EE 3801 Fall 2006

Midterm 2

November 21, 2006

INSTRUCTIONS:

- **Carry only one side of a $8\frac{1}{2}$" $\times$ 11" note and a pencil or a pen with you. The exam is closed-book, closed-note. No calculator is allowed.**
- **The duration of the exam is 75 minutes.**

Problem	Points
I	/10
II	/20
III	/10
IV	/10
V	/10
VI	/20
VII	/20
Total	/100

Name:

Problem I [10 pts]
Calculate the following convolution:

$$y(t) = e^{2t}u(-t) * u(t).$$

Problem II [20 pts]

1. Find the exponential Fourier series of the following signal

$$f(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

2. Using your answer above and the property of Fourier series, find the exponential Fourier series of the following signal

$$g(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0 - T_0/2)$$

3. Find the Fourier transform of $g(t)$.

Problem III [10 pts]
Suppose $f(t) \overset{\mathcal{F}}{\longleftrightarrow} F(\omega)$. Find the Fourier transform of the signal

$$f(at + b)$$

with a and b being some real-valued constants.

Problem IV [10 pts]

Define $\text{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1, & |t| \leq \frac{\tau}{2}, \\ 0, & |t| > \frac{\tau}{2}. \end{cases}$ Given the Fourier transform $\text{rect}\left(\frac{t}{\tau}\right) \overset{\mathcal{F}}{\longleftrightarrow} \tau\text{sinc}\left(\frac{\omega\tau}{2}\right)$, find the Fourier transform of the signal

$$f(t) = \text{sinc}(Wt)$$

with W being some constant, by making use of the duality property of the Fourier transform.

Problem V [10 pts]
An LTI system has an impulse response

$$h(t) = e^{-at}u(t)$$

with $a > 0$. The input signal is given by

$$f(t) = e^{-bt}u(t)$$

with $b > 0$. Find the magnitude spectrum and the phase spectrum of the output signal.

Problem VI [20 pts]
Find the Fourier transforms of the following signals:
(a) $f(t) = \sin(\omega_0 t)$;
(b) $g(t) = (1+t)[u(t+1) - u(t)] + (1-t)[u(t) - u(t-1)]$.

Problem VII [20 pts] A signal $f(t)$ has the spectrum $F(\omega) = u(\omega + 200\pi) - u(\omega - 200\pi)$.

1. What is the Nyquist sampling rate of $f(t)$?

2. It is sampled at a rate of (a) 150Hz (b) 200 Hz (b) 300Hz. For each of the three cases: If the sampled signal is passed through an ideal lowpass filter of bandwidth 100Hz and unit gain, sketch the spectrum of the output signal.