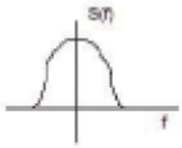
Solutions Homework #3
Introduction to Communication Systems – E3701

Problem 1:
If we have a generic signal s(t) whose Fourier spectrum looks like the following figur



Now we know that the analytical signal of s(t) is defined as $s_a(t)=s(t)+j\,s\hat{}(t)$, where s is the Hilbert transform of s(t). The complex conjugate of the analytical signal is $s_a^*(t)=s(t)-j\,s\hat{}(t)$

$\mathcal{F}(s_a^*(t))$ looks like:

Then $\mathcal{F}(s_a^*(t)\ e^{j2\pi f_o t}\ )$ will look like:

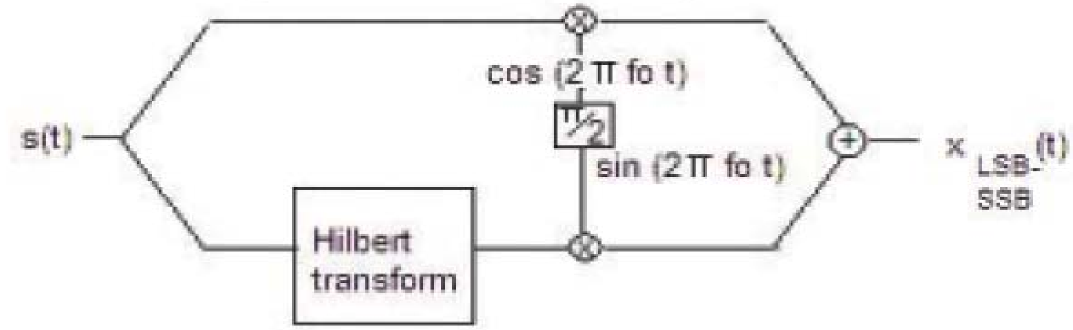
Now $s_a^*(t)\ e^{j2\pi f_o t} + (s_a^*(t)\ e^{j2\pi f_o t})^* = 2\,Re(s_a^*(t)\ e^{j2\pi f_o t})$ (**)
At the same time, $s_a^*(t)\ e^{j2\pi f_o t} + (s_a^*(t)\ e^{j2\pi f_o t})^* = s_a^*(t)\ e^{j2\pi f_o t} + s_a(t)\ e^{-j2\pi f_o t}$, and

$\mathcal{F}(s_a^*(t)\ e^{j2\pi f_o t} + s_a(t)\ e^{-j2\pi f_o t})$ looks like:

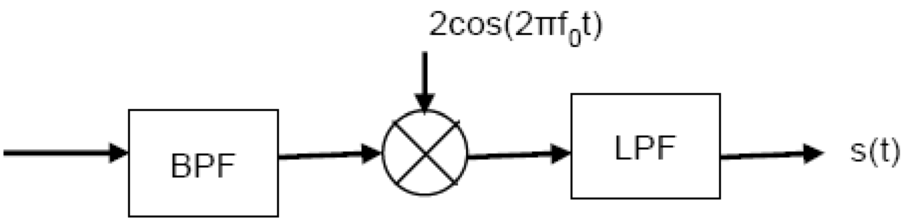
which is just the LSB-SSB we are looking for.

Now the right hand side of equality (**) is
$2\,Re(s_a^*(t)\ e^{j2\pi f_o t}) = 2\,Re\{\ (s(t)-j\,s\hat{}(t))\ (cos(2\pi f_o t) + j\,sin(2\pi f_o t))\} =$
$= s(t)\,cos(2\pi f_o t) + s\hat{}(t)\,sin(2\pi f_o t)$, which is therefore also the LSB-SSB signal sought.

Therefore, to generate the LSB-SSB signal:



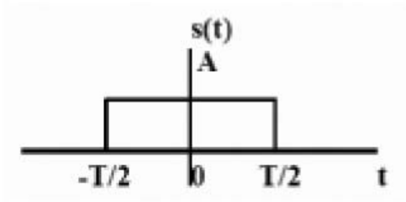
b) The received signal should first pass through a BPF filter, followed by a LO, and then pass through a LPF, as shown by the following figure.



c) To make sure the baseband signal can be separated from the $2f_0$ signal, we must have $2f_0 - w > w$. Hence, we have $f_0 > w$.

Problem 2:

The Hilbert transform of the pulse:



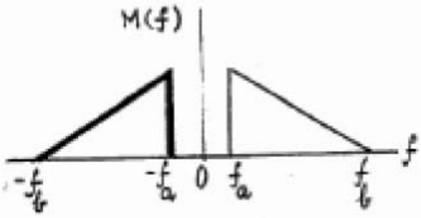
is, by definition: $\frac{1}{\pi}\int_{-T/2}^{T/2}\frac{A}{t-\tau}d\tau = \lim\{\ \frac{1}{\pi}\int_{-T/2}^{t-\varepsilon}\frac{A}{t-\tau}d\tau + \frac{1}{\pi}\int_{t+\varepsilon}^{T/2}\frac{A}{t-\tau}d\tau\ \}=$

$$\frac{A}{\pi}\log\left(\frac{t+\frac{T}{2}}{t-\frac{T}{2}}\right)$$

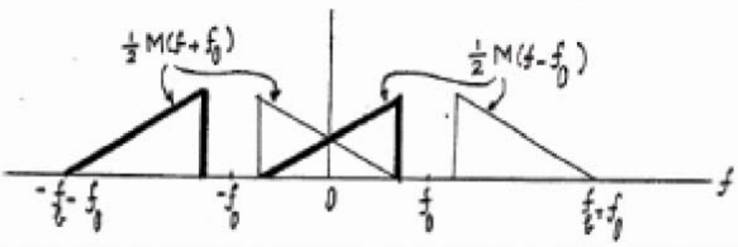
Problem 3:

Haykin's 2.18:

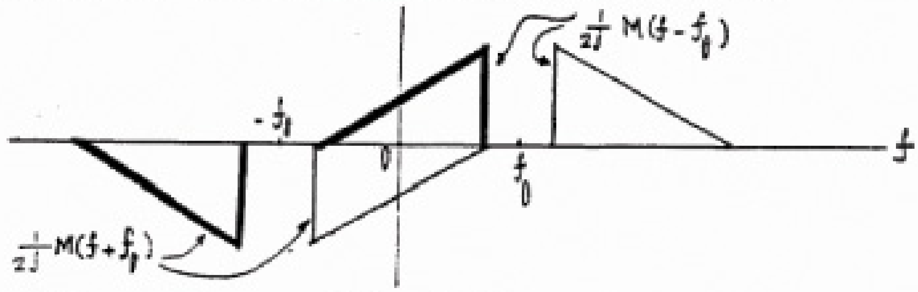
(a,b) The spectrum of the message signal is illustrated below:



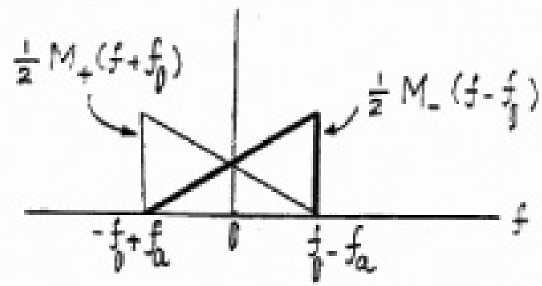
Correspondingly, the output of the upper first product modulator has the following spectrum:



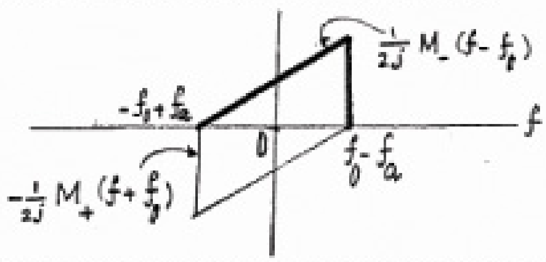
The output of the lower first product modulator has the spectrum:



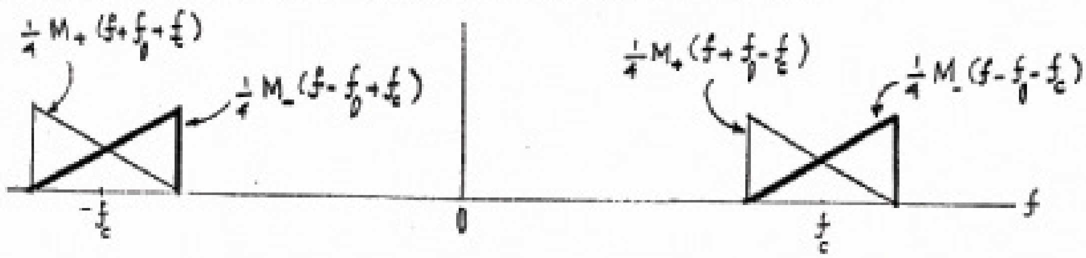
The output of the upper low pass filter has the spectrum:



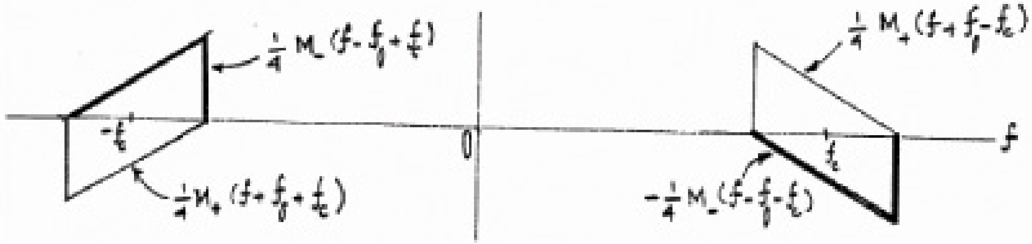
The output of the lower low pass filter has the spectrum:



The output of the upper second product modulator has the spectrum:



The output of the lower second product modulator has the spectrum:



Adding the two second product modulator outputs, their upper sidebands add constructively while their lower sidebands cancel each other.

(c) To modify the modulator to transmit only the lower sideband, a single sign change is required in one of the channels. For example, the lower first product modulator could multiply the message signal by $-\sin(2\pi f_0 t)$. Then, the upper sideband would be cancelled and the lower one transmitted.