy or power signal

<u>Ex:</u> $f(t) = t$

$$\frac{1}{T}\int_{-T/2}^{T/2} f(t)^2 \, dt$$

$$= \frac{1}{T}\underbrace{\int_{-T/2}^{T/2} t^2 \, dt}_{\frac{1}{3}t^3 \Big|_{-T/2}^{T/2}} = \frac{T^2}{12} \to \infty$$

⑥ Deterministic vs Random Signals

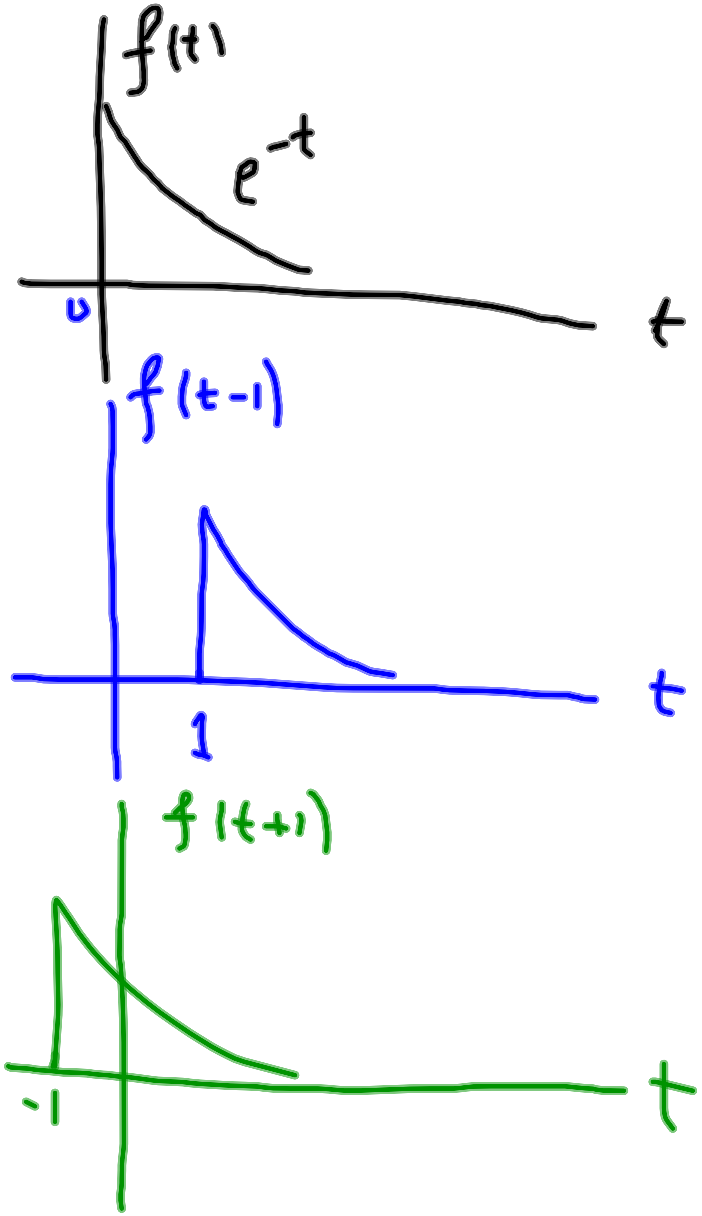
- deterministic signal: $f(t)$ is completely known for any t
- random signal: $f(t)$ is NOT known precisely but is known only probabilistically

§ 1.3 Useful Signal Operations

① Time shifting

$f(t) \longrightarrow \phi(t) = f(t - t_0)$

- $t_0 > 0$ shift to the right (delay)
- $t_0 < 0$ shift to the left (advance)


f(t)
e^-t
0
t
f(t-1)
1
t
f(t+1)
-1
t

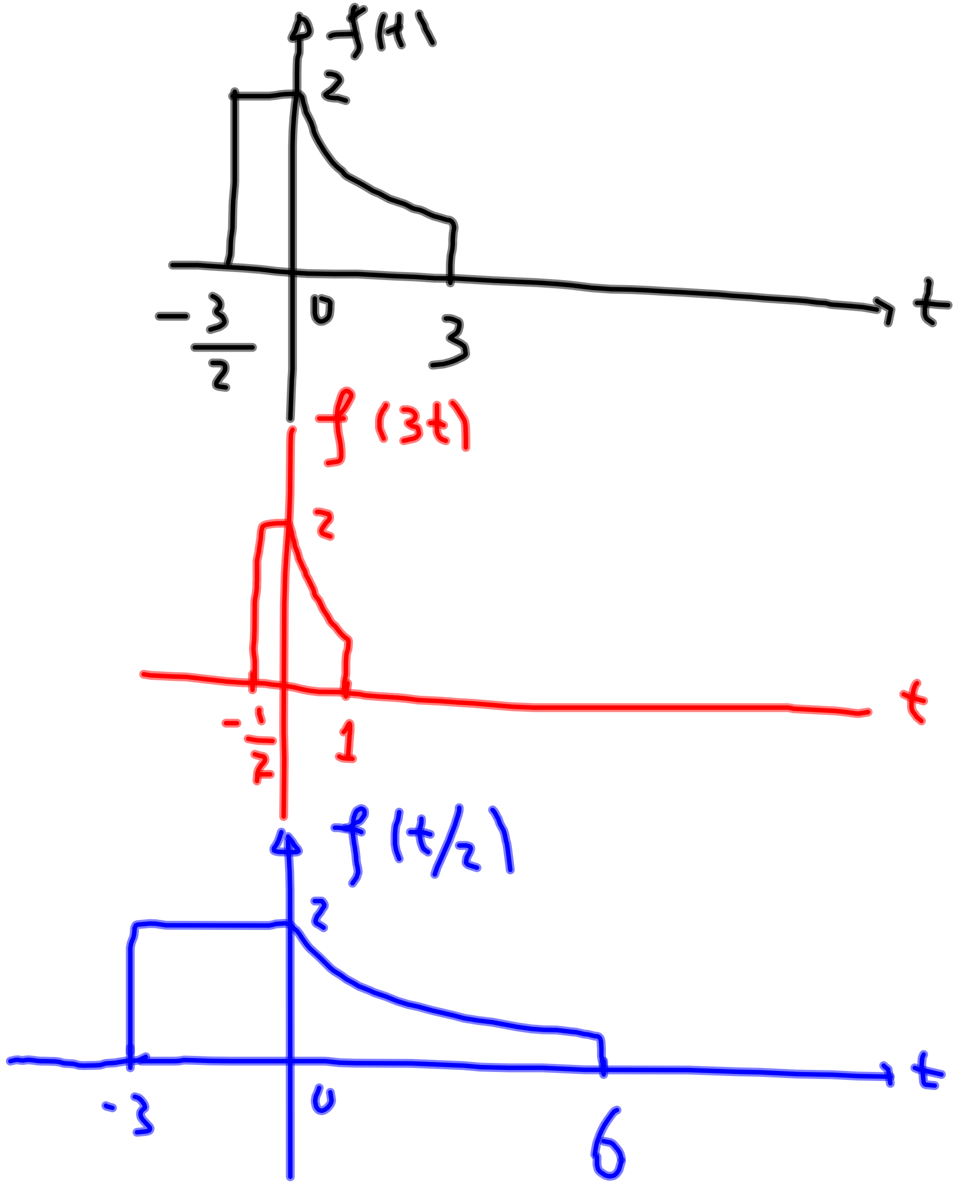
② Time scaling

$f(t) \longrightarrow \phi(t) = f(at) \quad a > 0$

- $a > 1$: shrink in time by a factor of a

- $0 < a < 1$: expand in time by a factor of $\frac{1}{a}$

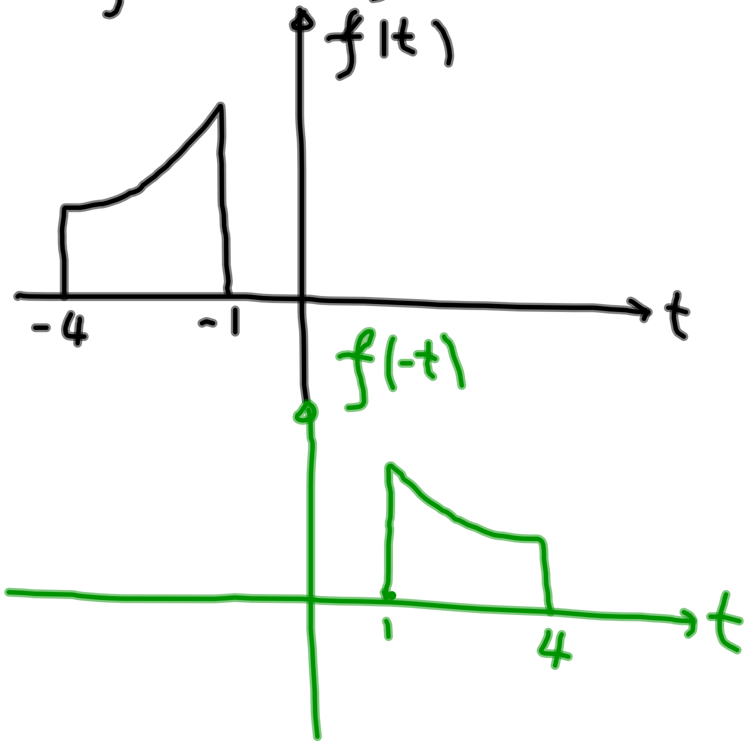
- origin $t = 0$ remains unchanged


f(t)
2
-3/2
0
3
t
f(3t)
2
-1/2
1
t
f(t/2)
2
-3
0
6
t

③ Time Reversal

$$f(t) \longrightarrow \phi(t) = f(-t)$$

- reflect $f(t)$ about the vertical axis



④ Combined Operations:

$$f(t) \longrightarrow \phi(t) = f(at-b)$$

$$a \neq 0$$