Solutions

Midterm-2006

Problem #1

a) $x_{FM}(t) = A\cos(2\pi f_0 t + \beta \sin 2\pi W t)$

$$\beta = \frac{\Delta f}{W} = \frac{h A_m}{W}$$

b) $\Delta f = h A_m$

c) $V_{out} = k\, \Delta f \cos 2\pi W t$

d) $x_{AM}(t) = A[1 + m s(t)]\cos 2\pi f_0 t$

$$s(t) = B \cos 2\pi W t$$

After Hard-Limiter & BPF_2

$$e(t) = k \cos 2\pi f_0 t$$

$$\frac{1}{2\pi}\frac{de(t)}{dt} = -\frac{1}{2\pi}(k\, 2\pi f_0)\sin 2\pi f_0 t$$

$$V_{env}(t) = k f_0$$

After Capacitor

$$V_{out}(t) = 0$$

e) For AM

SSB Receiver (and DSB Receiver)

$$e_o(t) = k \cos 2\pi f_o t$$

$H(f)$ passes $e_o(t)$

then

$$k \cos 2\pi f_o t \cdot \cos 2\pi f_o t =$$
$$k/2 \left[1 + \cos 2\pi (2f_o) t \right]$$

After LPF

$$\boxed{e_{out}(t) = k/2}$$

For FM (DSB Receiver

After $H(f) \Rightarrow$
$$e_o(t) = k \left[J_1(\beta) \cos 2\pi (f_o + W) t - J_1(\beta) \cos 2\pi (f_o - W) t \right]$$

then multiply by $\cos 2\pi f_o t$ + LPF

$$\Rightarrow k \frac{J_1(\beta)}{2} \cos 2\pi W t - k \frac{J_1(\beta)}{2} \cos 2\pi W t$$

$$= \boxed{0}$$

For FM (SSB Receiver)

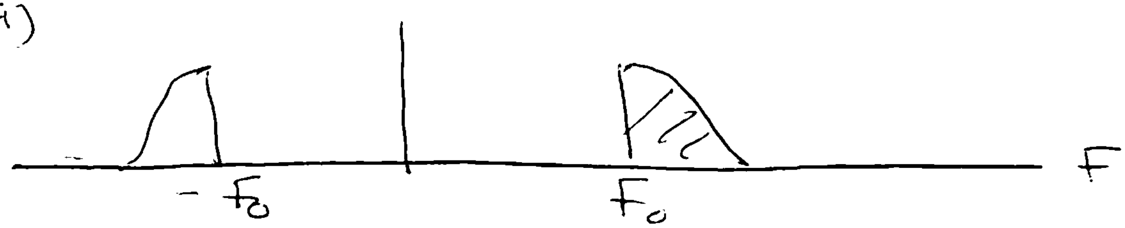
After $H(f)$
$$e_o(t) = k J_1(\beta) \cos 2\pi (f_o + W) t$$

then multiply by $\cos 2\pi f_o t$ + LPF

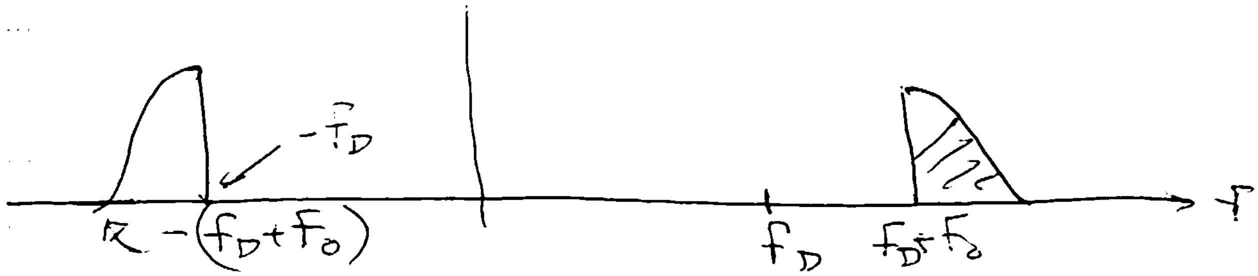
$$\Rightarrow \boxed{\frac{k J_1(\beta)}{2} \cos 2\pi W t}$$

Problem #2

a)

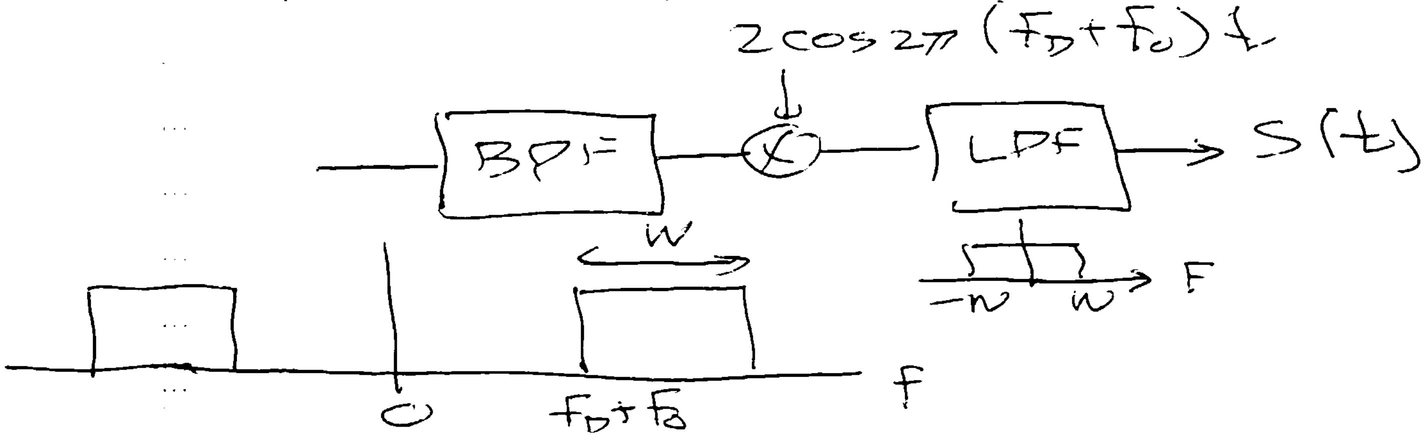


Single-Sideband



$$X_{USB-SSB}(t) = s(t)\cos 2\pi(f_D+f_0)t - \hat{s}(t)\sin 2\pi(f_D+f_0)t$$

b) Receiver



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If you do the actual analytic computation!

$$x(t) = s(t)\cos 2\pi f_0 t - \hat{s}(t)\sin 2\pi f_0 t$$

$$x_{SSB\text{-}USB}(t) = x(t)\cos 2\pi f_D t - \hat{x}(t)\sin 2\pi f_D t$$

$$= \left[s(t)\cos 2\pi f_0 t - \hat{s}(t)\sin 2\pi f_0 t\right]\cos 2\pi f_D t$$

$$- \widehat{\left[s(t)\cos 2\pi f_0 t - \hat{s}(t)\sin 2\pi f_0 t\right]}\sin 2\pi f_D t$$

$$- \left[s(t)\sin 2\pi f_0 t + \hat{s}(t)\cos 2\pi f_0 t\right]\sin 2\pi f_D t$$

$$\mathcal{H}\{r(t)\sin 2\pi f_0 t\} =$$

$$\mathcal{H}\left\{ r(t)\,\frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j} \right\}$$

$$= -j\,\frac{r(t)e^{j2\pi f_0 t}}{2j} - j\,\frac{e^{-j2\pi f_0 t}\,r(t)}{2j}$$

$$= -\,\frac{r(t)e^{j2\pi f_0 t} + r(t)e^{-j2\pi f_0 t}}{2}$$

$$= -\,r(t)\cos 2\pi f_0 t$$

$$= r(t)\,\mathcal{H}\{\sin 2\pi f_0 t\}$$

②

$$= s(t)\left[\cos 2\pi f_0 t \cos 2\pi f_D t - \sin 2\pi f_0 t \sin 2\pi f_D t\right.$$

$$- \hat{s}(t)\left[\sin 2\pi f_0 t \cos 2\pi f_D t + \cos 2\pi f_0 t \sin 2\pi f_D t\right]$$

$$= s(t)\left[\cos 2\pi (f_D + f_0) t\right]$$

$$- \hat{s}(t)\left[\sin 2\pi (f_D + f_0) t\right.$$