Solutions Homework #4
Introduction to Communication Systems – E3701

Problem 2.22

Consider an incoming narrow-band signal of bandwidth 10 kHz, and mid-band fre
which may lie in the range 0.535–1.605 MHz. It is required to translate this signal to
frequency band centered at 0.455 MHz. The problem is to determine the range of tuni
must be provided in the local oscillator.

Let f_c denote the mid-band frequency of the incoming signal, and f_l denote the loca
lator frequency. Then we may write

$$0.535 < f_c < 1.605$$

and

$$f_c - f_l = 0.455$$

where both f_c and f_l are expressed in MHz. That is,

$$f_l = f_c - 0.455$$

When $f_c = 0.535$ MHz, we get $f_l = 0.08$ MHz; and when $f_c = 1.605$ MHz, we get $f_l = 1.15$
Thus the required range of tuning of the local oscillator is 0.08–1.15 MHz.

Problem 2.28

(a) The envelope of the FM wave s(t) is

$$a(t) = A_c \sqrt{1+\beta^2 \sin^2(2\pi f_m t)}$$

The maximum value of the envelope is

$$a_{max} = A_c \sqrt{1+\beta^2}$$

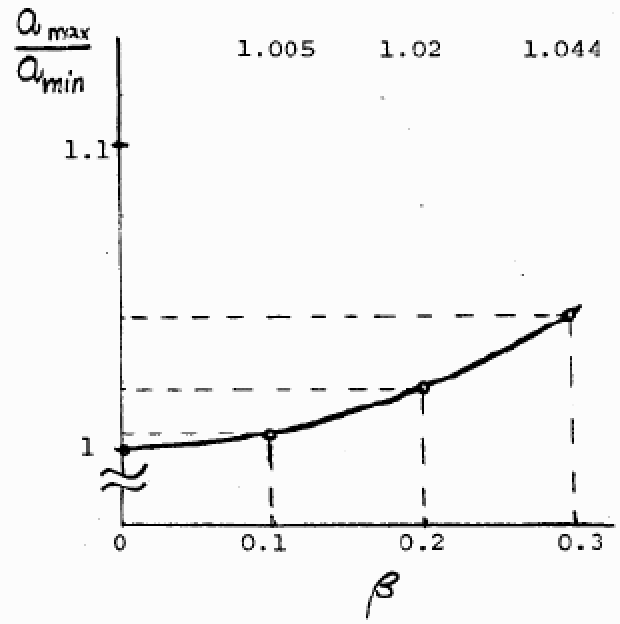
and its minimum value is

$$a_{min} = A_c$$

Therefore,

$$\frac{a_{max}}{a_{min}} = \sqrt{1+\beta^2}$$

This ratio is shown plotted below for $0 < \beta < 0.3$:



(b) Expressing s(t) in terms of its frequency components:

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} \beta A_c \cos[2\pi(f_c+f_m)t] - \frac{1}{2} \beta A_c \cos[2\pi(f_c-f_m)t]$$

The mean power of s(t) is therefore

$$P_1 = \frac{A_c^2}{2} + \frac{\beta^2 A_c^2}{8} + \frac{\beta^2 A_c^2}{8}$$

$$= \frac{A_c^2}{2} (1 + \frac{\beta^2}{2})$$

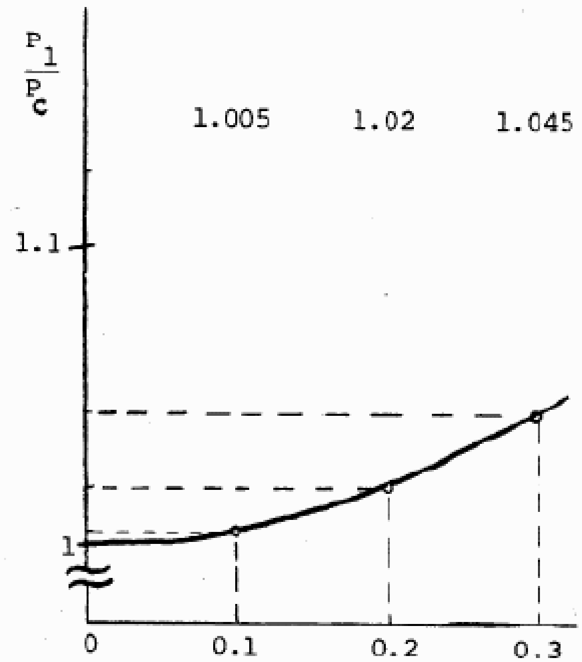
The mean power of the unmodulated carrier is

$$P_c = \frac{A_c^2}{2}$$

Therefore,

$$\frac{P_1}{P_c} = 1 + \frac{\beta^2}{2}$$

which is shown plotted below for $0 \le \beta \le 0.3$:



(c) The angle $\theta_i(t)$, expressed in terms of the in-phase component, $s_I(t)$, and the quadrature component, $s_Q(t)$, is:

$$\theta_i(t) = 2\pi f_c t + \tan^{-1}\left[\frac{s_I(t)}{s_Q(t)}\right]$$

$$= 2\pi f_c t + \tan^{-1}[\beta\sin(2\pi f_m t)]$$

Since $\tan^{-1}(x) = x - x^3/3 + \ldots$,

$$\theta_i(t) = 2\pi f_c t + \beta\sin(2\pi f_m t) - \frac{\beta^3}{3}\sin^3(2\pi f_m t)$$

The harmonic distortion is the power ratio of the third and first harmonics:

$$D_h = \left(\frac{\frac{1}{3}\beta^3}{\beta}\right)^2 = \frac{\beta^4}{9}$$

For $\beta = 0.3$, $D_h = 0.09\%$

Problem 2.31

(a) From Table A4.1, we find (by interpolation) that $J_0(\beta)$ is zero for

$\beta = 2.44$,

$\beta = 5.52$,

$\beta = 8.65$,

$\beta = 11.8$,

and so on.

(b) The modulation index is

$$\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

Therefore,

$$k_f = \frac{\beta f_m}{A_m}$$

Since $J_0(\beta) = 0$ for the first time when $\beta = 2.44$, we deduce that

$$k_f = \frac{2.44 \times 10^3}{2}$$

$$= 1.22 \times 10^3 \text{ hertz/volt}$$

Next, we note that $J_0(\beta) = 0$ for the second time when $\beta = 5.52$. Hence, the corresponding value of A_m for which the carrier component is reduced to zero is

$$A_m = \frac{\beta f_m}{k_f}$$

$$= \frac{5.52 \times 10^3}{1.22 \times 10^3}$$

$$= 4.52 \text{ volts}$$

Problem 4:

The SSB-FM signal seen in class was: $A\, e^{-\hat{\phi}(t)} \cos(2\pi f_o t + \Phi(t)) =$

$A\, Re[\, e^{j2\pi f_o t} e^{j(\phi(t)+j\hat{\phi}(t))}\,] = A\, Re[\, e^{j2\pi f_o t} e^{j(\phi_a(t))}\,]$, where $\hat{\phi}(t)$ is the Hilbert transform of $\Phi(t)$. This signal, as seen in class, contains frequencies only above f_o. We now want only frequencies above f_o. From the development done in class, we saw that the reason why the signal above contains only frequencies f_o is that the Fourier transform of $\Phi_a(t)$ only has non-zero frequency components above zero. We know that the Frourier transform of $\Phi_a^*(t)$ only has non-zero frequency components below zero, so if create a signal like the one above but which includes $\Phi_a^*(t)$ instead of $\Phi_a(t)$, we will have want we want, which is a SSB-FM signal with non-zero components only below f_o. Thus the desired SSB-FM signal is $A\, Re[\, e^{j2\pi f_o t} e^{j(\phi_a^*(t))}\,] = A\, Re[\, e^{j2\pi f_o t} e^{j(\phi(t)-j\hat{\phi}(t))}\,] =$

$A\, e^{\hat{\phi}(t)} \cos(2\pi f_o t + \Phi(t))$

Problem 5

a)

$$x_{FM} = A\cos\left(2\pi f_o t + \beta_1 \sin(2\pi W_1 t) + \beta_2 \sin(2\pi W_2 t)\right)$$
$$= A\,\mathrm{Re}\left[e^{j2\pi f_o t} e^{j\beta_1 \sin(2\pi W_1 t)} e^{j\beta_2 \sin(2\pi W_2 t)}\right]$$

b)

According to what was seen in class, we can expand $e^{j\beta_1 \sin(2\pi W_1 t)}$ in Fourier series, and we would obtain $e^{j\beta_1 \sin(2\pi W_1 t)} = \sum_{n=-\infty}^{\infty} J_n(\beta_1) e^{j2\pi n W_1 t}$, and in the same way

$e^{j\beta_2 \sin(2\pi W_2 t)} = \sum_{m=-\infty}^{\infty} J_n(\beta_2) e^{j2\pi m W_2 t}$

Thus, $x_{FM}(t) = A\, Re\, [2\pi f_o t \sum_{n=-\infty}^{\infty} J_n(\beta_1) e^{j2\pi n W_1 t} \sum_{m=-\infty}^{\infty} J_n(\beta_2) e^{j2\pi m W_2 t}\,] =$

$= A \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_n(\beta_1) J_m(\beta_2) \cos(2\pi(f_o + nW_1 + mW_2)t)$

c)

The maximum frequency deviation, $\Delta f = H_1 A_1 + H_2 A_2 = \Delta f_1 + \Delta f_2$.

d)

The highest frequency in the baseband signal we will assume is W_1 (and not W_2)

So the Carson bandwidth = $2\Delta f$ + 2 *(Highest Frequency in Baseband Signal)

Carson bandwidth = $2\Delta f + 2W_1$