EE 3801 Fall 2006

Final Exam

December 21, 2006

INSTRUCTIONS:

- **Carry only one $8\frac{1}{2}$" $\times$ 11" note (two sides) and a pencil or a pen with you. The exam is closed-book, closed-note. No calculator is allowed.**
- **The duration of the exam is 180 minutes.**

Problem	Points
I	/10
II	/10
III	/10
IV	/10
V	/10
VI	/10
VII	/10
VIII	/10
IX	/10
X	/10
Total	/100

Name:

Problem I [10 pts]
Consider a continuous-time system whose input-output relationship is given by

$$y(t) = \int_{-10}^{10} f(\tau)d\tau.$$

Classify this system according to the following criteria. *Justify your answers.*

1. Is this system linear or nonlinear?

2. Is this system time-invariant or time-variant?

3. Is this system instantaneous or dynamic?

4. Is this system causal or noncausal?

5. Is this system BIBO stable or BIBO unstable?

Problem II [10 pts]
Given
$$f_1(t) = e^{2t}[u(t+1) - u(t-1)], \quad \text{and} \quad f_2(t) = e^{-t}[u(t-2) - u(t-3)],$$
calculate the convolution $f_1(t) * f_2(t)$.

Problem III [10 pts]
Consider the signal $f(t) = \cos(3\pi t) \sin(5\pi t)$.

1. Find the Fourier transform of $f(t)$.

2. Find the exponential Fourier series of $f\left(t - \frac{1}{10}\right)$.

Problem IV [10 pts] (Note: we solved this problem in class.)
Given the following assumptions about $f(t)$, find $f(t)$.
(a) $f(t)$ is real;
(b) $f(t)$ is periodic with period $T = 4$;
(c) $F_n = 0$ for $|n| > 1$;
(d) The signal with Fourier coefficients $G_n = e^{-j\pi n/2} F_{-n}$ is odd;
(e) $\frac{1}{T}\int_T |f(t)|^2 = \frac{1}{2}$.

Problem V [10 pts]
An LTI system has the following transfer function

$$(D^2 - 5D - 8)y(t) = D(3D - 4)f(t).$$

Draw the block diagrams corresponding to the following implementations of this system, and write down the corresponding state-space equations.

1. Direct form.
2. Cascade form.
3. Parallel form.

Problem VI [10 pts]
An LTI system has the following transfer function

$$H(s) = \frac{s^3(1+e^{-2s})}{(s+1)^2(s+2).}$$

1. Find the input-output relationship in terms of a differential equation.

2. Find the impulse response $h(t)$ of this system.

Problem VII [10 pts]
An LTI system is described by the following differential equation and initial conditions

$$(D^2 + 4D + 4)y(t) = Df(t), \quad \text{and} \quad y(0^-) = \dot{y}(0^-) = 1.$$

The input signal is $f(t) = e^{-3t}u(t)$. Calculate the zero-input response and the zero-state response.

Problem VIII [10 pts]
An LTI system has the following transfer function

$$H(s) = \frac{s+5}{s^3+5s^2+6s}.$$

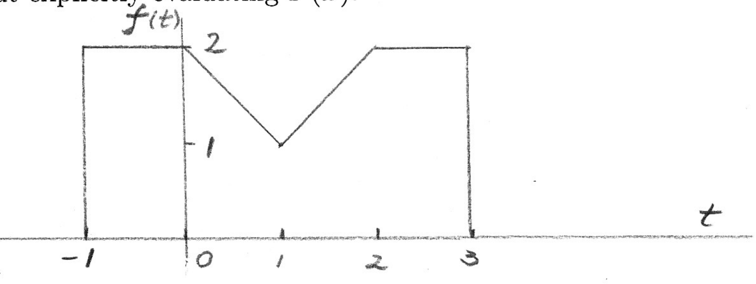
1. Characterize the asymptotic stability and BIBO stability of this system.

2. If the input signal $f(t) = \left[3 + 2\cos\left(3t + \frac{\pi}{4}\right) - \sin\left(t - \frac{\pi}{5}\right)\right] u(t)$, calculate the steady state output signal $y_{ss}(t)$.

Problem IX [10 pts]

The signal $f(t) = \cos\omega_0 t$ is sampled at the angle frequency ω_s, and then passed through an idea lowpass filter with cutoff frequency $\omega_c = \omega_s/2$. Find the output signal $y(t)$ for the following sampling frequencies: (a) $\omega_s = 3\omega_0$, and (b) $\omega_s = 3\omega_0/2$.

Problem X [10 pts]
Let $F(\omega)$ denote the Fourier transform of the signal $f(t)$ shown below. Perform the following evaluations without explicitly evaluating $F(\omega)$.



1. Find $\angle F(\omega)$.

2. Find $F(0)$.

3. Find $\int_{-\infty}^{\infty} F(\omega) d\omega$.

4. Sketch the inverse Fourier transform of $\Re\{F(\omega)\}$.