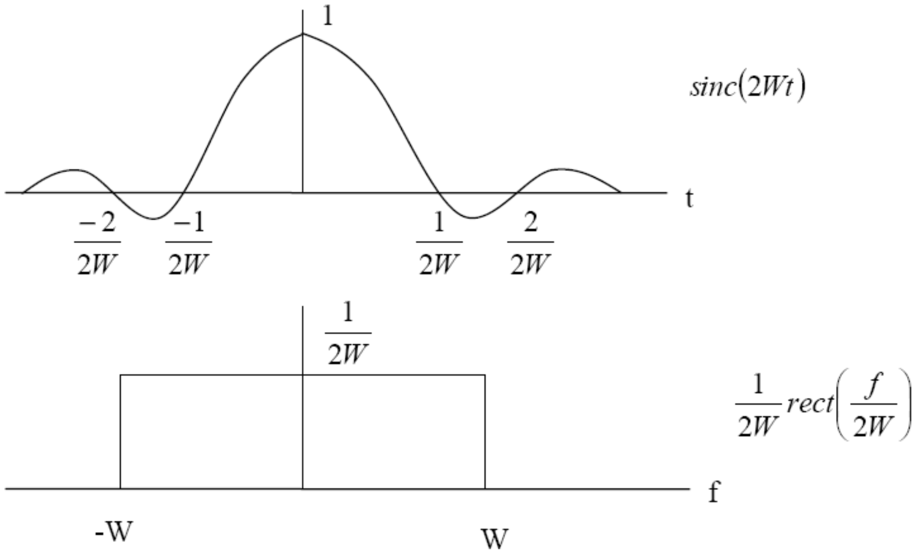
Solutions Homework #6
Introduction to Communication Systems – E3701

<u>Problem 3.3</u>

a. From table A6.3 in Appendix 6 we know that the relationship between the time and frequency domains of the sinc function is as follows:

$$sinc(2Wt) \Leftrightarrow \frac{1}{2W} rect\left(\frac{f}{2W}\right)$$



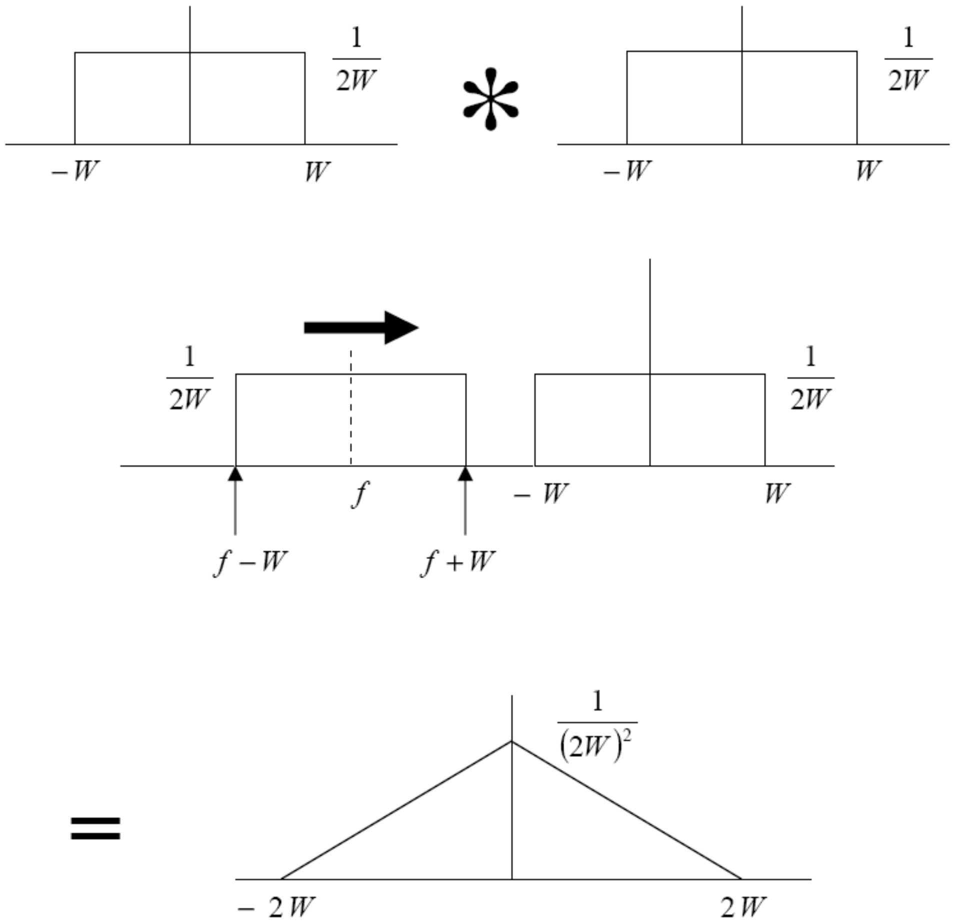
Note also that $sinc(2Wt) = \frac{\sin(2\pi Wt)}{2\pi Wt}$. From this diagram we can see that the bandwidth of the sinc signal is W, therefore in the case of $sinc(200t)$ we have W = 100. Recall that Nyquist states that in order to avoid aliasing we must sample at a *minimum* of twice the highest frequency present in our baseband signal. Thus our Nyquist rate and Nyquist interval are

$$F_N = 200Hz \quad T_N = \frac{1}{F_s} = 5ms$$

Part b

$sinc^2(200t) = sinc(200t)sinc(200t)$

If we evaluate this in the frequency domain, recalling that multiplication in one domain is convolution in the other domain, we have the convolution of two rect functions.



The result of convolving two rect functions is to create a triangle function with peak value equal to the square of the amplitude of the original rect functions and with a base twice as wide as the original rect functions. Thus our new Nyquist rate and interval are

$$F_N = 400Hz \quad T_N = \frac{1}{F_s} = 2.5ms$$

Part c

The Nyquist rate is always twice the highest frequency present in the signal. We know the highest frequencies present in both of the functions in our sum from parts a and b. Thus, the highest frequency in our signal is 200Hz. Our Nyquist frequency and interval are identical to part b.

$$F_N = 400Hz \quad T_N = \frac{1}{F_s} = 2.5ms$$

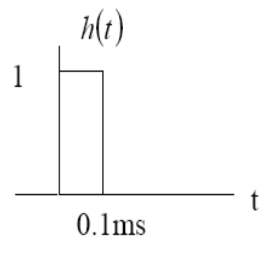
Problem 3.5

Part a

The majority of this problem can be solved by following the derivation in section 3.3 of the text, *Pulse-Amplitude Modulation*. If you follow the derivation of PAM you end up with the spectrum

$$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)H(f)$$

where f_s is our sampling frequency, 1 kHz, M(f) is the given spectrum of our baseband signal and H(f) is the spectrum of our pulse of unit amplitude and duration 0.1ms. It is of use to examine this pulse signal more closely.



The pulse in the time domain is shown to the right. This is obviously a standard rectangle function with the difference that it is no longer centered at zero, but has been shifted right by 0.05ms or T/2 in the more general case. With this shift in mind, we can write a single pulse as

$$h(t) = rect\left(\frac{t - T/2}{T}\right) = rect\left(\frac{t - 10^{-4}/2}{10^{-4}}\right).$$

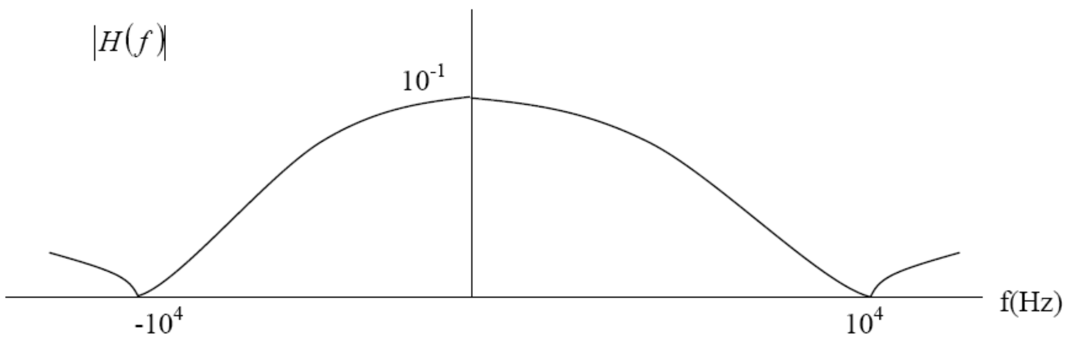
We can look up the Fourier transforms of the rect function in the Appendix, Table A6.3. In addition we need to use the time shifting property found in Table A6.2 of the Appendix. Using this information we can write our single pulse in the frequency domain.

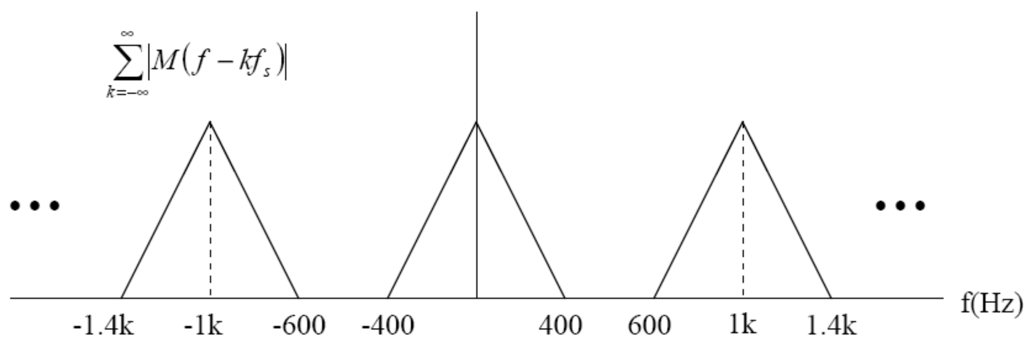
$$H(f) = Tsinc(fT)e^{-j2\pi f\frac{T}{2}} = 10^{-4}sinc(10^{-4}f)e^{-j10^{-4}\pi f}$$

If we go back and examine the equation for our spectrum, S(f), we see that the sum is over the dummy variable k. Since there is no k inside the spectrum H(f) we can pull it out of the summation.

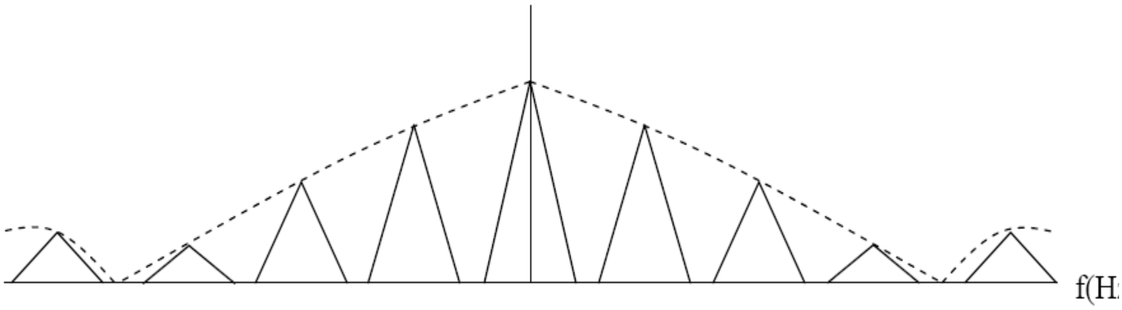
$$S(f) = f_s\sum_{k=-\infty}^{\infty}M(f-kf_s)H(f) = f_sH(f)\sum_{k=-\infty}^{\infty}M(f-kf_s)$$

Our final step is to sketch the spectrum S(f). The spectrum is obtain by multiplying the scaled spectrum, $f_sH(f)$ by the summation spectrum $\sum_{k=-\infty}^{\infty}M(f-kf_s)$. These two spectra are shown below.





The final step is to multiple these spectra together, taking note of where the zero crossings are in the H(f) spectrum.



Note that this is not to scale. There will be many more triangles “inside” of the dotted line.

Problem 3.8

Part a

An 8 kHz sampling rate means a sampling period of 1/8000 = 125 μs. We have twenty four voice signals and plus on extra pulse to fit into this length of time. Each pulse is 1 μs in duration. That means there are 125 / 25 = 5 μs between each pulse. Since each pulse lasts 1 μs, that leaves 4 μs between each individual pulse.

4 μs

Part b

The Nyquist rate is twice our highest frequency or 2*3.4 kHz = 6.8 kHz. Our new sampling period is 1 / 6800 = 147 μs. This allows 147 / 25 = 5.88 μs between each pulse. The pulses have 1 μs duration, so the time between pulses is

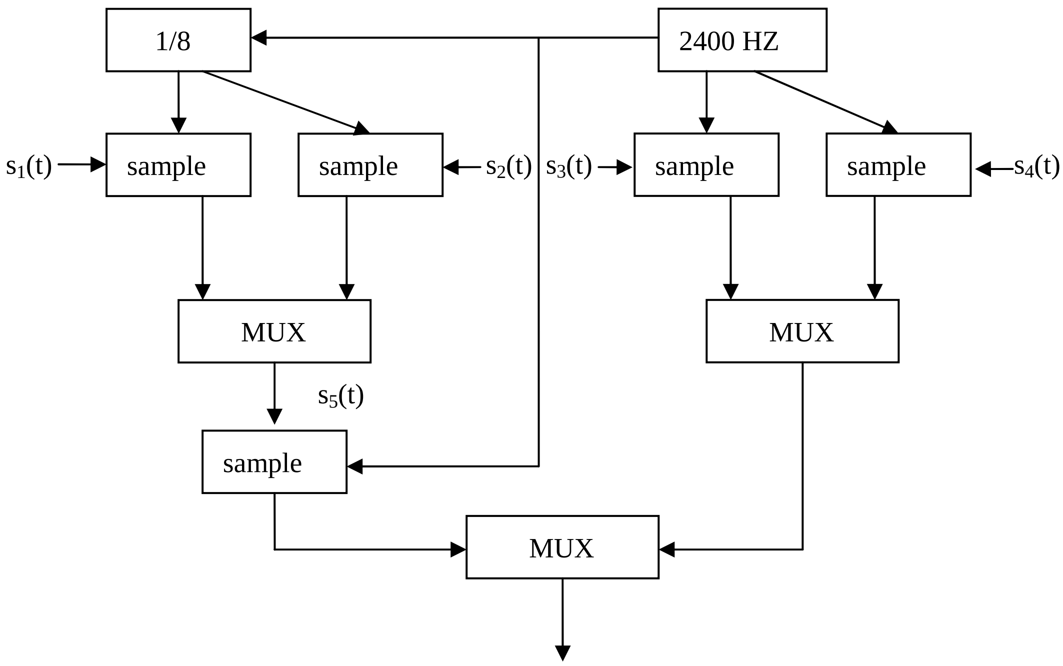
4.88 μs

Problem 3.10

Part a

2400/1600=15, which is between 2^3 and 2^4. Hence R=3.

Part b



Problem 3.17

Part a

A Gaussian random variable X has a probability density function (PDF)

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(x-\mu)^2} ,$$

where μ is the mean and σ^2 is the variance. We are given the mean and variance for input and asked to find the probability that the input lies outside of the range [-4 , 4]. can accomplish this by finding the area under the function p(x) from (-∞ , -4] plus [4 or, equivalently, one minus the area from [-4 , 4].

$$\begin{aligned} Prob\begin{pmatrix} input\ outside \\ range\ of\ -4\ to\ 4 \end{pmatrix} &= P(x \le -4 \quad \cup \quad x \ge 4) \\ &= 1 - P(-4 \le x \le 4) \\ &= 1 - \int_{-4}^{4} p(x)dx = 1 - \frac{1}{\sqrt{2\pi}} \int_{-4}^{4} e^{-\frac{1}{2}x^2} dx \\ &= 1 - 0.9999367 \end{aligned}$$

$$\boxed{prob = 63.3 * 10^{-6}}$$

Part b

Equation 3.33 on page 197 of the text provides us with an equation for signal-to-nois ratio of a quantizer

$$(SNR)_O = \left(\frac{3P}{m_{\max}^2} \right) 2^{2R}$$

where P is the average power of the input signal, m_{max} is the maximum amplitude of input signal and R is the number of bits used to quantize the signal. We are given th $m_{max} = 4$ and that variance $\sigma^2 = 1$. Recall that the variance of a random variable representing a signal can be thought of as the average power of the signal. Expressir ratio in dB is obtained by multiplying the base 10 logarithm of the ratio by 10.

$$10\log_{10}\left(\left(\frac{3}{16} \right) 2^{2R} \right) = 10\log_{10}\left(\frac{3}{16} \right) + 2R * 10\log_{10}(2) = 6R - 7.2 \quad dB$$

Problem 3.18

Part a

We have 50 x 10^6 bits/sec and each sample has 7 bits.

$$\frac{50 \cdot 10^6 \ bits}{1 \sec} \cdot \frac{1 \ sample}{7 \ bits} = \frac{50 \cdot 10^6}{7} \ samples/\text{sec} = 7.14 \ MHz$$

Since this is our sampling frequency, Nyquist tells us that our maximum message bandwidth is half of this or

$$\boxed{W = 3.57 \ MHz}$$

Part b

For a sine wave the average power P = $A^2/2$. We can again use equation 3.33

$$(SNR)_O = \left(\frac{3P}{m_{\max}^2}\right)2^{2R} = \left(\frac{3\,A^2/2}{A^2}\right)2^{2\cdot 7} = \frac{3}{2}2^{14}$$

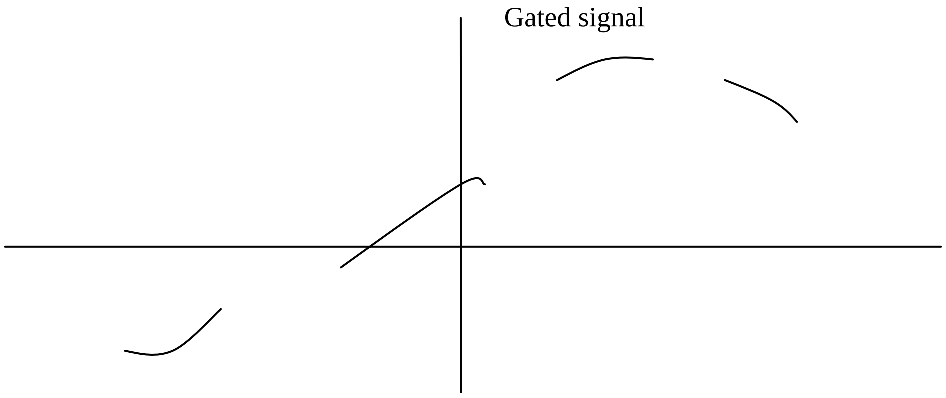
$$\boxed{(SNR)_O = 24{,}576 \qquad or \qquad 43.9 \ dB}$$

Problem 7

Part a

The spectrum of the gated signal is the convolution s(t) and $p_{gate}(t)$.

Part b



Part c

If the gate signal frequency 1/T is larger than twice of the maximum frequency components of signal s(t), we can recover the s(t) exactly.

Part d

The sampling frequency has to be twice as big as the maximum frequency in the sampled signal s(t).