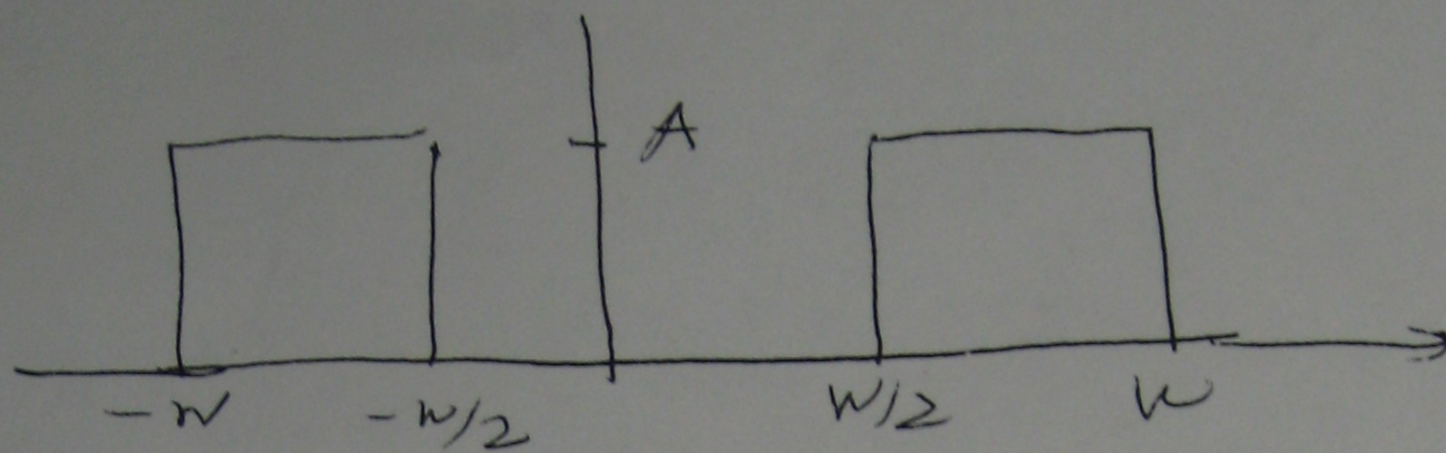
EM E3701 - Spring 2007

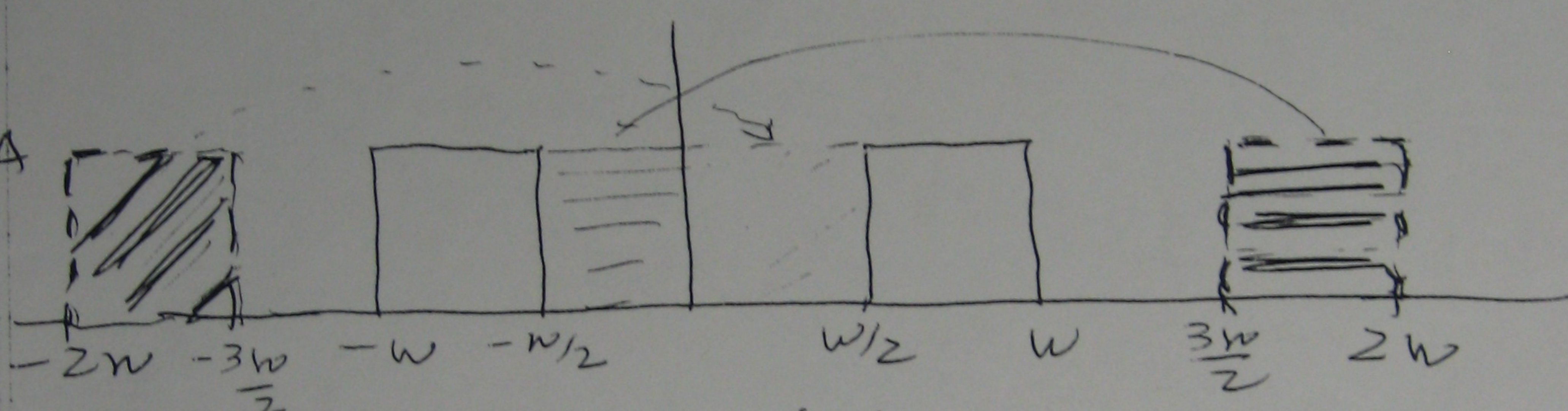
Solutions to Final

(1)



(15) a) No

b)



One Solution

(10)

1- Sampling Frequency

$f_s > 2(100\,K) = \boxed{200\ \text{Ksamples/sec}}$

2- We must cover full range of amplitudes $\underline{\pm 100V}$

$$\frac{S}{N_q} = \frac{3}{2}2^{2n} \Rightarrow 1.77dB + 6n > 82\,dB$$

14

$= 85.77dB$ $\boxed{n \geq 14\ \text{bits}}$

3- $\boxed{R_b = 200K \times 14 = 2800\ \text{Kbps}}$

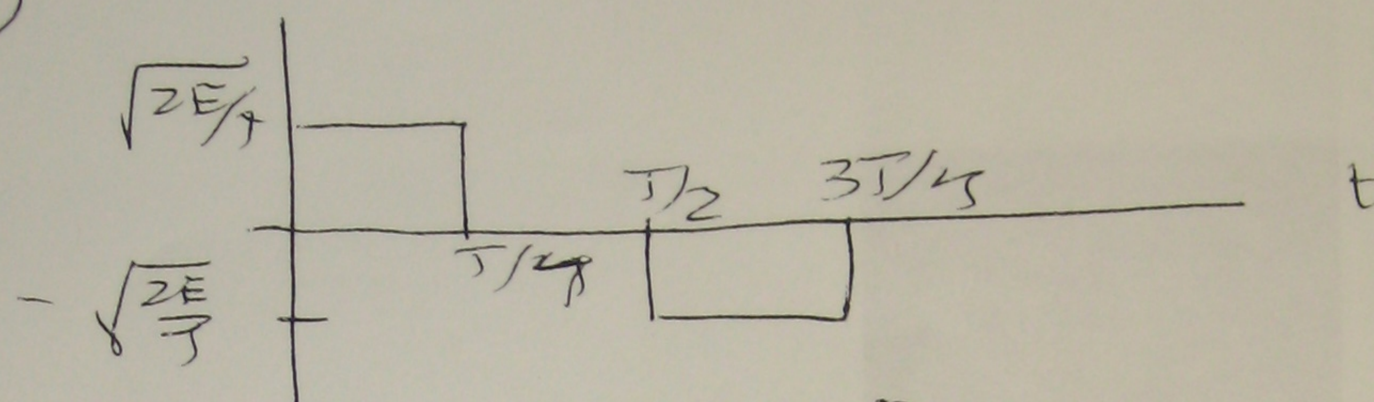
$(n = 13;\ \frac{S}{N_q} = 79.77dB)$

(9)

Now for the ~~smallest~~ music signal if $S/N_q >$ 75 dB, and the voice signal

is only $\pm 10v$; we have lost $(10)^2$ in average power (20dB) and $S/N_q \geq 65.77$

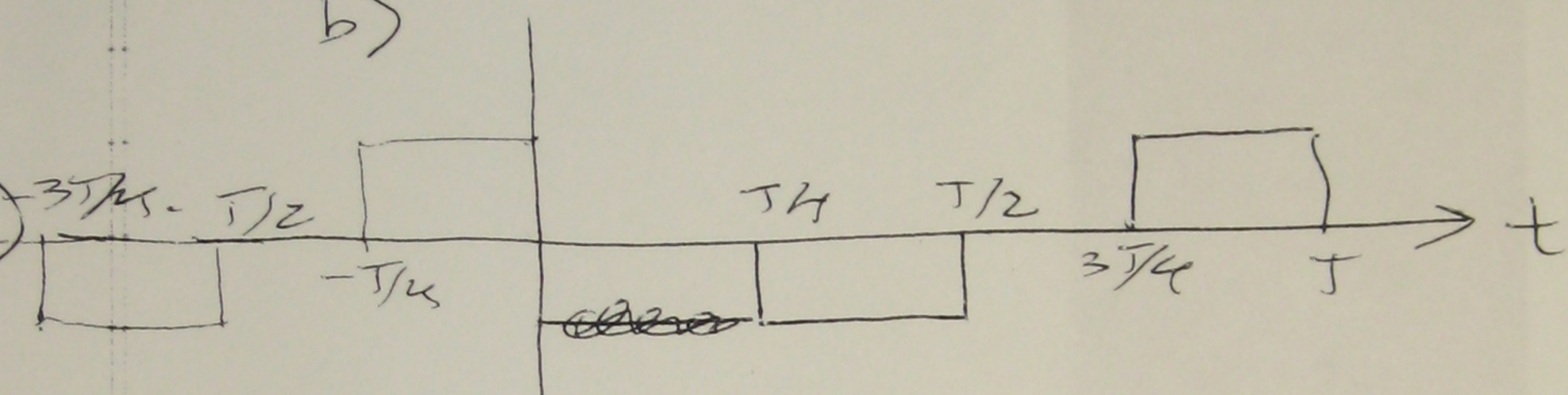
2



3 a) $E = \left[\frac{2E}{T}\left(\frac{T}{4}\right)\right]^2 = E$

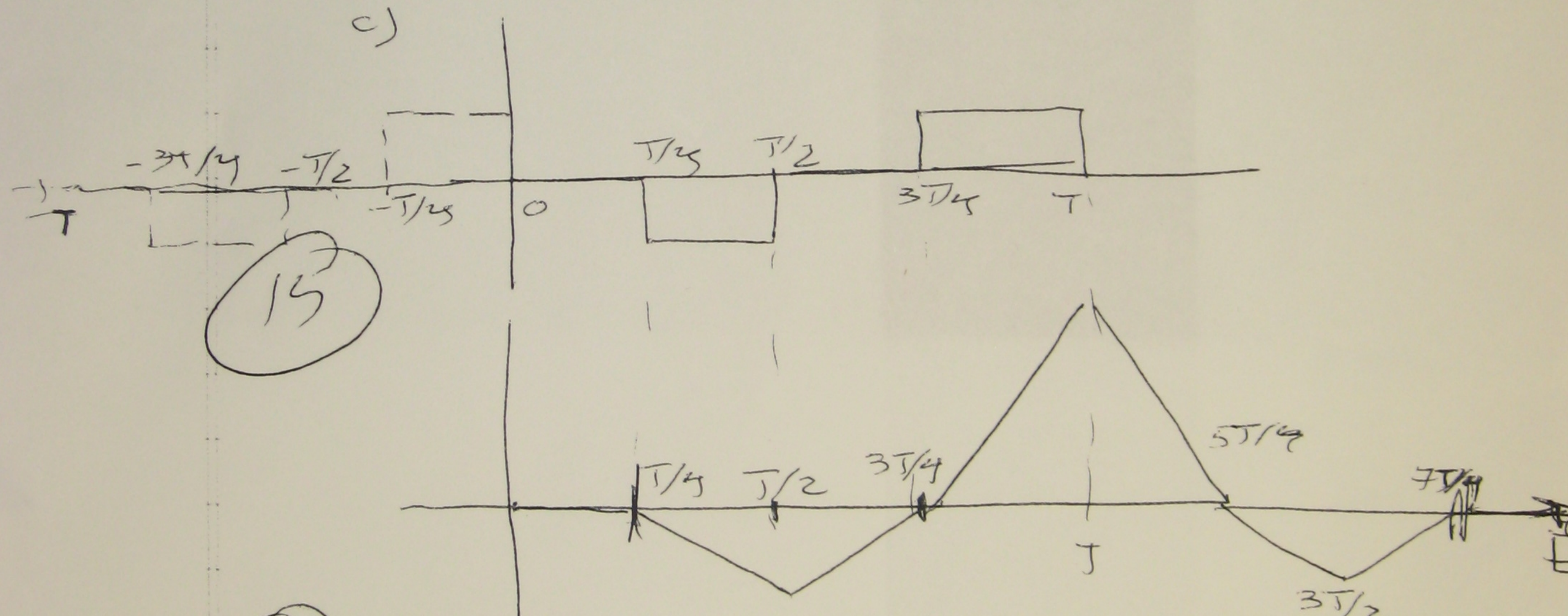
b)

10



c)

15



5 d) $\Pr\{\varepsilon\} = Q\left(\sqrt{\frac{2E}{N_0}}\right)$

(3) System A 16 sets of 16 QAM

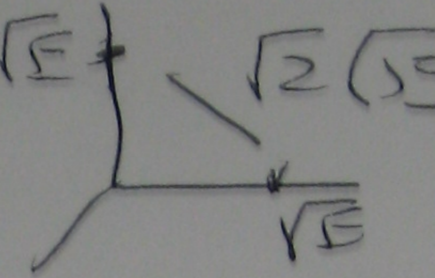
System B 256 Orth FSK

(10) a) QAM- $2 \times 16 = 32$

MFSK $= 256$

(5) b) $d = \sqrt{2}\sqrt{E_{sFSK}}$

255 at this distance



(3) c) $E_{QAM} = \frac{M-1}{6} d^2 = \frac{5}{2} d^2$.

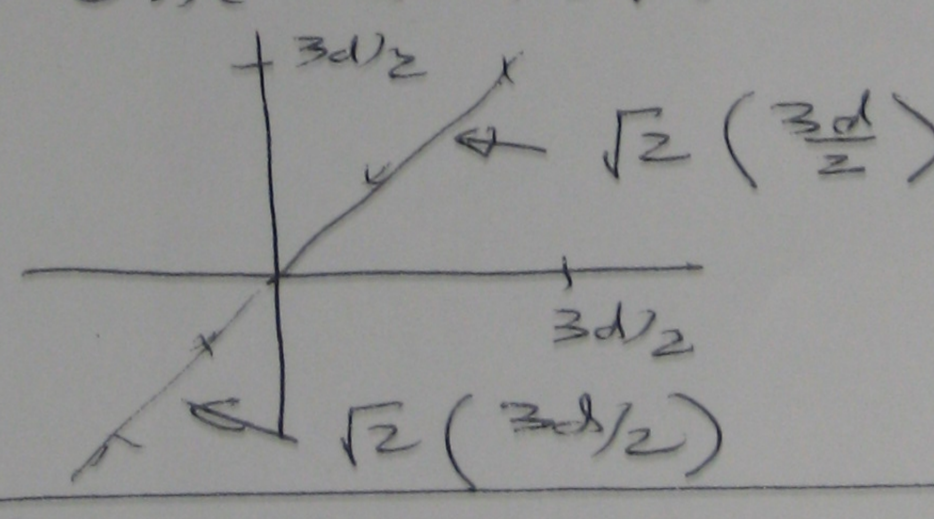
(3) d) $d_{min} = d$.

(3) e) QAM; $d = \sqrt{\frac{2}{5} E_s}$ MFSK $d = \sqrt{2}\sqrt{E_{FSK}}$

~~QAM~~ MFSK

(5) f) $d_{max} = d_{min} = \sqrt{2}\sqrt{E}$

(5) g) In one QAM set



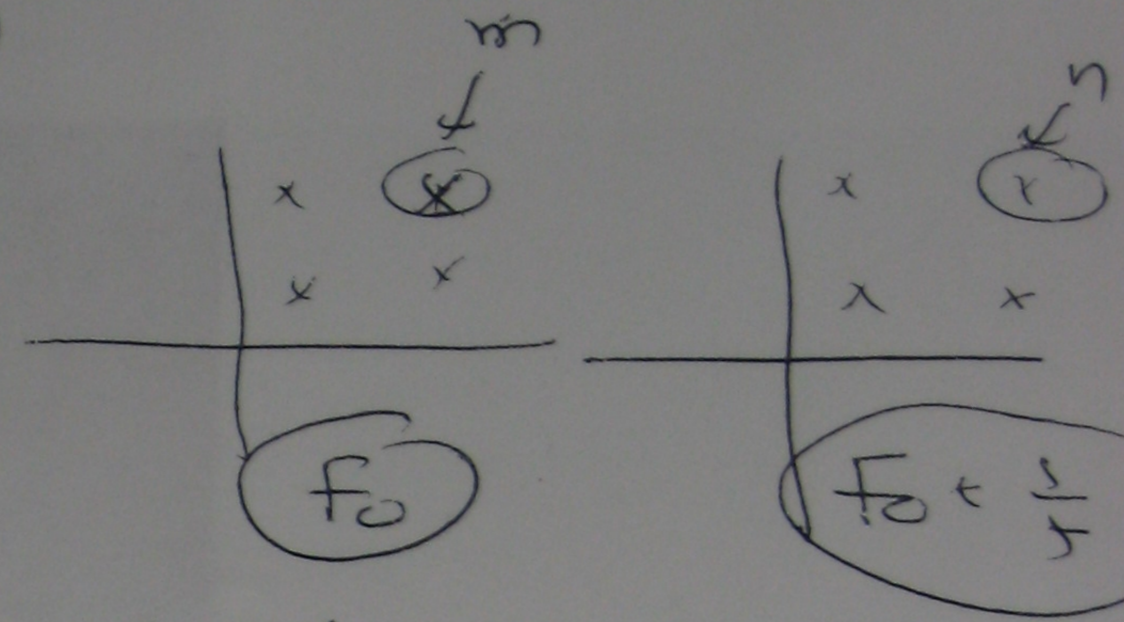
$$\boxed{d_{max} = 2\left(\sqrt{2}\,\frac{3d}{2}\right) = 3\sqrt{2}\,d}$$

Between biggest points in two different QAM sets is a four dimensional measurement

$$d_{max}^2 = \left(\frac{3d}{2}^2, \frac{3d}{2}^2, \frac{3d}{2}^2, \frac{3d}{2}^2\right)$$

$$= 4\left(\frac{3d}{2}\right)^2$$

$$d_{max} = 2\left(\frac{3d}{2}\right) = 3d$$



$$\boxed{3d < 3\sqrt{2}\,d}$$

$$m = \left(\frac{3d}{2}, \frac{3d}{2}, 0, 0\right.$$

$$n = \left(0, 0, \frac{3d}{2}, \frac{3d}{2}\right)$$

The First result is the best result for d_{max}

$$d_{max} = 3\sqrt{2}\,d$$

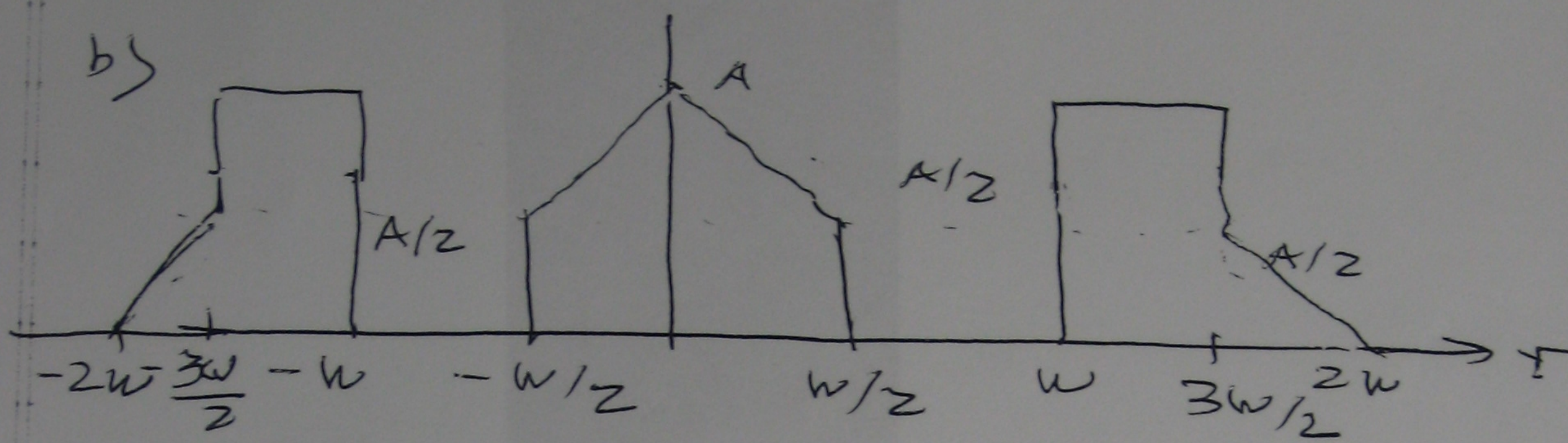
ELEN E3701

2005 - Final

Prob #1

a) No, not flat

b)



Prob # 3

$$E_s = \frac{M-1}{6} d^2 = \frac{5}{2} d^2$$

a) QAM - 2
QPSK - 8
MFSK - 16

b) $d = \sqrt{2}\sqrt{E_s}$

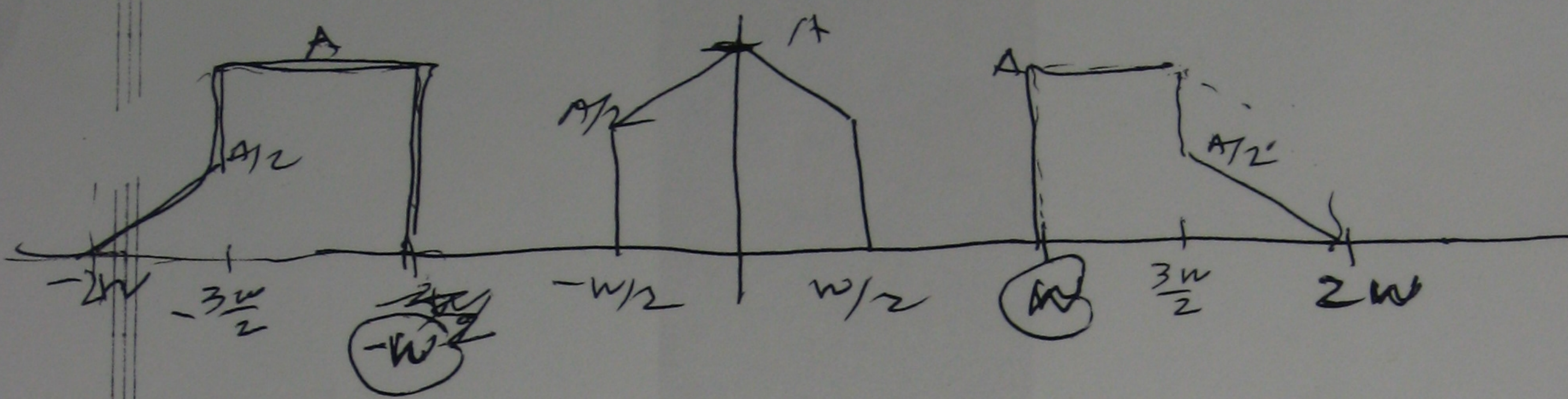
c) QAM: $E_s = 5/2\, d^2$
QPSK: $E_s = d^2/2$
MFSK: $E_s = d^2/2$

d) QAM: $d_{min}^2 = 2/5\, E_s$

e) QAM; $d_{max} = \sqrt{2}\left(\frac{3d}{2}\right) * 2 = \sqrt{2}\left(\frac{3d}{2}\right)\sqrt{\frac{2}{5}E_s} = \frac{6}{\sqrt{5}}\sqrt{E_s}$
QPSK: $d_{max} = 2\sqrt{E_s} = \sqrt{2}\, d$
MFSK: $d_{max} = d_{min} = \sqrt{2}\sqrt{E_s}$

1

Problem #1



5 a) No, Not Flat

15 b) See above

5 c) $f_s > 2(20K) = 40K$

5 d) $f_b = 12 \times 40K = 480$ Kbps

5 e) $\frac{S}{N} = \frac{A^2/2}{\Delta^2/12} = 6\left(\frac{A}{\Delta}\right)^2 = \frac{6}{4}\left(\frac{M}{2}\right)^2 = \frac{3}{2}2^{2n}$

$= 1.77 + 6n = 1.77 + 6(12) = 73.77$ dB

For the Voice Signal

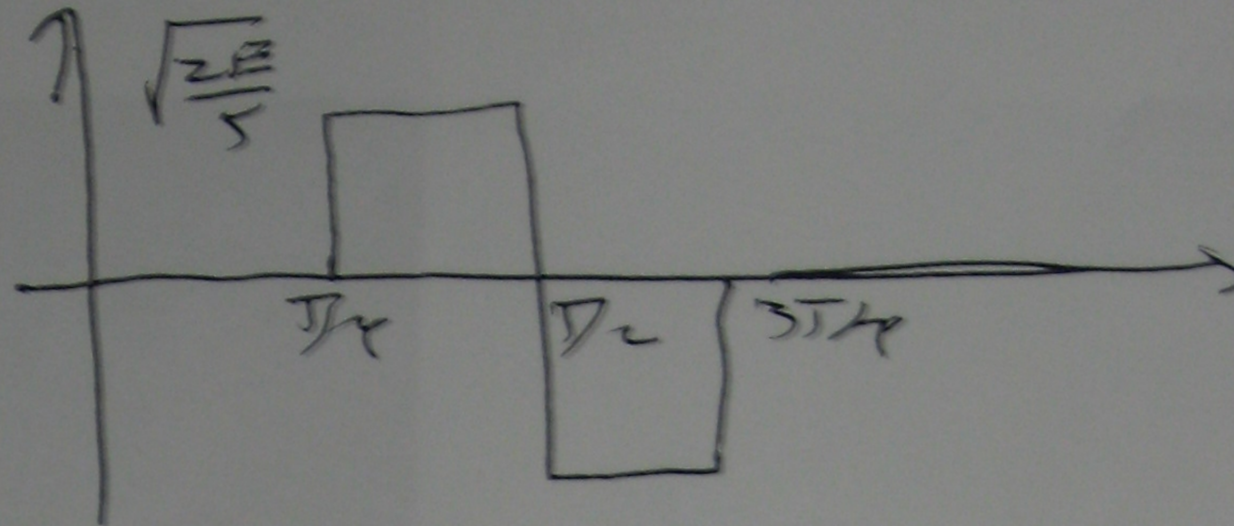
$S = \frac{(A/2)^2}{2}$ instead of $A^2/2$

$= \frac{A}{2}\left(\frac{1}{4}\right)$ (6 dB less)

$\frac{S}{N} = 73.77 - 6 = 67.77$ dB

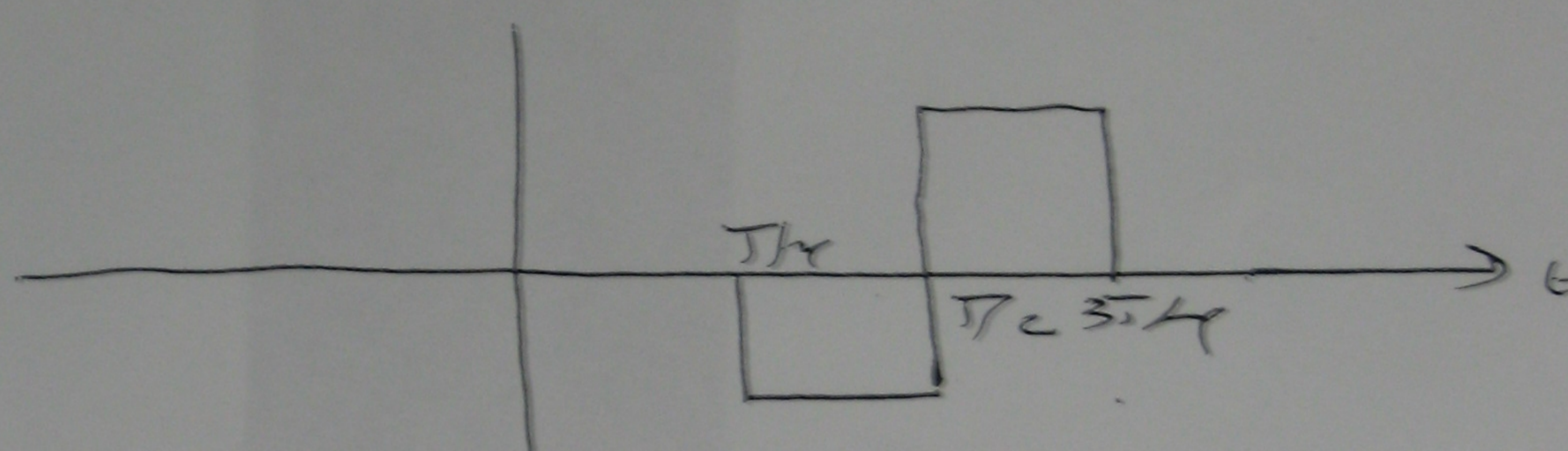
2 3

2)

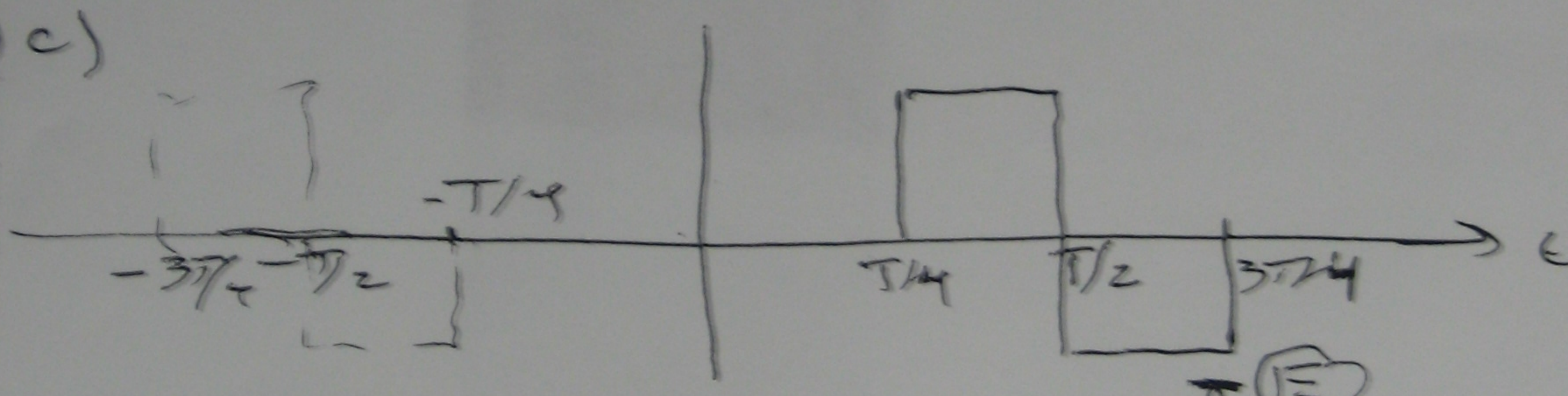


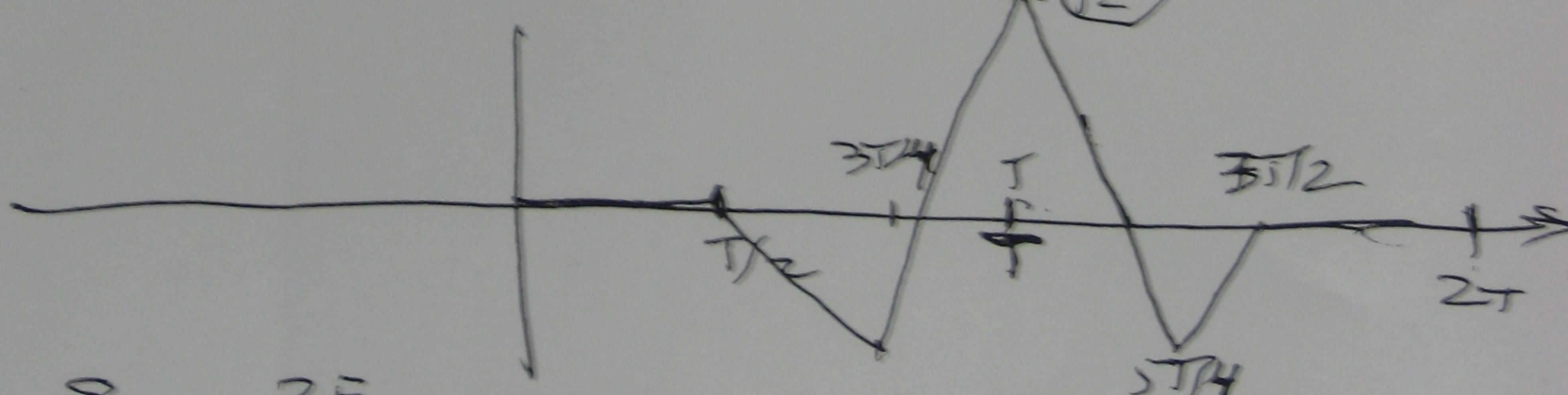
3 a) $= \frac{2E}{T} \times \frac{T}{4} + \frac{2E}{T} \times \frac{T}{4} = E$

5 b) $h_{mf}(t)$



5 c)





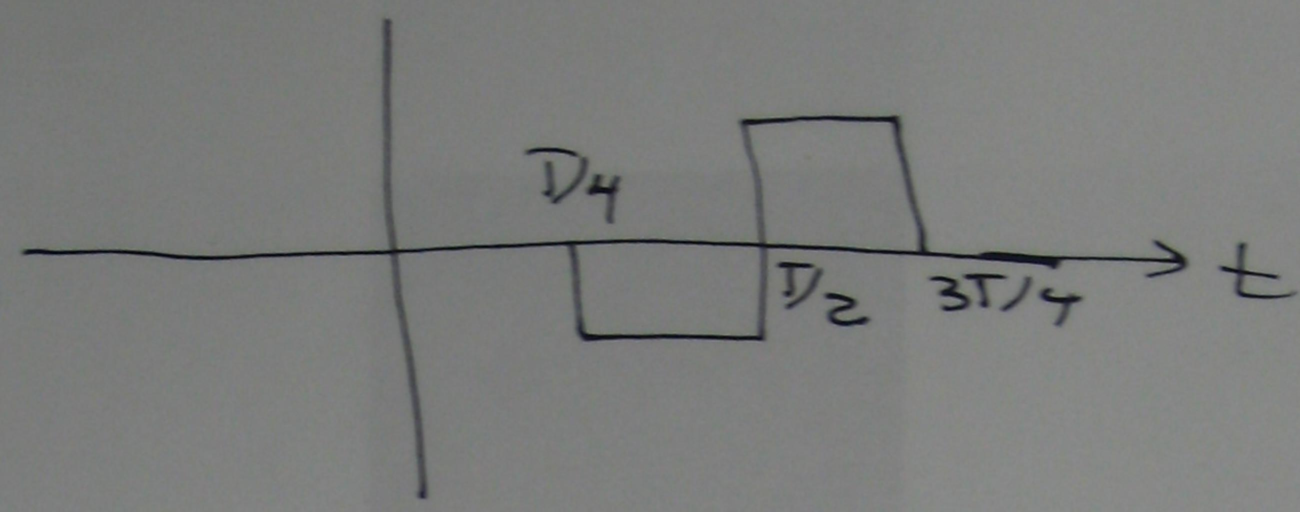
5 d) $\frac{S}{N} = \frac{2E}{N_0}$

e) $Pr\{E\} = Q\left(\sqrt{\frac{2E}{N_0}}\right) = Q\left(\sqrt{\frac{2E}{N_0}}\right)$

(3)

4

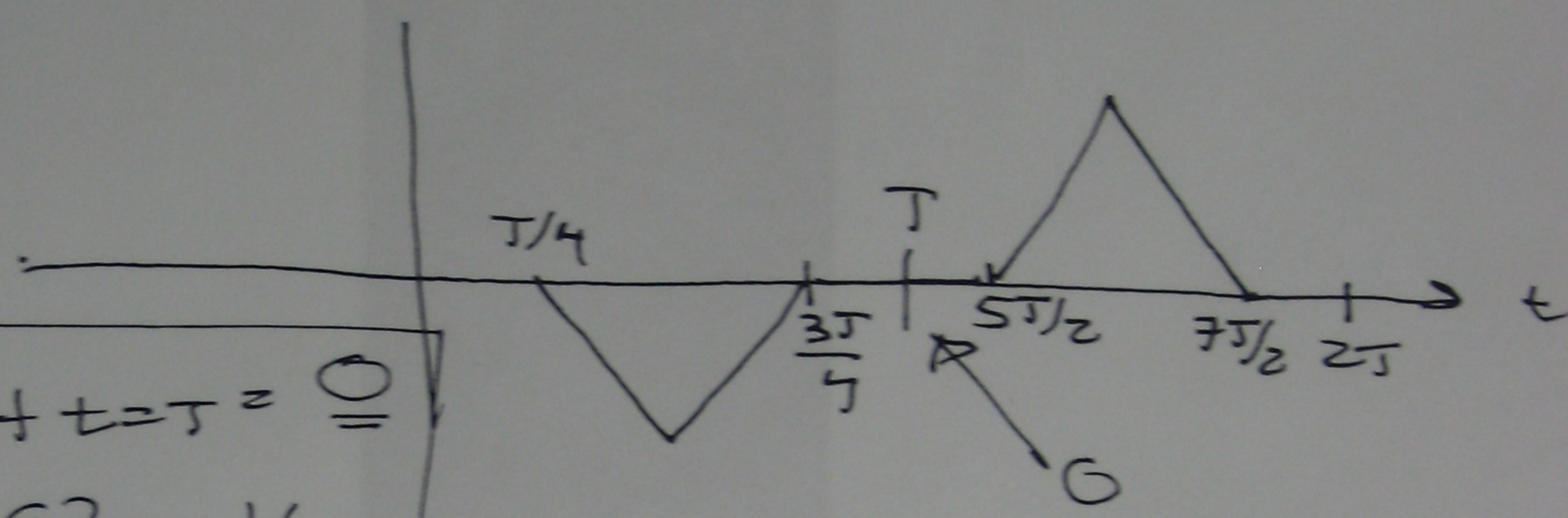
f)



(15)

(10)

(5)



$\boxed{\frac{S}{N}, \text{at } t=T = \underline{\underline{0}} \quad Pr\{\varepsilon\} = 1/2}$

5

#4

$$E_s = \frac{16-1}{6} d^2 = \frac{5}{2} d^2$$

10 a) QAM - 2

QPSK - 8

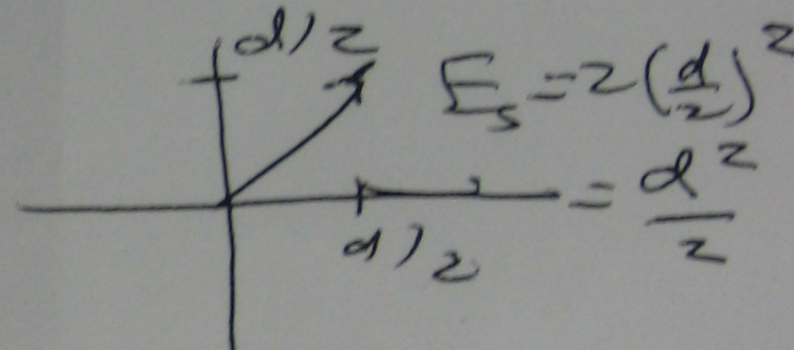
MFSK - 16

3 b) $d = \sqrt{2}\sqrt{E_s}$

5 c) QAM $= E_s = \frac{5}{2} d^2$

QPSK ; $E_s = \frac{d^2}{2}$

MFSK ; $E_s = \frac{d^2}{2}$



5 d) QAM, $\boxed{d^2_{min} = \frac{2}{5} E_s}$

8 e) QAM

$d_{max} = \boxed{\sqrt{2}\left(\frac{3d}{2}\right) \times 2 = 3\sqrt{2}\, d}$

$= 3\sqrt{2}\sqrt{\frac{2}{5}E_s} = \frac{6}{\sqrt{5}}\sqrt{E_s}$

QPSK $d_{max} = \boxed{2\sqrt{E_s} = 2\sqrt{\frac{d^2}{2}} = \sqrt{2}\, d}$

MFSK $d_{max} = \boxed{d_{min} = \sqrt{2}\sqrt{E_s}}$

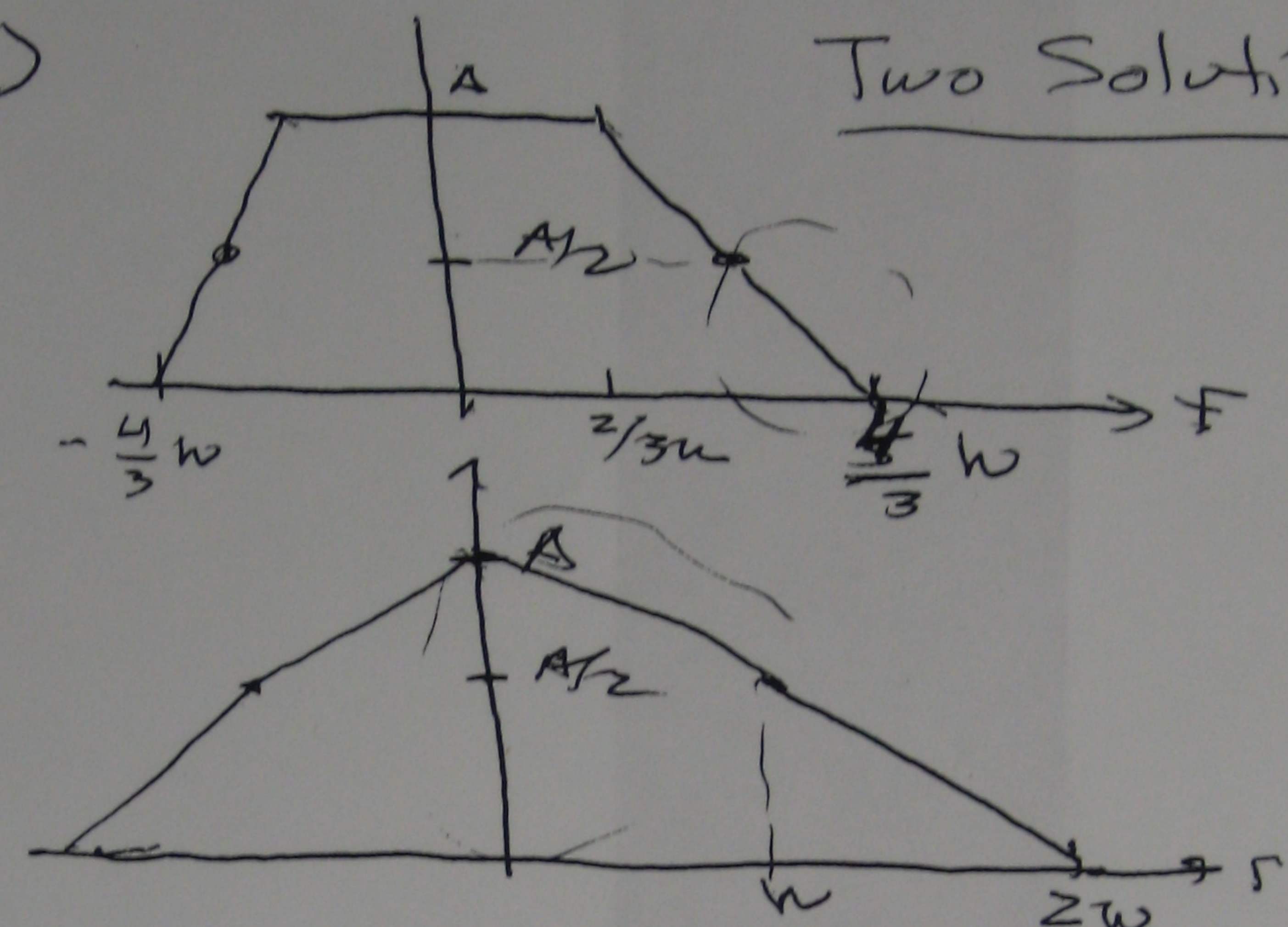
Spring 2006 – Final Exam (1 of 6)

Solutions (ELEN E 3701

1) a) NO

$$\sum_{n=-\infty}^{\infty} S(f + n2W) = 1 \ (\text{or } K) \qquad |f| \le W$$

b) Two Solutions



c) min BW = $\frac{4}{3}W$

d) $f_s = 12$ Ksamples/sec

$2A = 200$ v

$n = 8$; $M = 2^8$

For the signal which is ± 100v

(signal 3) $\frac{S}{N} = \frac{3}{2} 2^{2n} \Rightarrow 1.77 + 6n$, dB

$\frac{S}{N} = 1.77 + 6(8) = 49.77\text{dB} > 20\text{dB}$

but

$F_s \not> 2(10) = 20$ K

For Signals 1 & 2

$12k = f_s > 2(1)$

$12k = f_s > 2(5)$

Signal 2

$\frac{S}{N} = 49{,}77 - 20 = 29{,}77 > 20dB$ ✓

10 v instead of 100 v means
signal power decreases by $\left(\frac{100}{10}\right)^2 = 100$
$\Rightarrow + 20dB$

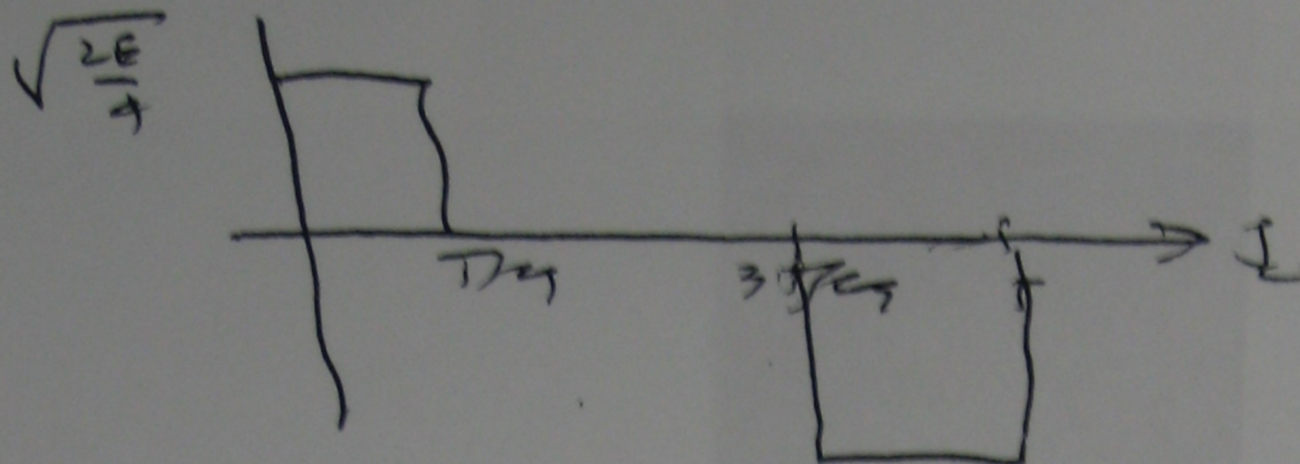
Signal 1

$\frac{S}{N} = 49{,}77 - 40 = 9{,}77dB < 20dB$

$1V \rightarrow \left(\frac{100}{1}\right)^2 = 10000 \Rightarrow + 40dB$

Only Signal 2 satisfies
Condition

Problem #2

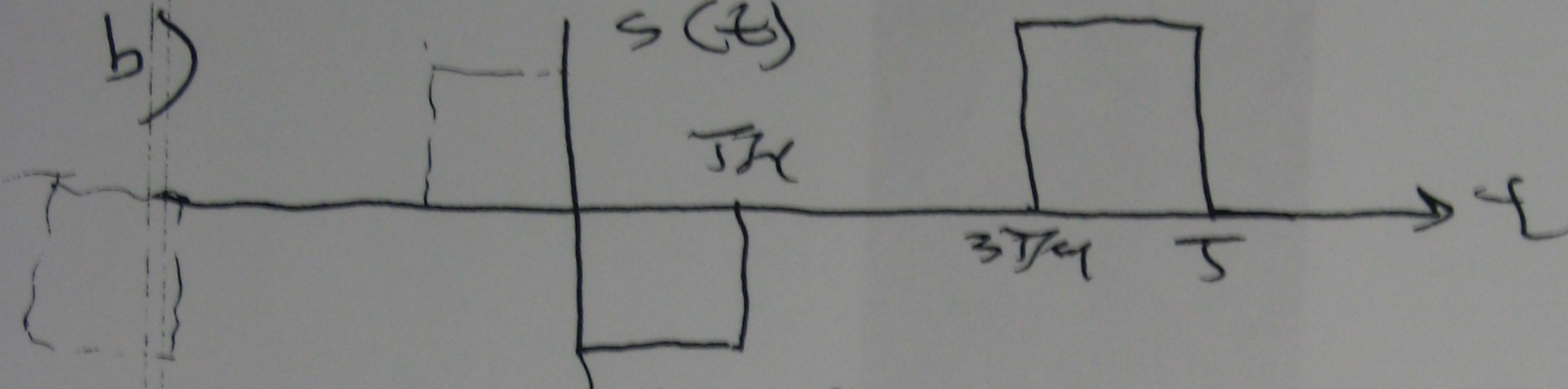


a)

$$E_s = \left[\left(\frac{2E}{T}\right)\frac{T}{4}\right] \times 2 = E$$

8

b)



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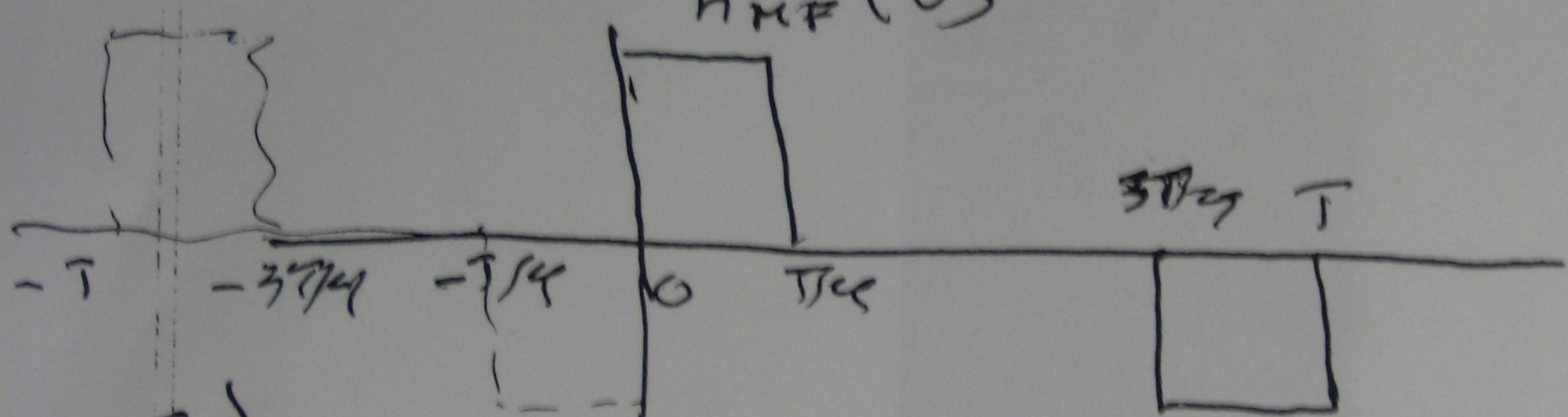
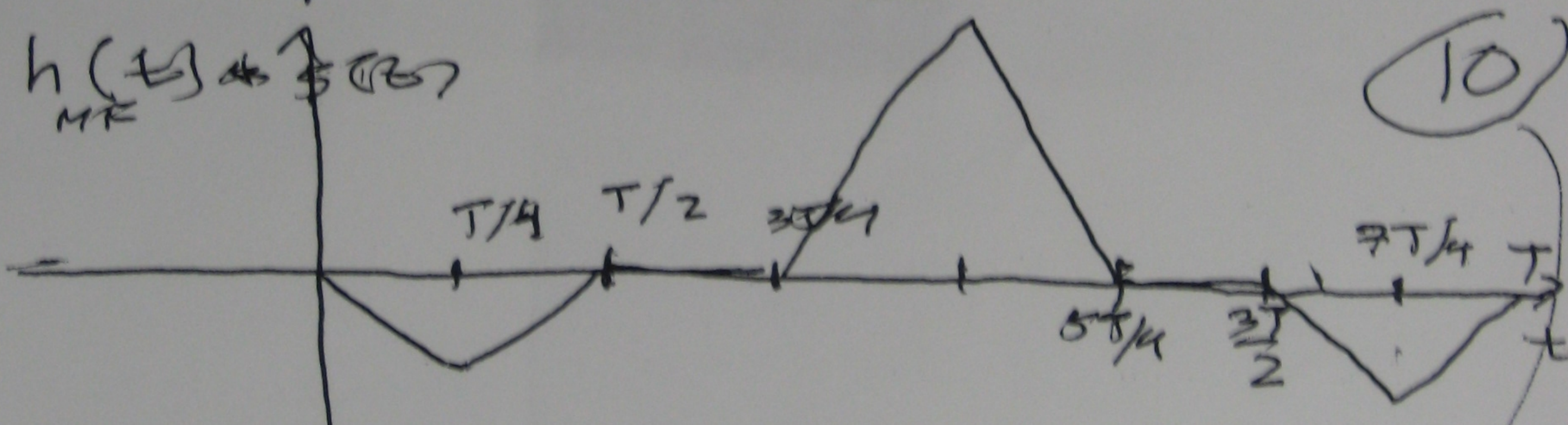


c)

$h_{MF}(t) * s(t)$



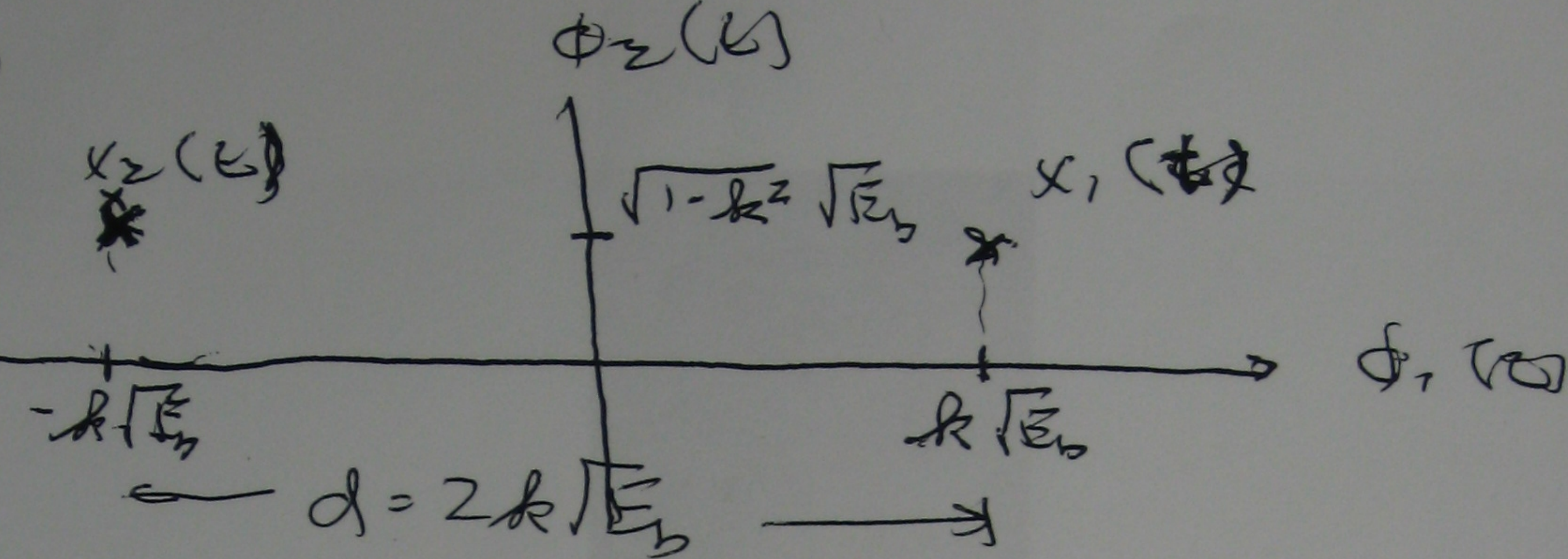
10

d) $Pr\{\varepsilon\} = Q\left(\sqrt{\frac{2E}{N_0}}\right)$

1

e)

1)



2)

$$d = 2k\sqrt{E_b}$$

$$\Pr\{\varepsilon\} = Q\left(\frac{d}{\sqrt{2N_0}}\right)$$

$$= Q\left(\frac{2k\sqrt{E_b}}{\sqrt{2N_0}}\right)$$

$$= Q\left(k\sqrt{\frac{2E_b}{N_0}}\right)$$

12

3) IF $k=1$

$$\Pr\{\varepsilon\} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

4

IF $k=0$

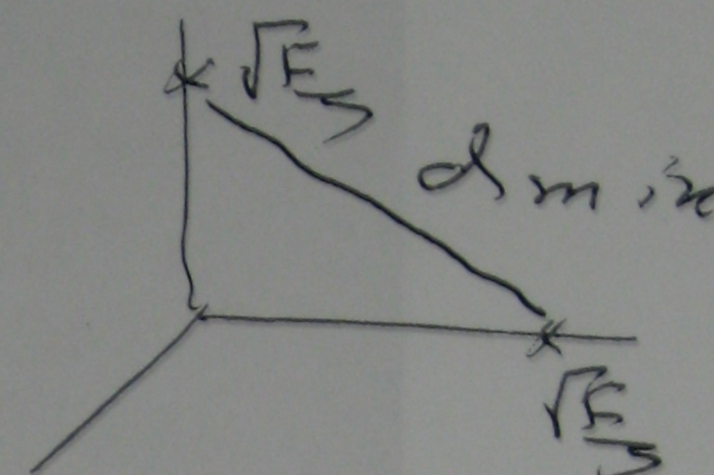
$$\Pr\{\varepsilon\} = 1/2$$

Problem #3

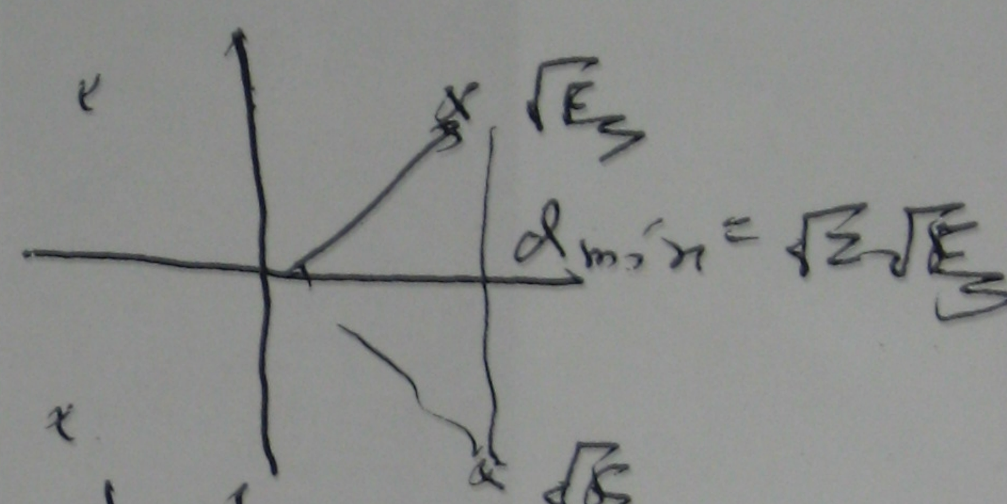
a) 16-QPSK - $2 \times 16 = 32$ D

64-FSK - 64 D

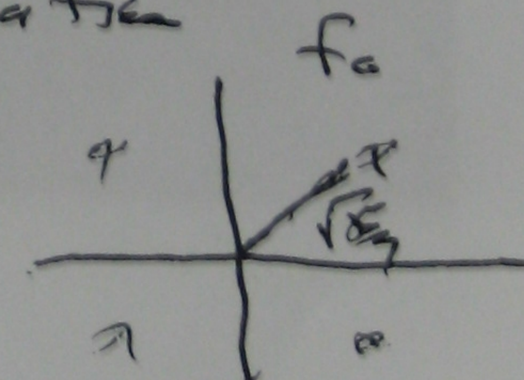
b) $\boxed{d_{min} = \sqrt{2}\sqrt{E_s}}$



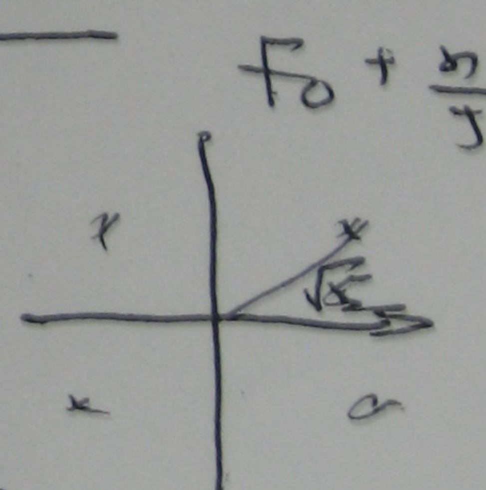
c) $\boxed{d_{min} = \sqrt{2}\sqrt{E_s}}$



We also have dmin between one point in one constellation and one point in another constellation



All points have same energy, E_s



$$d^2 = E_s + E_s - 2\rho\sqrt{E_s}^2$$

$\rho = 0$ because they are all orthogonal

$$d^2 = 2E_s$$

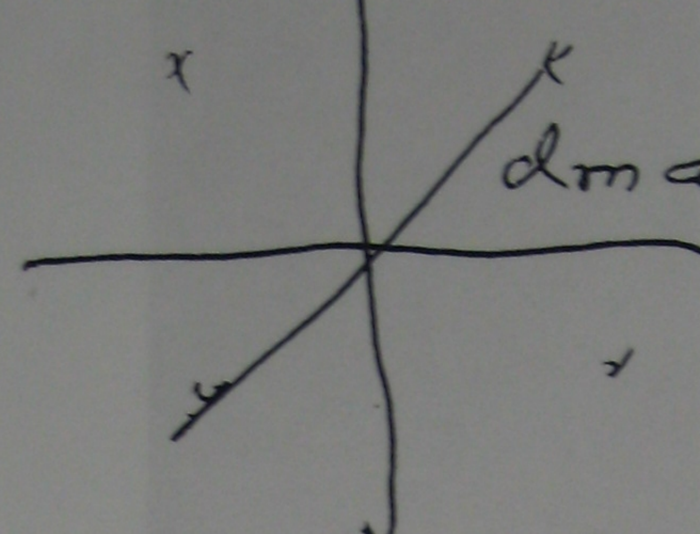
$$d = \sqrt{2}\sqrt{E_s}$$

d) Bath are the same.

e) $d_{max} = d_{min} = \sqrt{2}\sqrt{E_s}$

f)

$d_{max} = 2\sqrt{E_s}$



$d_{max} = 2\sqrt{E_s}$

$d_{min} = \sqrt{2}\sqrt{E_s}$

$d_{max} = \sqrt{2}\, d_{min}$